

# Cryptography Overview

*Acknowledgments: Lecture slides are from the Computer Security course thought by Dan Boneh and John Mitchell at Stanford University. When slides are obtained from other sources, a reference will be noted on the bottom of that slide. A full list of references is provided on the last slide.*

# Cryptography

## ◆ Is

- A tremendous tool
- The basis for many security mechanisms

## ◆ Is not

- The solution to all security problems
- Reliable unless implemented properly
- Reliable unless used properly
- Something you should try to invent or implement yourself

# Kerckhoff's principle



A cryptosystem should be secure even if **everything** about the system, except the secret key, **is public knowledge.**

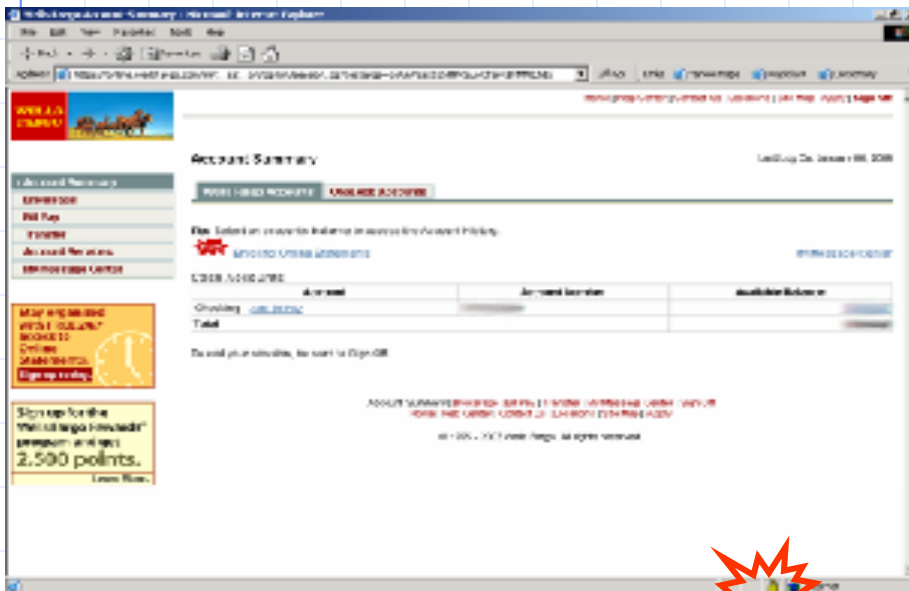
# Goal 1: secure communication

Step 1: Session setup to exchange key

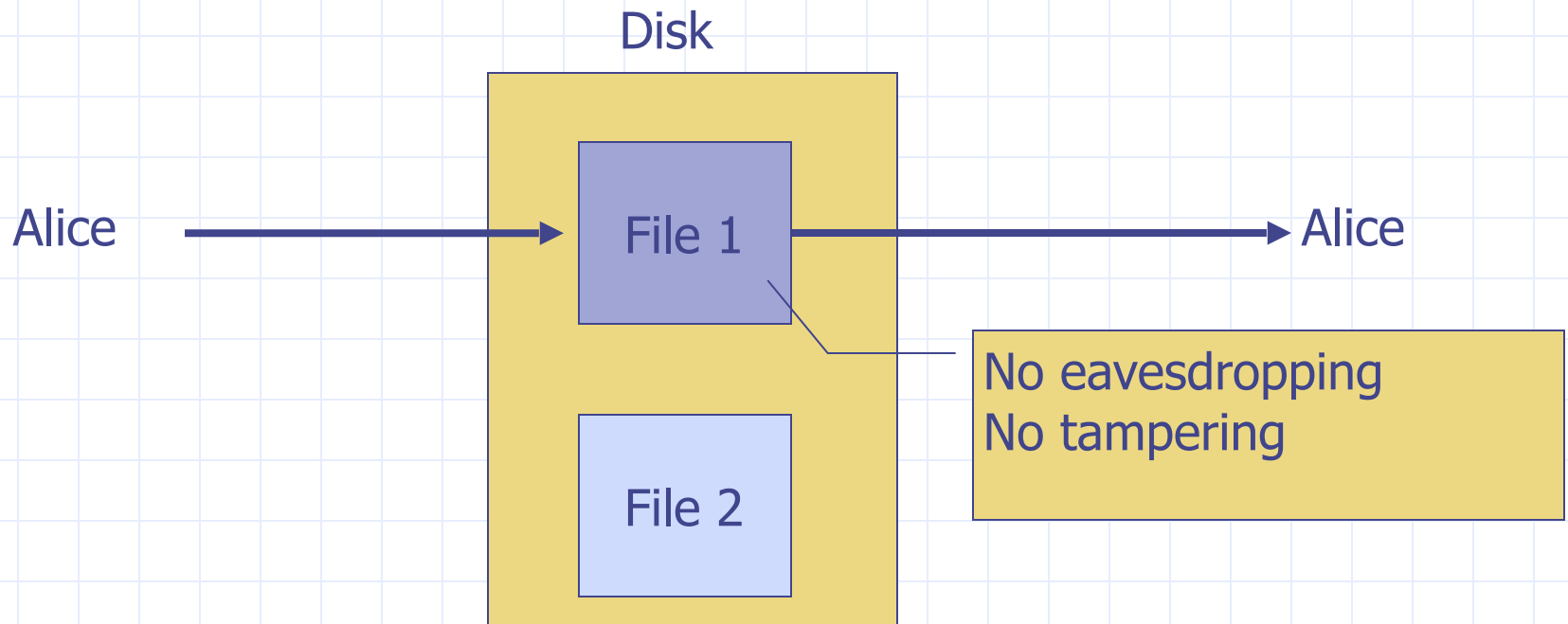
Step 2: encrypt data



HTTPS



# Goal 2: Protected files



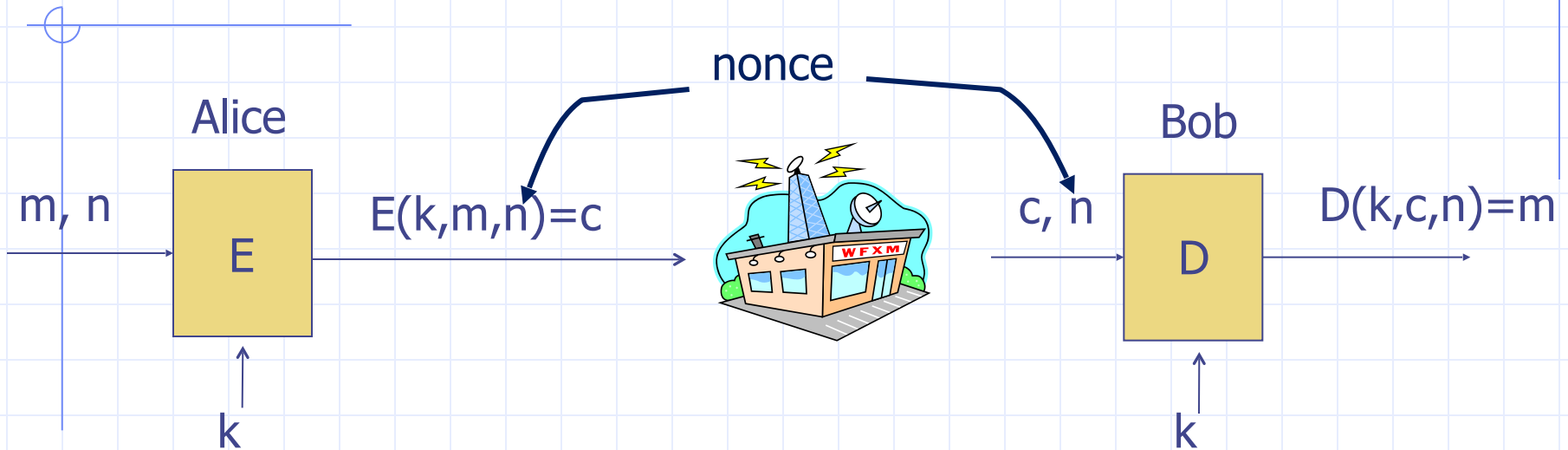
Analogous to secure communication:

Alice today sends a message to Alice tomorrow

# Symmetric Cryptography

Assumes parties already  
share a secret key

# Building block: sym. encryption



E, D: cipher      k: secret key (e.g. 128 bits)

m, c: plaintext, ciphertext      n: nonce (aka IV)

Encryption algorithm is **publicly known**

- Never use a proprietary cipher

# Use Cases

## **Single use key:** (one time key)

- Key is only used to encrypt one message
  - encrypted email: new key generated for every email
- No need for nonce (set to 0)

## **Multi use key:** (many time key)

- Key used to encrypt multiple messages
  - files: same key used to encrypt many files



# First example: One Time Pad

(single use key)

## ◆ Vernam (1917)

Key:

0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

Plaintext:

1	1	0	0	0	1	1	0	0	0
---	---	---	---	---	---	---	---	---	---

⊕

Ciphertext:

1	0	0	1	1	0	1	0	1	0
---	---	---	---	---	---	---	---	---	---

## ◆ Shannon '49:

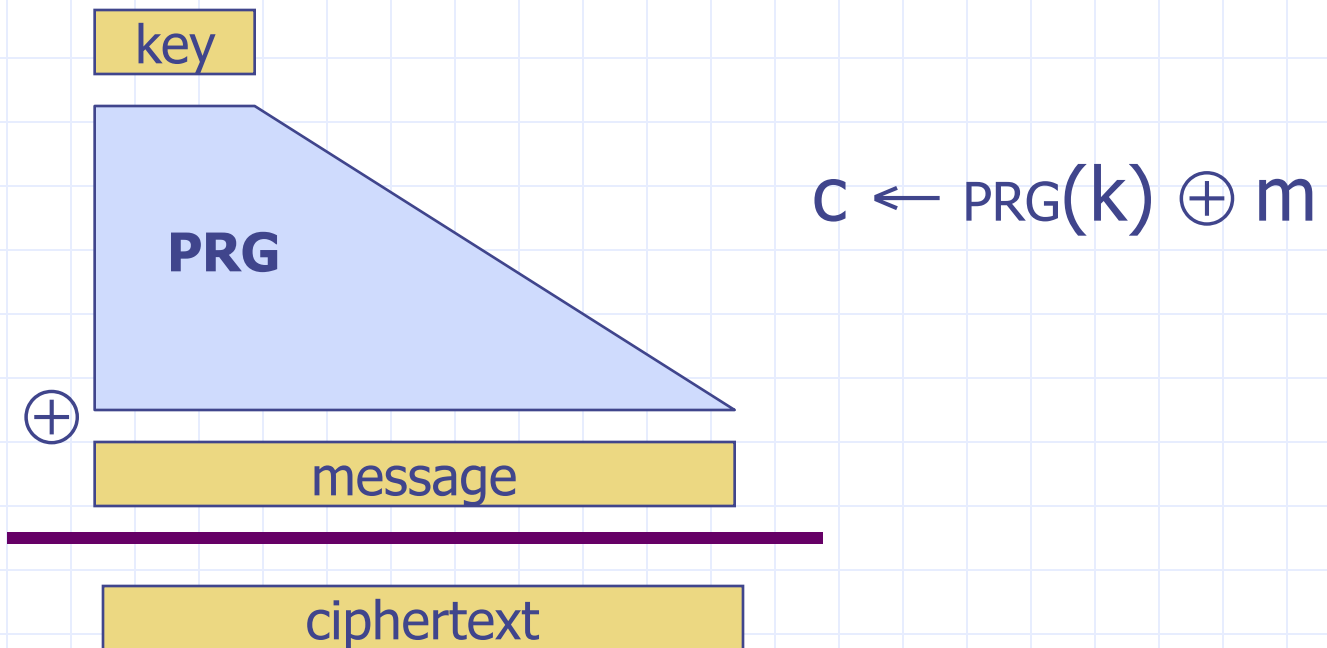
- OTP is "secure" against ciphertext-only attacks

# Stream ciphers

(single use key)

Problem: OTP key is not as long as the message

Solution: Pseudo random key -- stream ciphers



Stream ciphers: RC4 (126 MB/sec) , Salsa20/12 (643 MB/sec)

# Dangers in using stream ciphers

One time key !!

“Two time pad” is insecure:

$$\left\{ \begin{array}{l} C_1 \leftarrow m_1 \oplus \text{PRG}(k) \\ C_2 \leftarrow m_2 \oplus \text{PRG}(k) \end{array} \right.$$

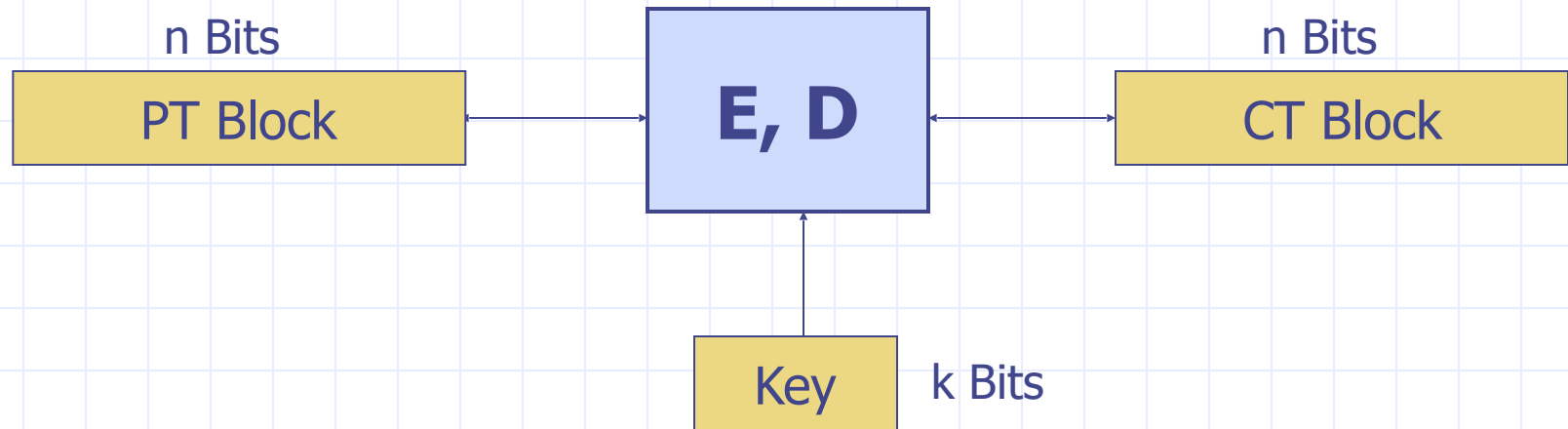
Eavesdropper does:

$$C_1 \oplus C_2 \rightarrow m_1 \oplus m_2$$

Enough redundant information in English that:

$$m_1 \oplus m_2 \rightarrow m_1, m_2$$

# Block ciphers: crypto work horse



Canonical examples:

1. 3DES:  $n = 64$  bits,  $k = 168$  bits
2. AES:  $n = 128$  bits,  $k = 128, 192, 256$  bits

IV handled as part of PT block

# Building a block cipher

Input:  $(m, k)$

Repeat simple “mixing” operation several times

- DES: Repeat 16 times:

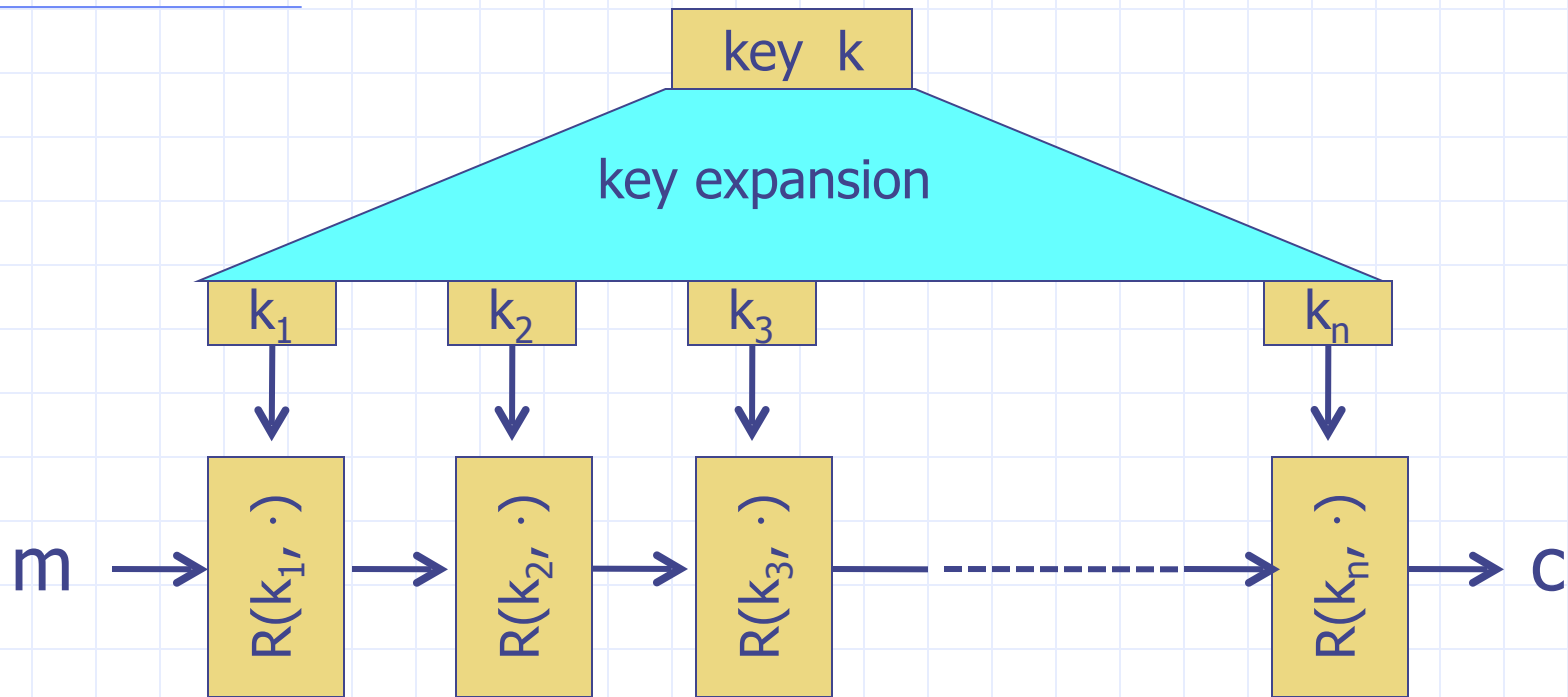
$$\begin{cases} m_L \leftarrow m_R \\ m_R \leftarrow m_L \oplus F(k, m_R) \end{cases}$$

- AES-128: Mixing step repeated 10 times

Difficult to design: must resist subtle attacks

- differential attacks, linear attacks, brute-force, ...

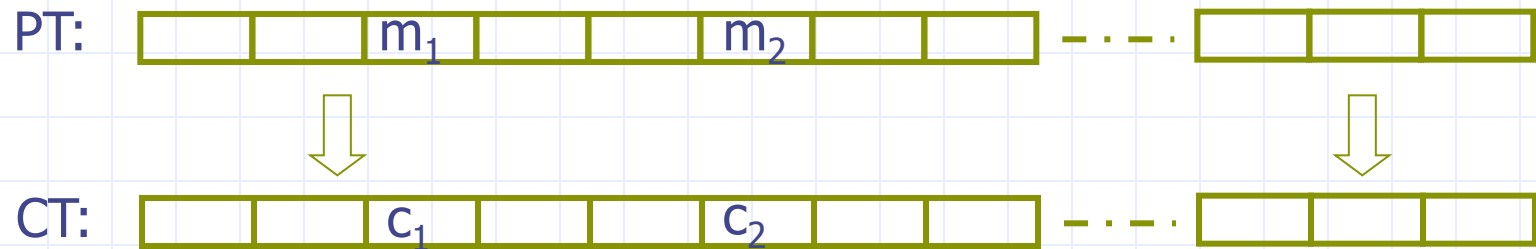
# Block Ciphers Built by Iteration



$R(k, m)$ : round function  
for DES ( $n=16$ ), for AES-128 ( $n=10$ )

# Incorrect use of block ciphers

## Electronic Code Book (ECB):



## Problem:

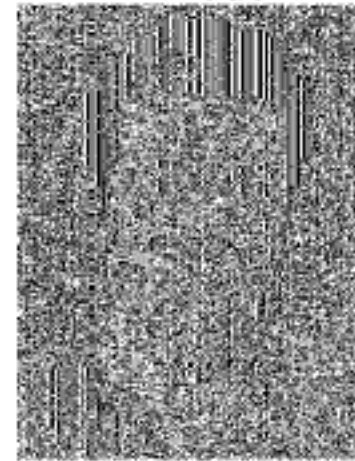
- if  $m_1 = m_2$  then  $c_1 = c_2$

# In pictures

An example plaintext



Encrypted with AES in ECB mode

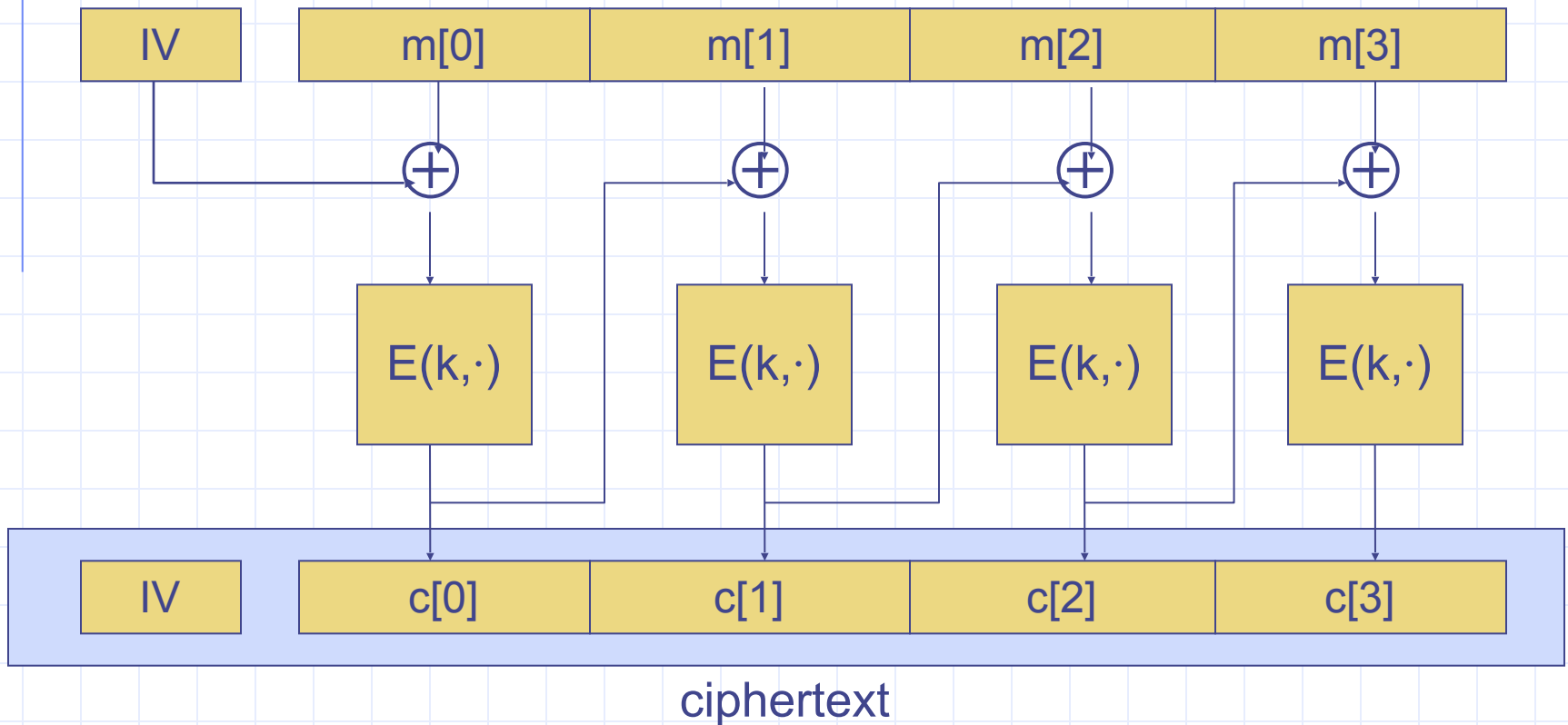




# Correct use of block ciphers I: CBC mode

$E$  a secure PRP.

Cipher Block Chaining with random IV:



Q: how to do decryption?

# Use cases: how to choose an IV

Single use key: no IV needed (IV=0)

Multi use key: (CPA Security)

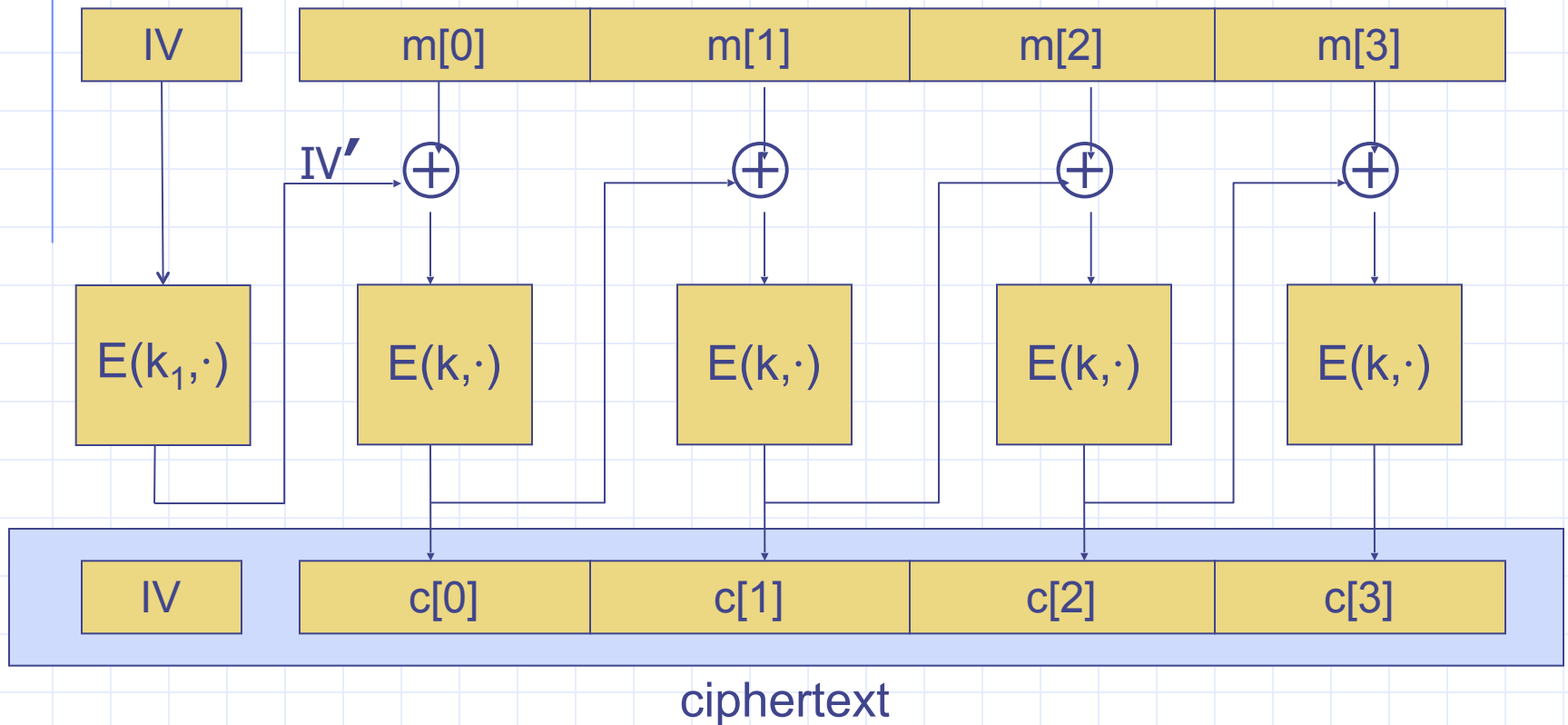
Best: use a fresh random IV for every message

Can use unique IV (e.g. counter)

but then first step in CBC must be  $IV' \leftarrow E(k_1, IV)$

# CBC with Unique IVs

unique IV means:  $(k, IV)$  pair is used for only one message.  
generate unpredictable  $IV'$  as  $E(k, IV)$

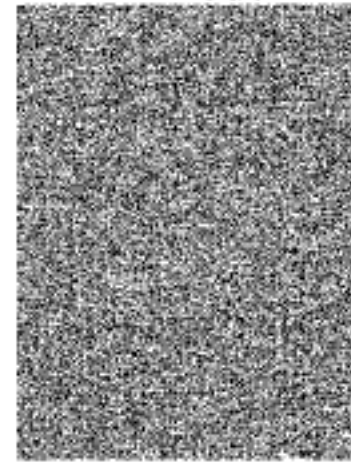


# In pictures

An example plaintext

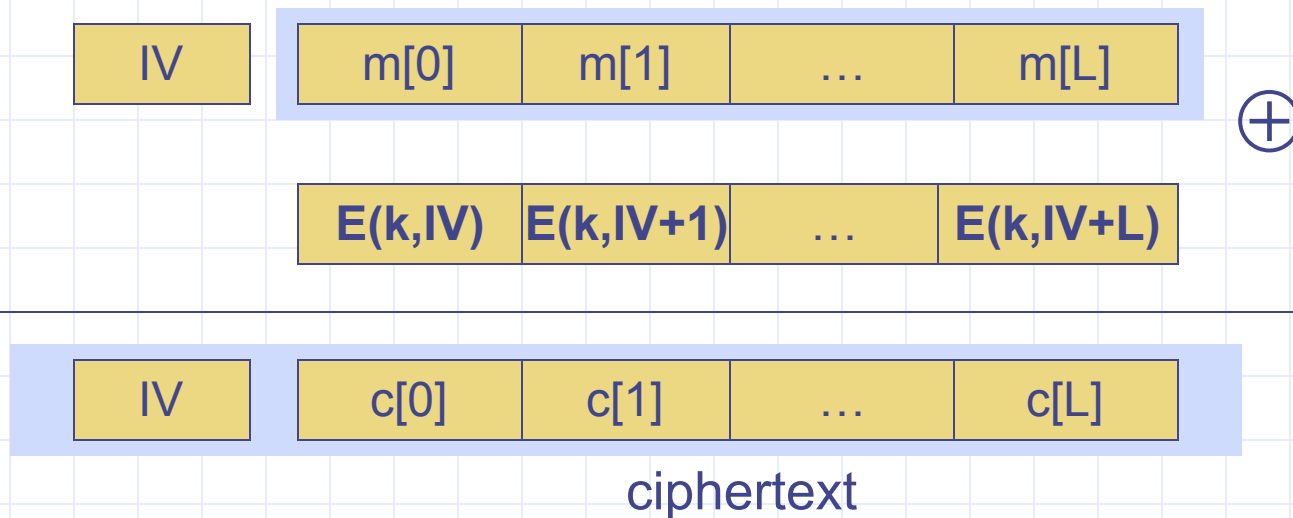


Encrypted with AES in CBC mode



## Correct use of block ciphers II: CTR mode

Counter mode with a random IV: (parallel encryption)



# Performance:

Crypto++ 5.6.0 [ Wei Dai ]

Intel Core 2 (on Windows Vista)

<u>Cipher</u>	<u>Block/key size</u>	<u>Speed (MB/sec)</u>
RC4		126
Salsa20/12		643
3DES	64/168	10
AES/GCM	128/128	102

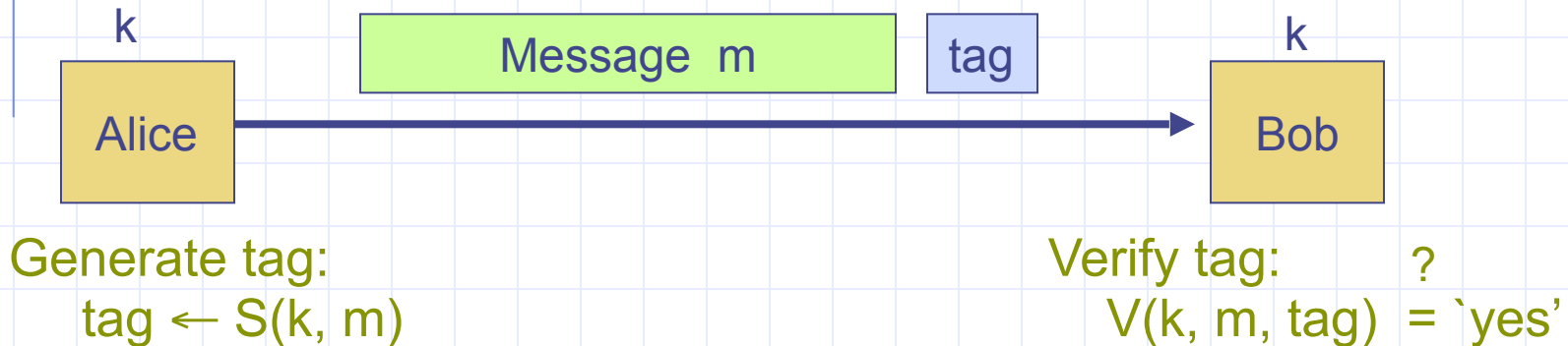
AES is about 8x faster with AES-NI : Intel Westmere and onwards



# Data integrity

# Message Integrity: MACs

- ◆ Goal: message integrity. No confidentiality.
  - ex: Protecting public binaries on disk.



note: non-keyed checksum (CRC) is an insecure MAC !!



# Secure MACs

◆ Attacker information: chosen message attack

- for  $m_1, m_2, \dots, m_q$  attacker is given  $t_i \leftarrow S(k, m_i)$

◆ Attacker's goal: existential forgery.

- produce some **new** valid message/tag pair  $(m, t)$ .

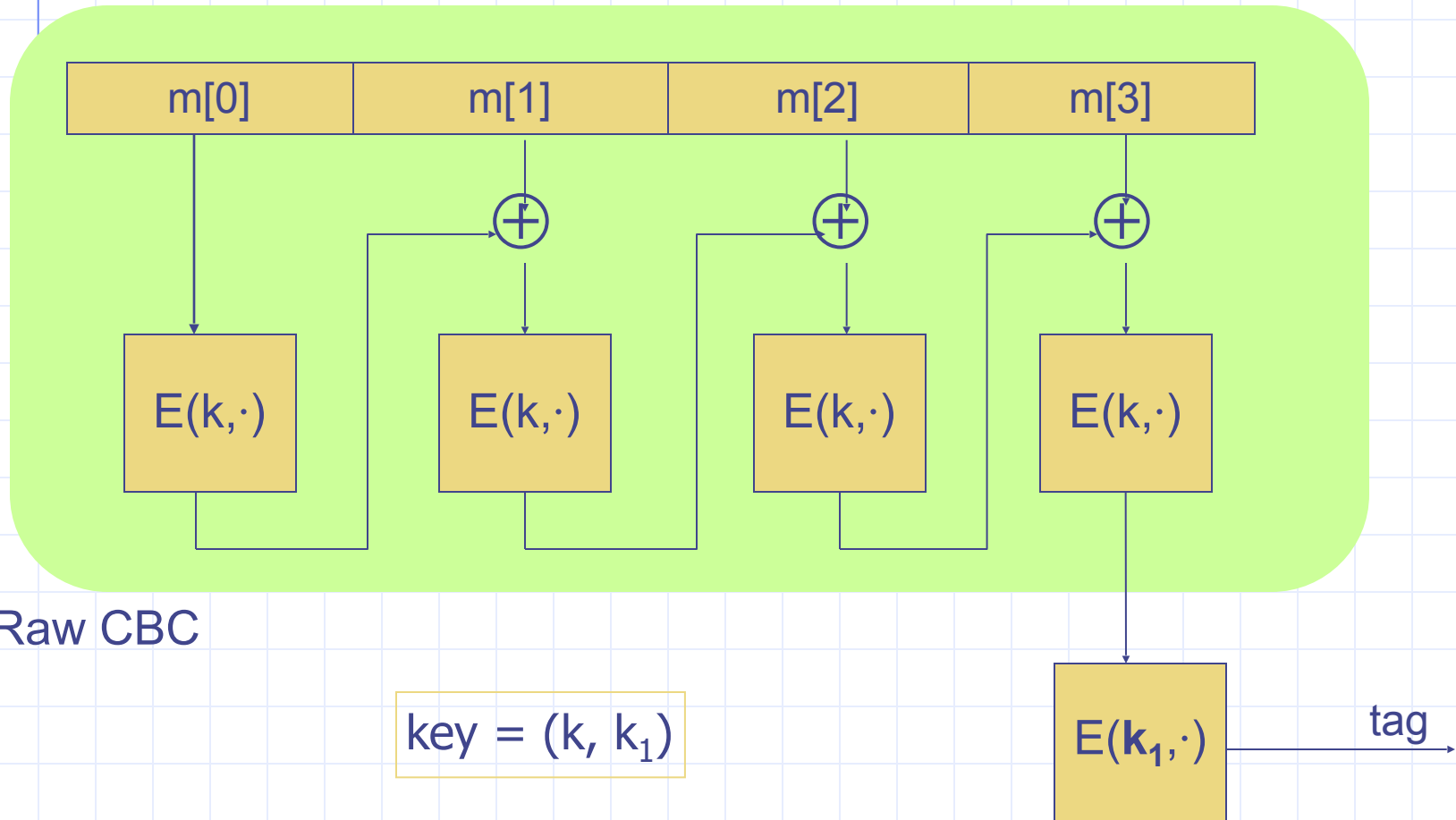
$$(m, t) \notin \{ (m_1, t_1), \dots, (m_q, t_q) \}$$

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◆ A secure PRF gives a secure MAC:

- $S(k, m) = F(k, m)$
- $V(k, m, t)$ : 'yes' if  $t = F(k, m)$  and 'no' otherwise.

# Construction 1: ECBC



# Construction 2: HMAC (Hash-MAC)

Most widely used MAC on the Internet.

H: hash function.

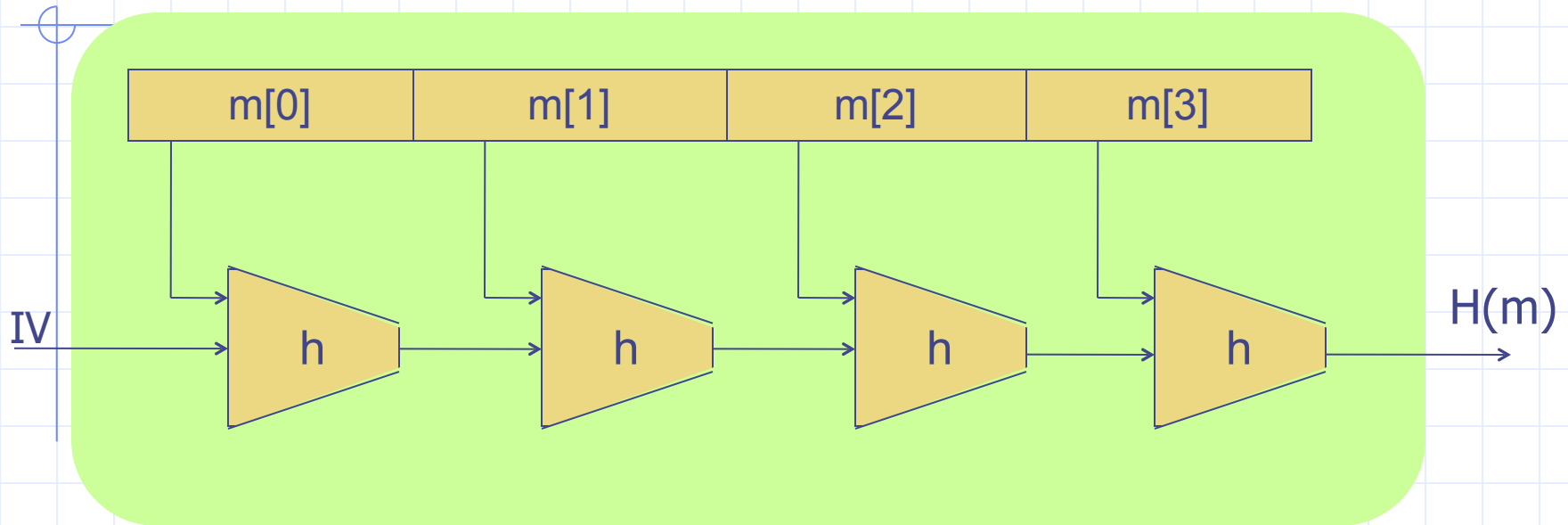
example: SHA-256 ; output is 256 bits

Building a MAC out of a hash function:

Standardized method: HMAC

$$S(k, m) = H(k \oplus \text{opad} \parallel H(k \oplus \text{ipad} \parallel m))$$

# SHA-256: Merkle-Damgard



$h(t, m[i])$ : compression function

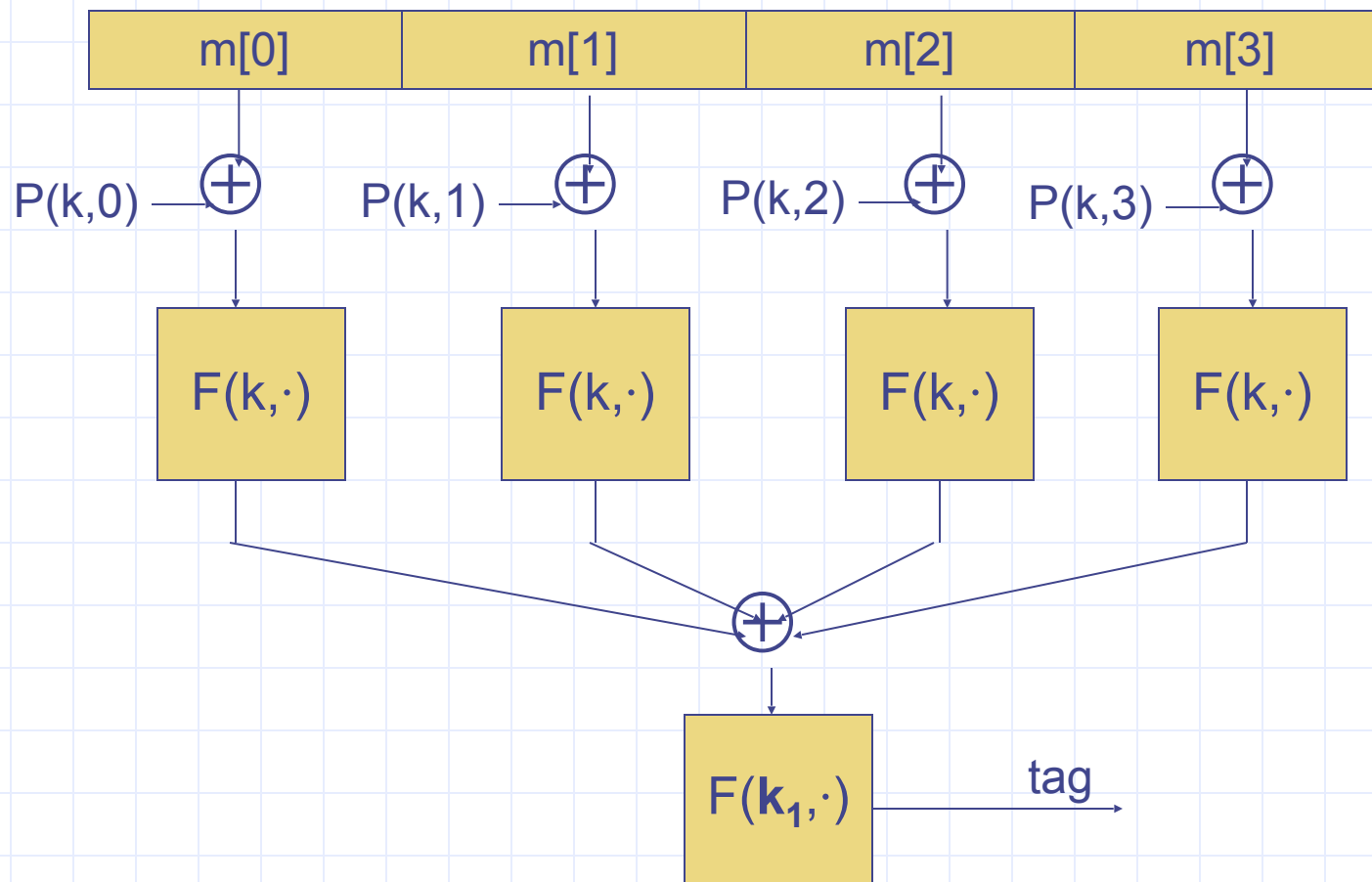
Thm 1: if  $h$  is collision resistant then so is  $H$

“Thm 2”: if  $h$  is a PRF then HMAC is a PRF

# Construction 3: PMAC – parallel MAC

ECBC and HMAC are sequential.

PMAC:



Why are these MAC constructions secure?

... not today – take 40-675

Why the last encryption step in ECBC?

- CBC (aka Raw-CBC) is not a secure MAC:
  - Given tag on a message  $m$ , attacker can deduce tag for some other message  $m'$
  - How: good crypto exercise ... take 40-675 ;)

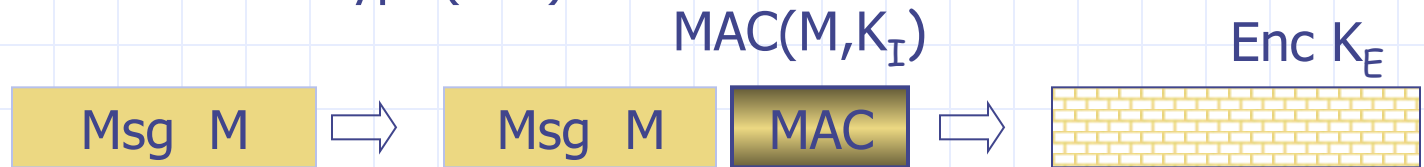


# Authenticated Encryption: Encryption + MAC

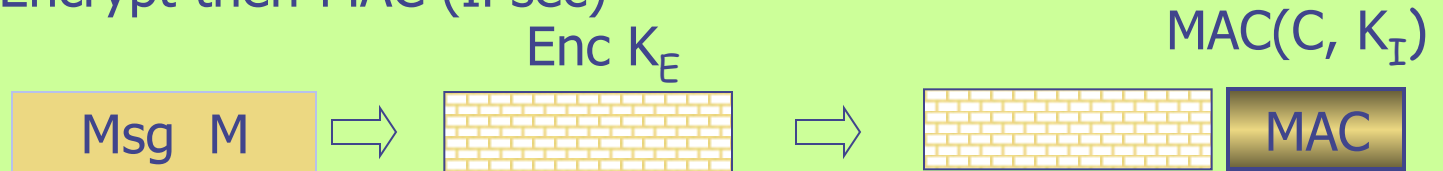
# Combining MAC and ENC (CCA)

Encryption key  $K_E$     MAC key =  $K_I$

## Option 1: MAC-then-Encrypt (SSL)



## Option 2: Encrypt-then-MAC (IPsec)



Secure for all secure primitives

## Option 3: Encrypt-and-MAC (SSH)

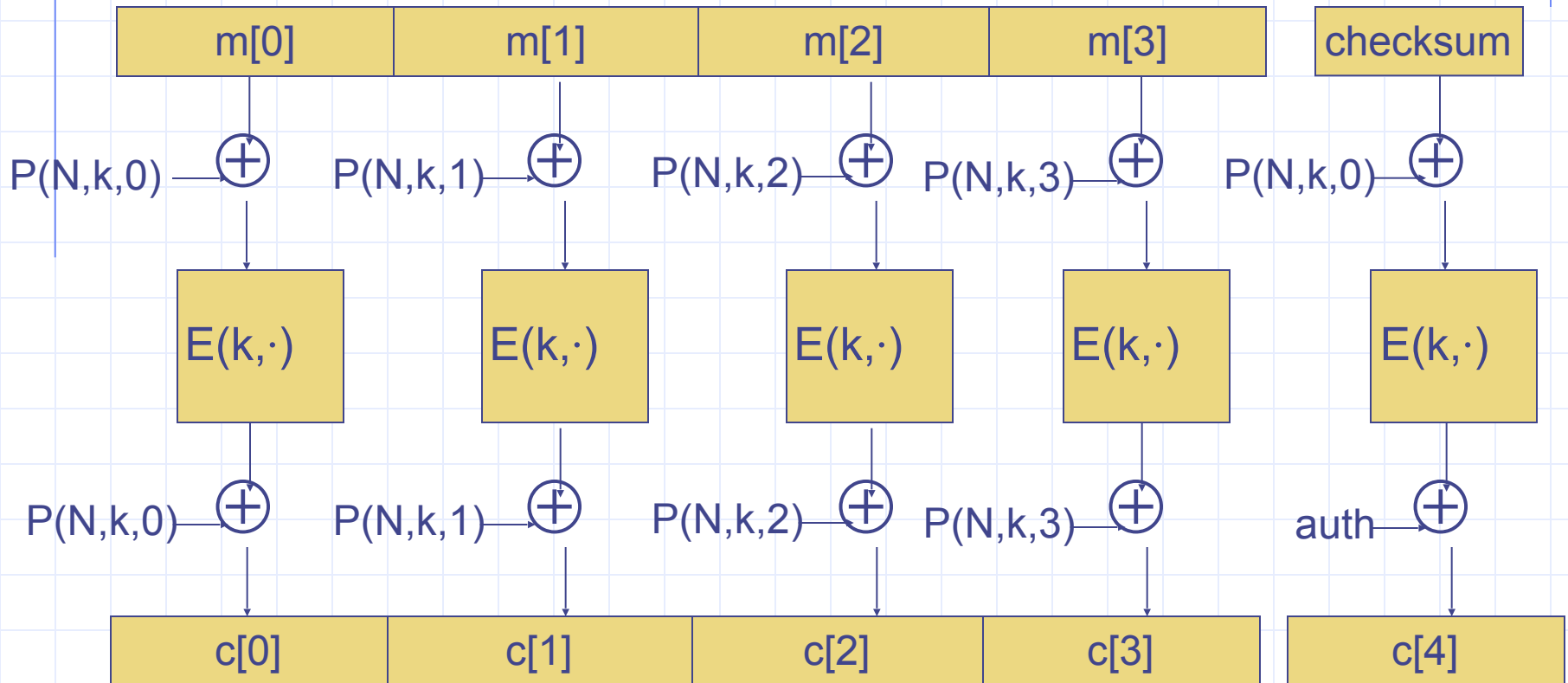




# OCB

offset codebook mode

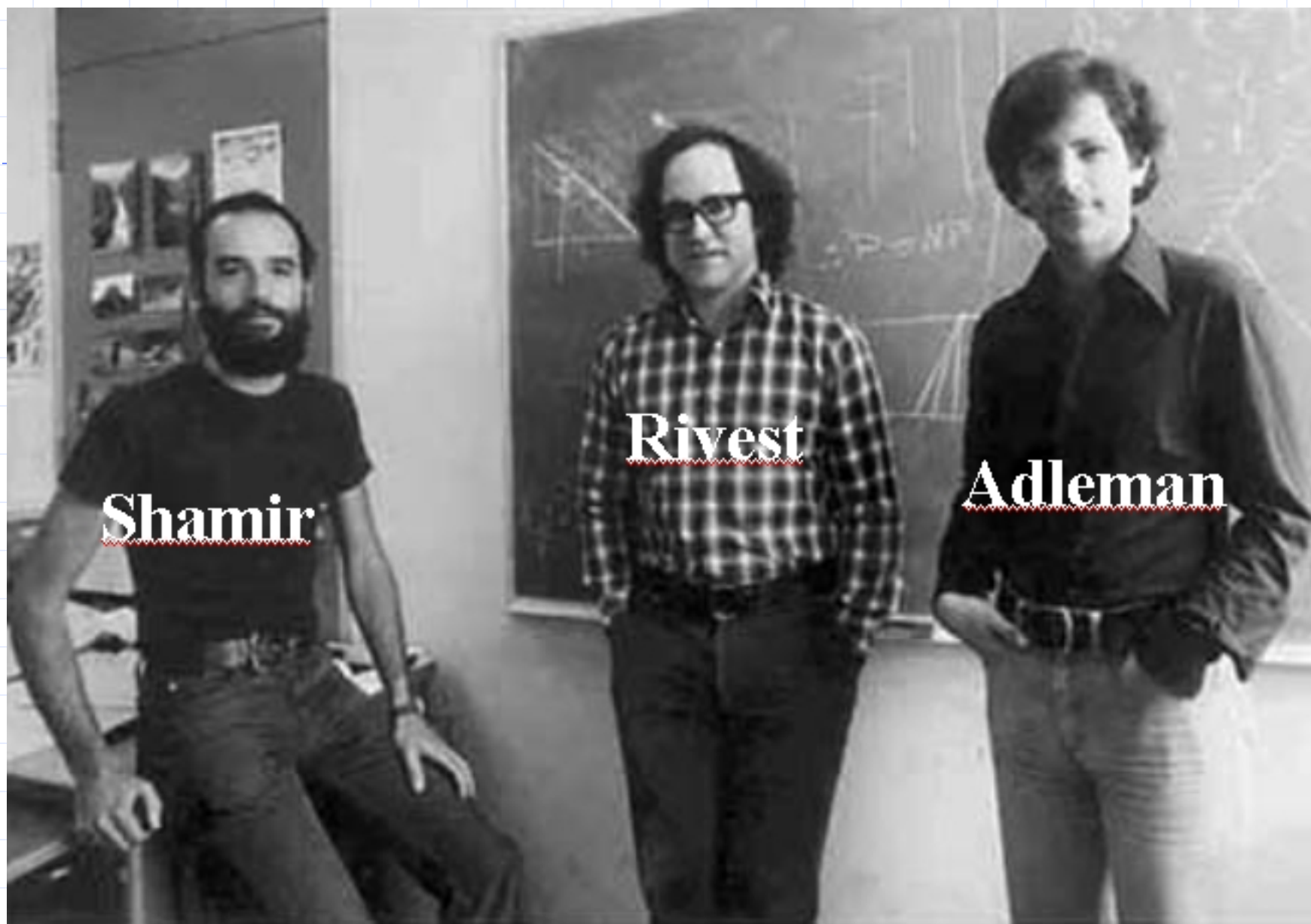
## More efficient authenticated encryption



Rogaway, ...



# Public-key Cryptography

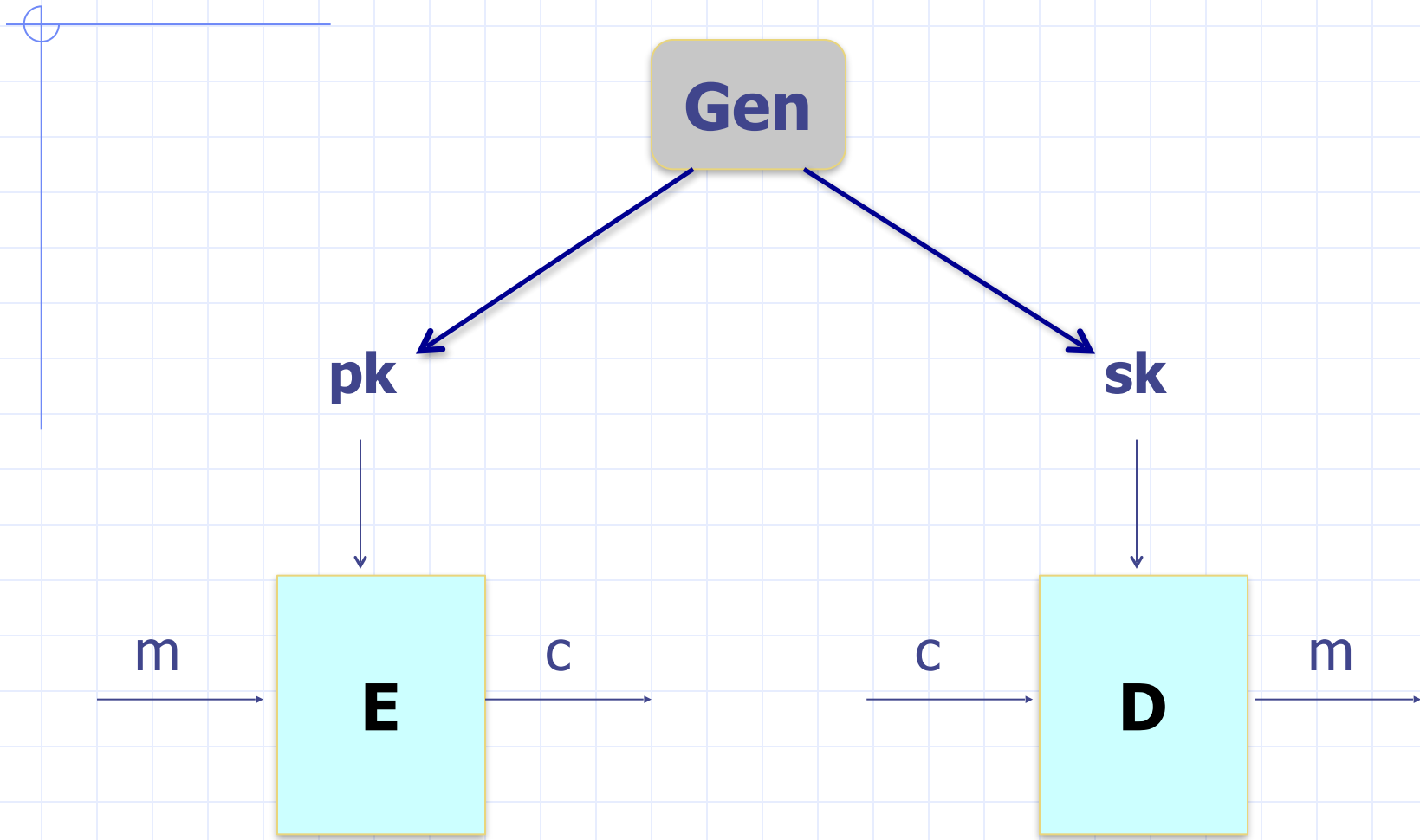


Shamir

Rivest

Adleman

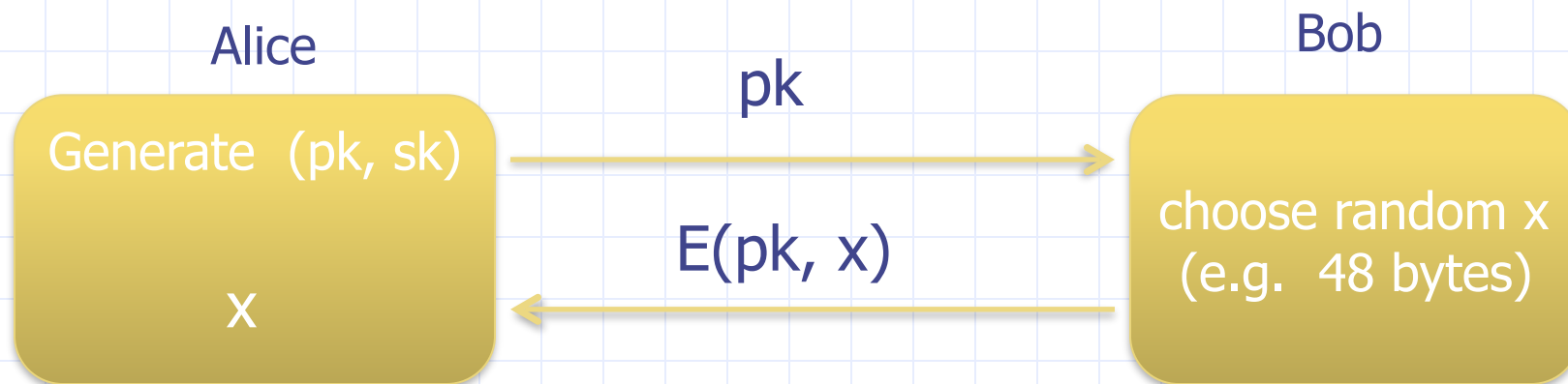
# Public key encryption: (Gen, E, D)



# Applications

## Session setup

(for now, only eavesdropping security)



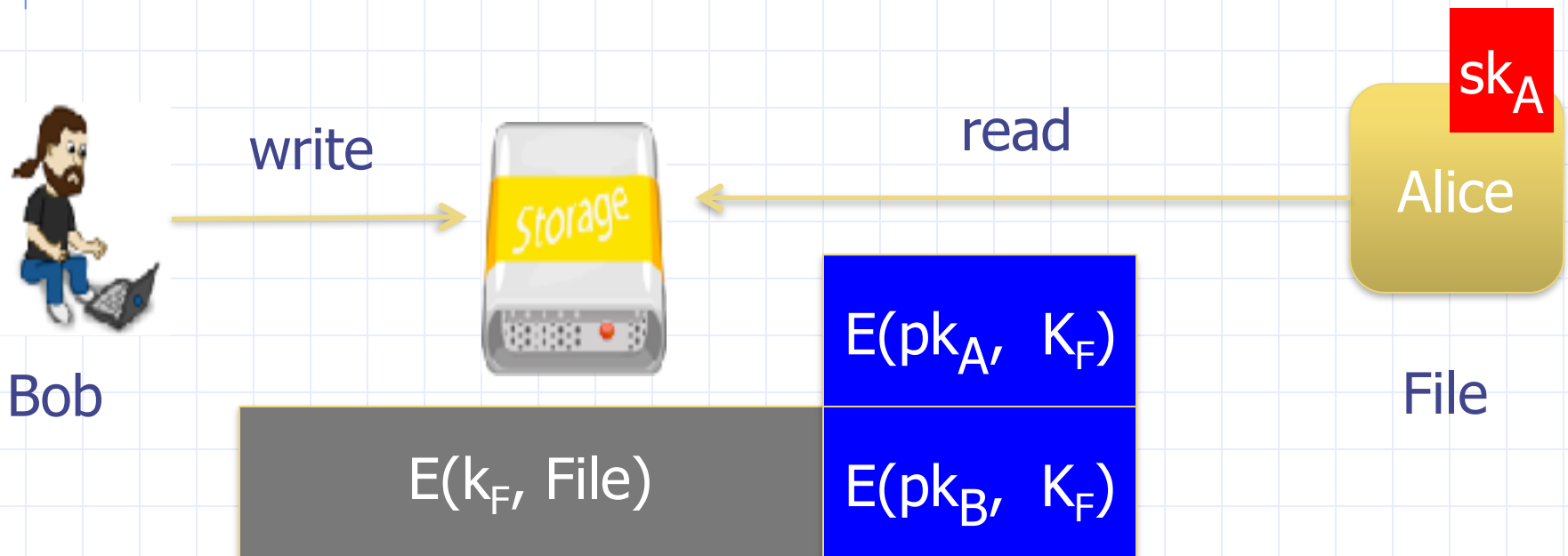
## Non-interactive applications: (e.g. Email)

- ◆ Bob sends email to Alice encrypted using  $pk_{\text{alice}}$
- ◆ Note: Bob needs  $pk_{\text{alice}}$  (public key management)

# Applications

Encryption in non-interactive settings:

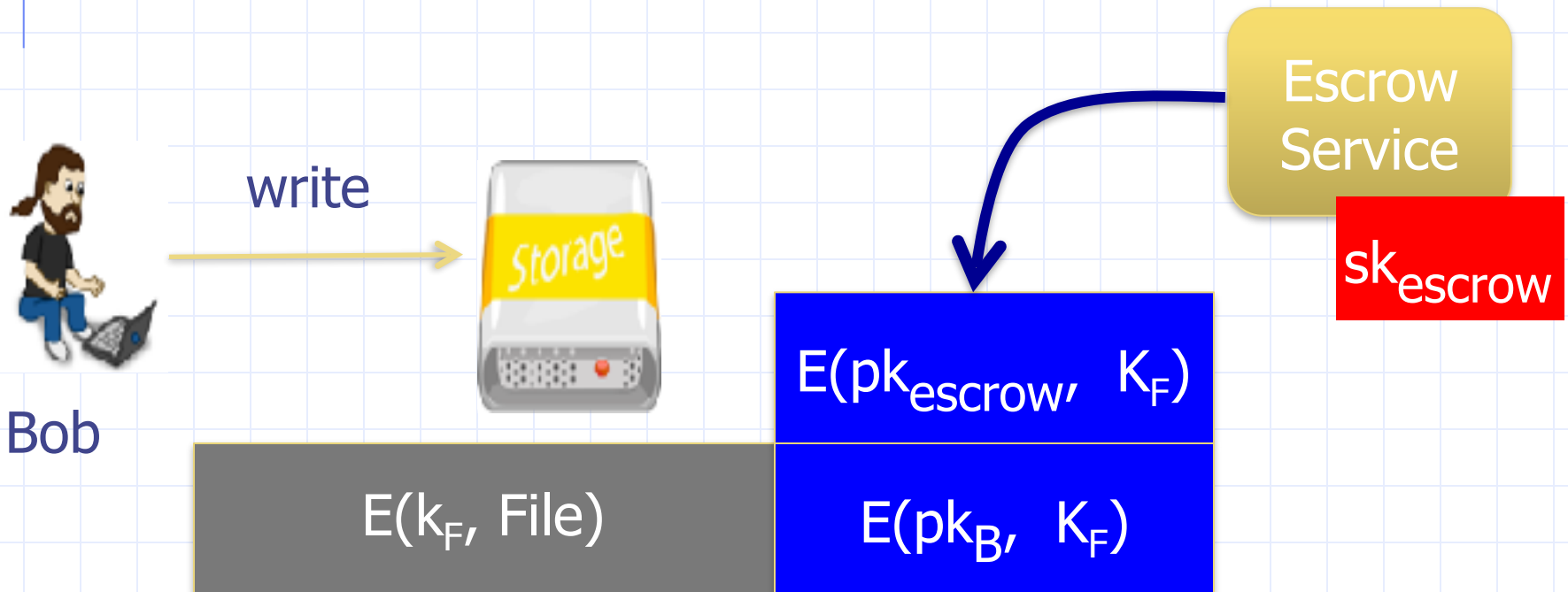
## ◆ Encrypted File Systems



# Applications

Encryption in non-interactive settings:

◆ Key escrow: data recovery without Bob's key



# Trapdoor functions (TDF)

**Def:** a trapdoor func.  $X \rightarrow Y$  is a triple of efficient algs.  $(G, F, F^{-1})$

- $G()$ : randomized alg. outputs key pair  $(pk, sk)$
- $F(pk, \cdot)$ : det. alg. that defines a func.  $X \rightarrow Y$
- $F^{-1}(sk, \cdot)$ : defines a func.  $Y \rightarrow X$  that  
inverts  $F(pk, \cdot)$

Security:  $F(pk, \cdot)$  is one-way without  $sk$



# Public-key encryption from TDFs

- $(G, F, F^{-1})$ : secure TDF  $X \longrightarrow Y$
- $(E_s, D_s)$ : symm. auth. encryption with keys in  $K$
- $H: X \longrightarrow K$  a hash function

We construct a pub-key enc. system  $(G, E, D)$ :

Key generation  $G$ : same as  $G$  for TDF

# Public-key encryption from TDFs

- $(G, F, F^{-1})$ : secure TDF  $X \longrightarrow Y$
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- $H: X \longrightarrow K$  a hash function

**$E(pk, m)$**  :

$x \xleftarrow{R} X, \quad y \longleftarrow F(pk, x)$

$k \longleftarrow H(x),$

$c \longleftarrow E_s(k, m)$

output  $(y, c)$

**$D(sk, (y, c))$**  :

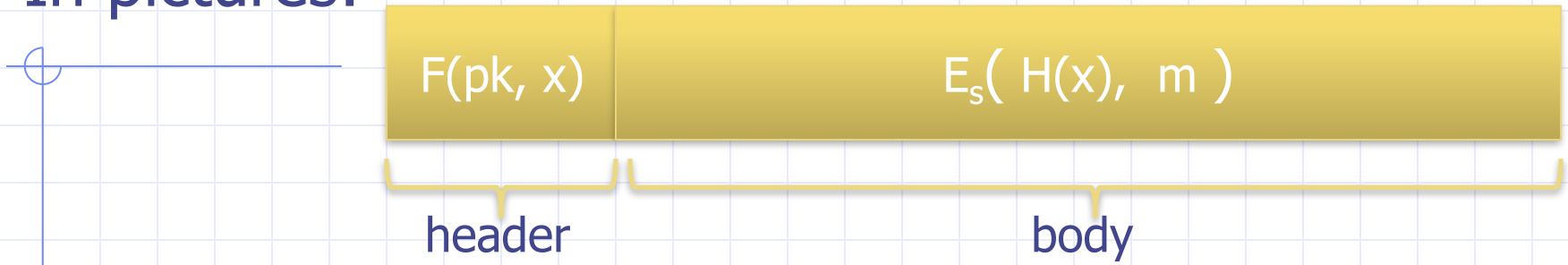
$x \longleftarrow F^{-1}(sk, y),$

$k \longleftarrow H(x),$

$m \longleftarrow D_s(k, c)$

output  $m$

In pictures:



## Security Theorem:

If  $(G, F, F^{-1})$  is a secure TDF,

$(E_s, D_s)$  provides auth. enc.

and  $H: X \rightarrow K$  is a "random oracle"

then  $(G, E, D)$  is  $CCA^{ro}$  secure.

# Digital Signatures

## ◆ Public-key encryption

- Alice publishes encryption key
- Anyone can send encrypted message
- Only Alice can decrypt messages with this key

## ◆ Digital signature scheme

- Alice publishes key for verifying signatures
- Anyone can check a message signed by Alice
- Only Alice can send signed messages

# Digital Signatures from TDPs

◆  $(G, F, F^{-1})$ : secure TDP  $X \rightarrow X$

◆  $H: M \rightarrow X$  a hash function

**Sign**(  $sk, m \in X$  ) :

output

$$\text{sig} = F^{-1}(sk, H(m))$$

**Verify**(  $pk, m, \text{sig}$  ) :

output

$$\begin{cases} 1 & \text{if } H(m) = F(pk, \text{sig}) \\ 0 & \text{otherwise} \end{cases}$$

Security: existential unforgeability under a chosen message attack (in the random oracle model)

# Public-Key Infrastructure (PKI)

- ◆ Anyone can send Bob a secret message
  - Provided they know Bob's public key
- ◆ How do we know a key belongs to Bob?
  - If imposter substitutes another key, can read Bob's mail
- ◆ One solution: PKI
  - Trusted root Certificate Authority (e.g. Symantec)
    - ◆ Everyone must know the verification key of root CA
    - ◆ Check your browser; there are hundreds!!
  - Root authority signs intermediate CA
  - Results in a certificate chain

# Limitations of cryptography

Cryptography works when used correctly !!

... but is not the solution to all security problems

