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Optoelectronics
EE/OPE 451, OPT 444
Fall 2009 Section 1: T/Th 9:30- 10:55 PM

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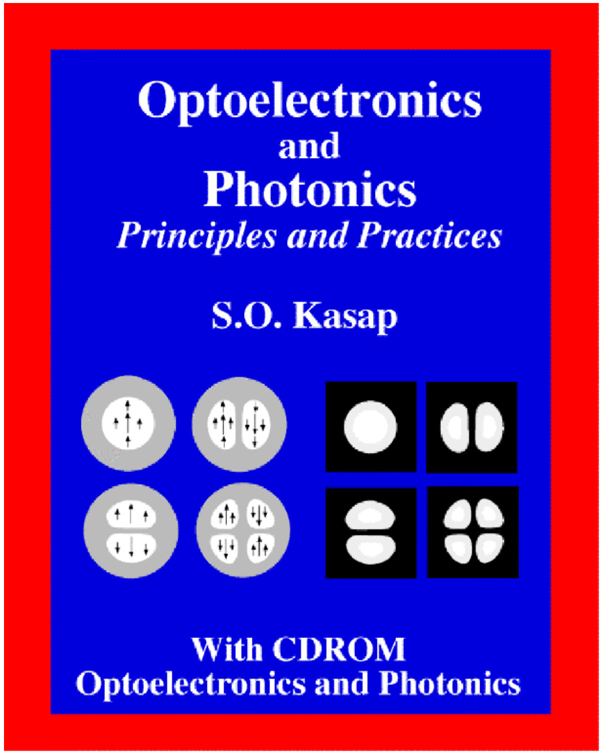
SEMICONDUCTOR SCIENCE AND LIGHT EMITTING DIODES

- **3.1 Semiconductor Concepts and Energy Bands**
  - A. Energy Band Diagrams
  - B. Semiconductor Statistics
  - C. Extrinsic Semiconductors
  - D. Compensation Doping
  - E. Degenerate and Nondegenerate Semiconductors
  - F. Energy Band Diagrams in an Applied Field
- **3.2 Direct and Indirect Bandgap Semiconductors: *E-k* Diagrams**
- **3.3 *pn* Junction Principles**
  - A. Open Circuit
  - B. Forward Bias
  - C. Reverse Bias
  - D. Depletion Layer Capacitance
  - E. Recombination Lifetime
- **3.4 The *pn* Junction Band Diagram**
  - A. Open Circuit
  - B. Forward and Reverse Bias
- **3.5 Light Emitting Diodes**
  - A. Principles
  - B. Device Structures
- **3.6 LED Materials**
- **3.7 Heterojunction High Intensity LEDs**
- **3.8 LED Characteristics**
- **3.9 LEDs for Optical Fiber Communications**

- **Chapter 3 Homework Problems: 1-11**



Optoelectronics
and
Photonics
Principles and Practices

S.O. Kasap

With CDROM
Optoelectronics and Photonics

Prentice-Hall Inc.

ISBN: 0-201-61087-6
http://photonics.usask.ca/

Energy Band Diagrams

- Quantization of the atom
- Lone atoms act like infinite potential wells in which bound electrons oscillate within allowed states at particular well defined energies
- The Schrödinger equation is used to define these allowed energy states

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{2m_e}{\hbar}\left[E - V(x)\right]\Psi = 0$$

E = energy, V = potential energy

- Solutions are in the form of waves oscillating at quantized energies and related propagation constants defined by the differential equation

$$\Psi(x) = Ce^{ikx} + De^{-ikx}$$

$$\therefore \Psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \qquad k^2 = \frac{2mE}{\hbar^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2} \qquad \text{since } \hbar = \frac{h}{2\pi}$$

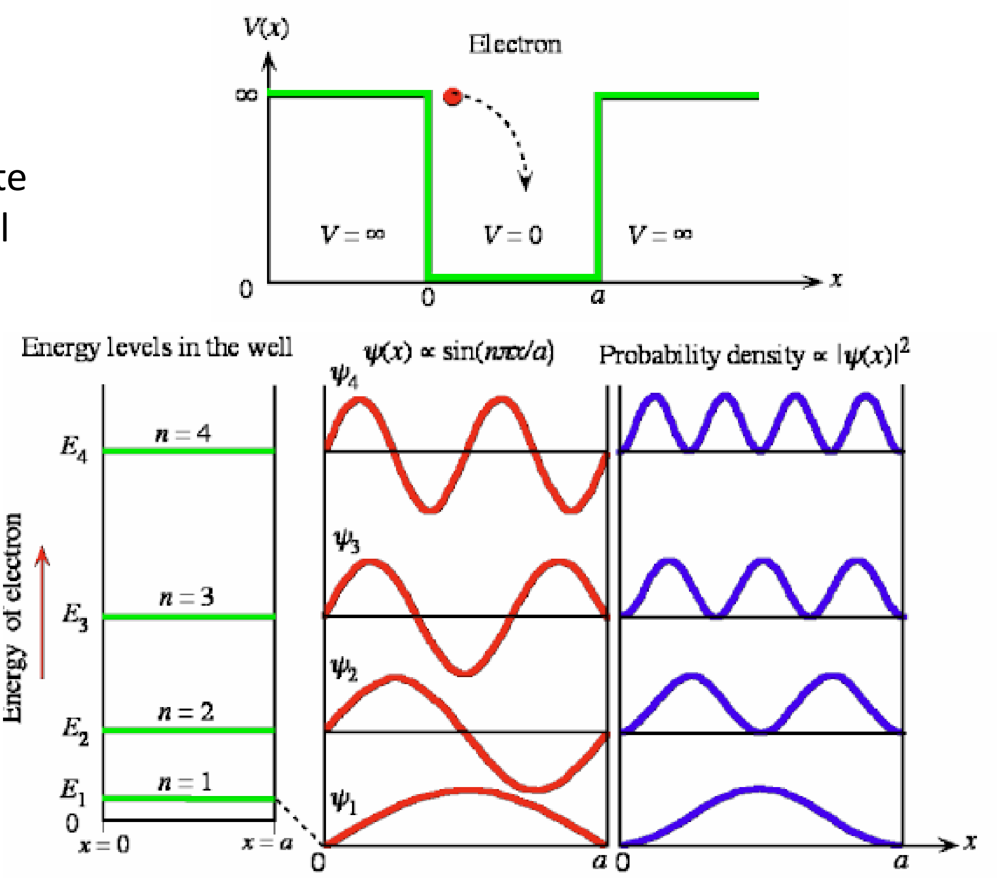


Fig. 3.15: Electron in a one-dimensional infinite PE well. The energy of the electron is quantized. Possible wavefunctions and the probability distributions for the electron are shown.

Quantum Mechanics and Molecular Binding

- All atoms have specific bound states
- Molecular binding requires filling of these allowed states in such a way as to reduce the amount of energy required to fill various states

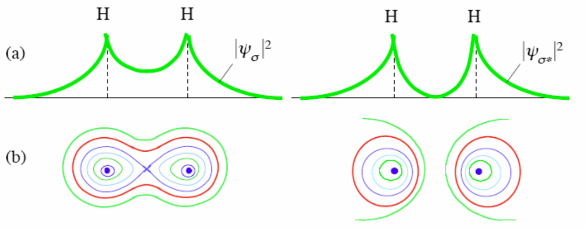
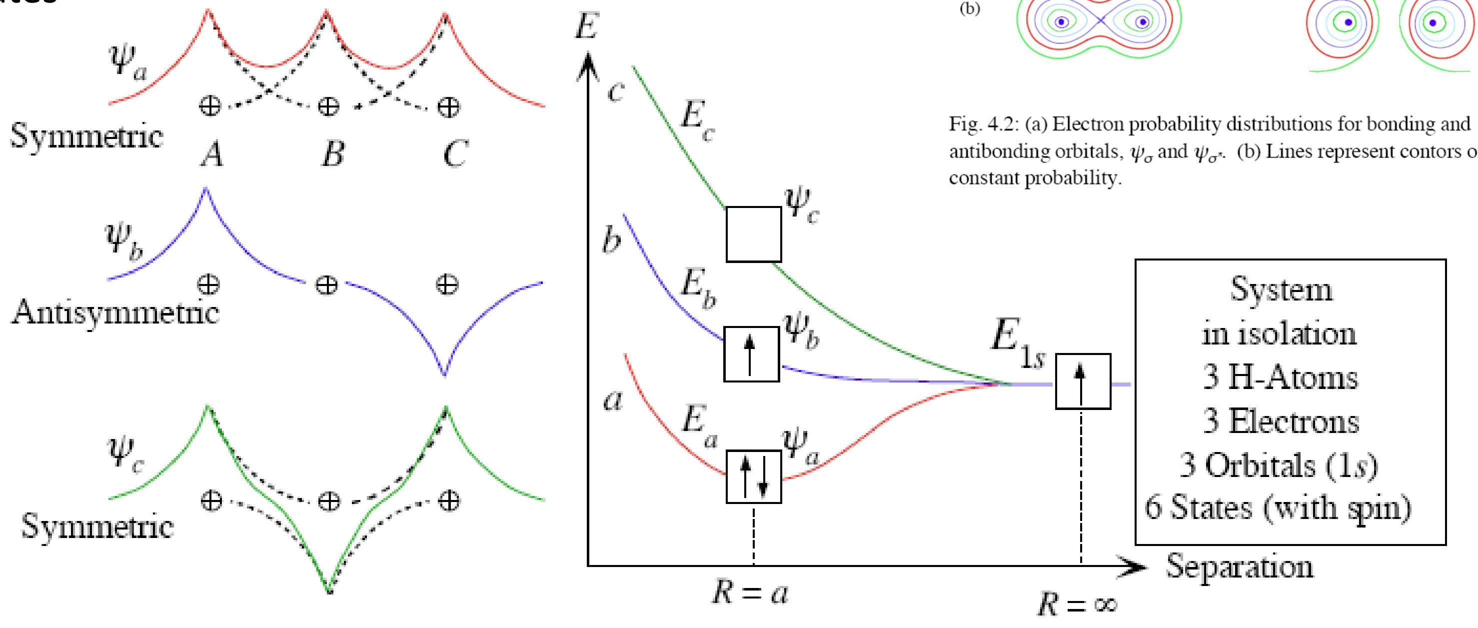


Fig. 4.2: (a) Electron probability distributions for bonding and antibonding orbitals, ψ_σ and ψ_{σ^*}. (b) Lines represent contors of constant probability.

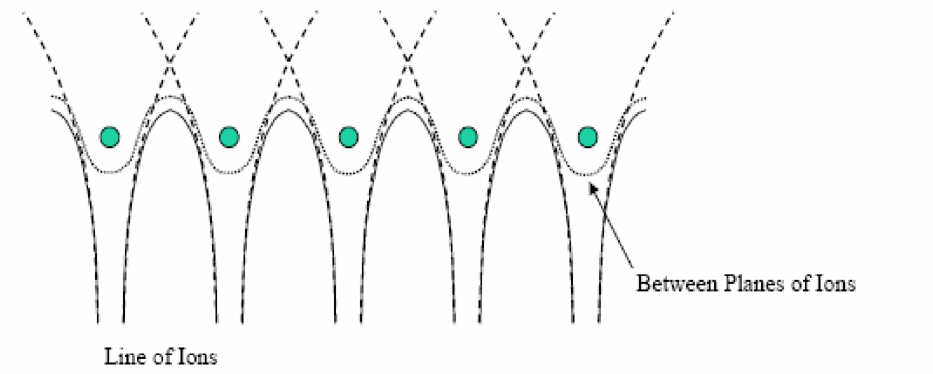


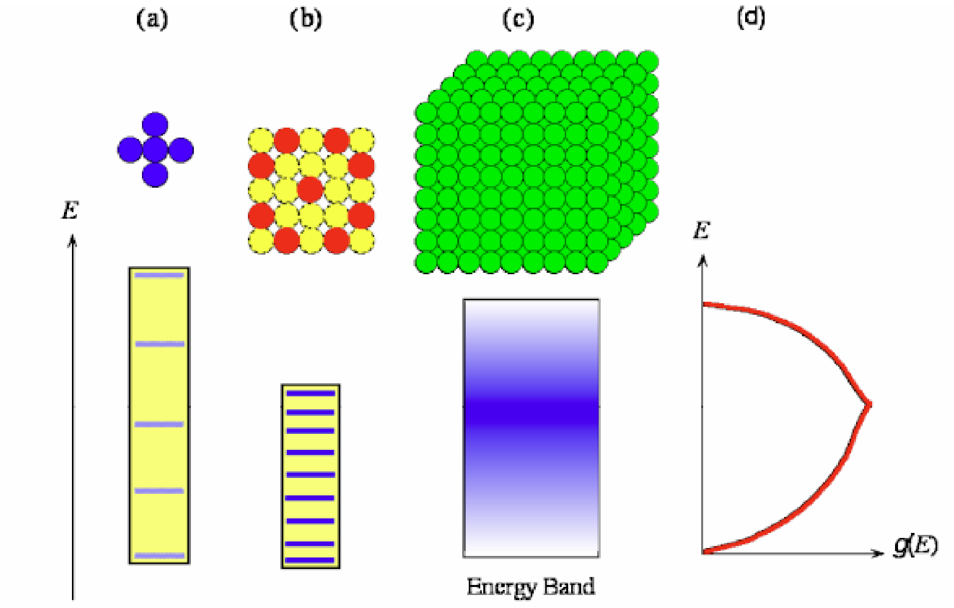
(a) (b)

Fig. 4.7: (a) Three molecular orbitals from three ψ_{1s} atomic orbitals overlapping in three different ways. (b) The energies of the three molecular orbitals labeled as a, b and c in a system with 3 H atoms (highly simplified).

Effect of Periodicity in Solid Systems

- The compression of quantum states into periodic structures results overlap of available quantum states
- Pauli exclusion principle states that:
  - electrons must fill available quantum states from lowest to highest potential
  - allowed states are defined by orbital solutions obtained from quantum mechanics
- As atoms come closer together, orbitals become shared allowing electrons to fill exchange orbitals between the materials
- As the number of atoms brought into proximity increases degeneracies occur resulting in allowable energy bands
- These bands can be modeled using the Kronig Penning model with solutions that give an electron Density of States





[Applet detailing different crystallographic binding types](http://jas.eng.buffalo.edu/education/solid/genUnitCell/index.html)
http://jas.eng.buffalo.edu/education/solid/genUnitCell/index.html

Quantized States in Solid Li

Allowable Quantum States in Li

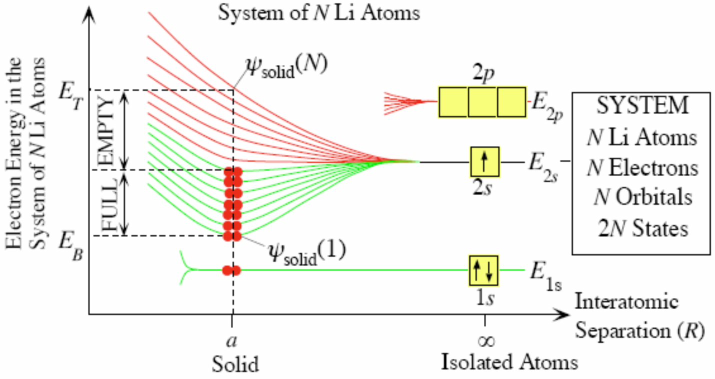


Fig. 4.8: The formation of a $2s$-energy band from the $2s$-orbitals when N Li atoms come together to form the Li solid. The are N $2s$-electrons but $2N$ states in the band. The $2s$-band therefore is only half full. The atomic $1s$ orbital is close to the Li nucleus and remains undisturbed in the solid. Thus each Li atom has a closed K-shell (full $1s$ orbital).

Overlapping Orbitals in 1 mol of Li

1 mol = 10^{23} atoms

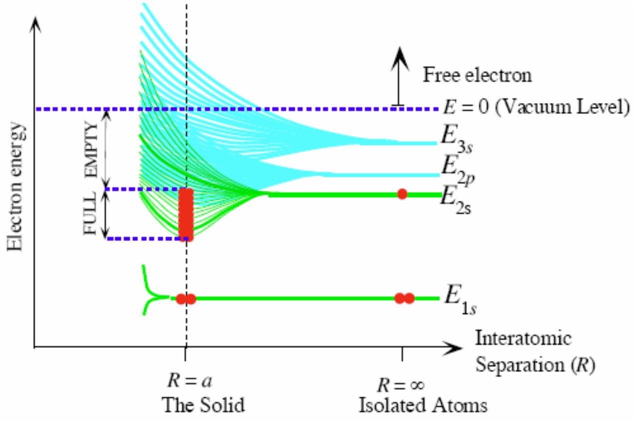


Fig. 4.9: As solid atoms are brought together from infinity, the atomic orbitals overlap and give rise to bands. Outer orbitals overlap first. The $3s$ orbitals give rise to the $3s$ band, $2p$ orbitals to the $2p$ band and so on. The various bands overlap to produce a single band in which the energy is nearly continuous.

Metal Energy Bands

- Overlapping energy degeneracies in metals
- Lead to continuous energy bands
- Statistically stable energy for electrons lies within these overlapping bands and only slight excitations lead to conduction b/c the variation in allowable quantum states is nearly continuous

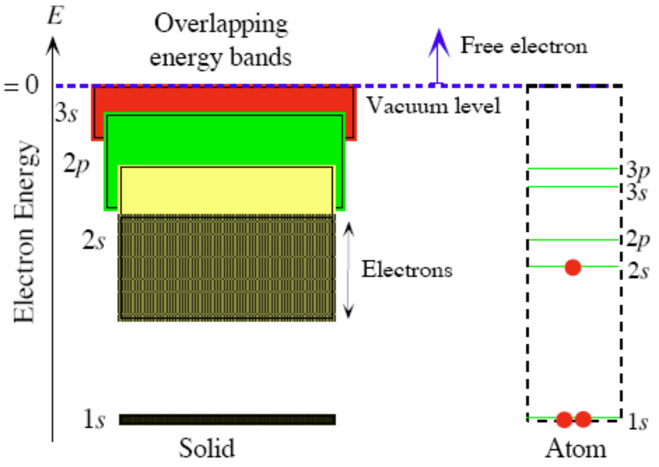


Fig. 4.10: In a metal the various energy bands overlap to give a single band of energies that is only partially full of electrons. There are states with energies up to the vacuum level where the electron is free.

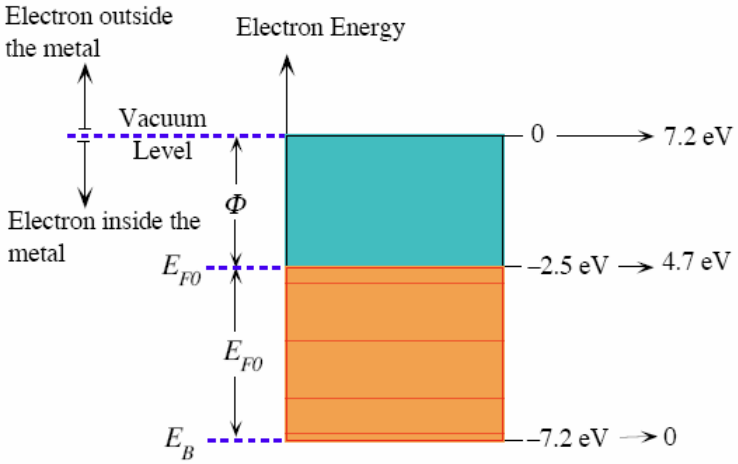
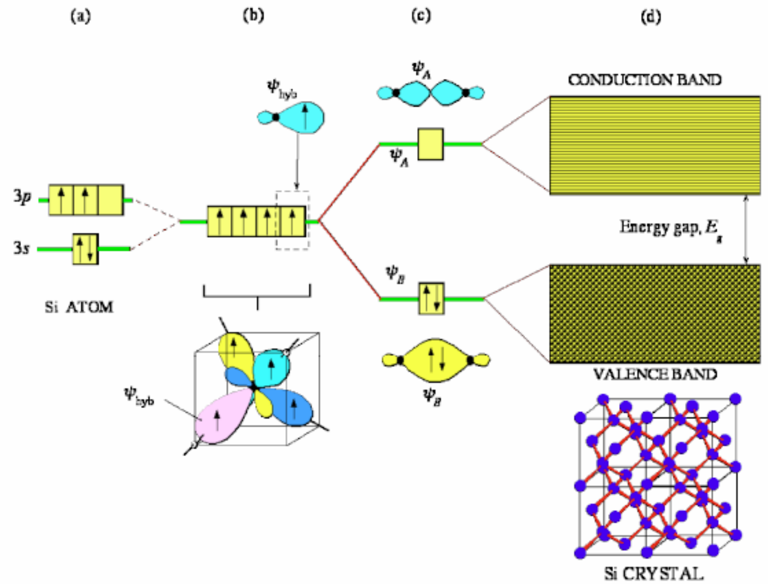


Fig. 4.11: Typical electron energy band diagram for a metal All the valence electrons are in an energy band which they only partially fill. The top of the band is the vacuum level where the electron is free from the solid ($PE = 0$).

Semiconductors

- Semiconductors are distinctly different
- In Semiconductors there is no overlapping degeneracy between conduction and valence bands
- The result is a bandgap, E_g, that is present between bound and conducting electron states $E_g = E_c - E_v$
- The width of the conduction band is called the electron affinity, χ
- At energies above the $E_c + \chi$ electrons can be ejected from the material
- In silicon for example, all of the valence electrons are used to fill the binding orbitals located in the valance band



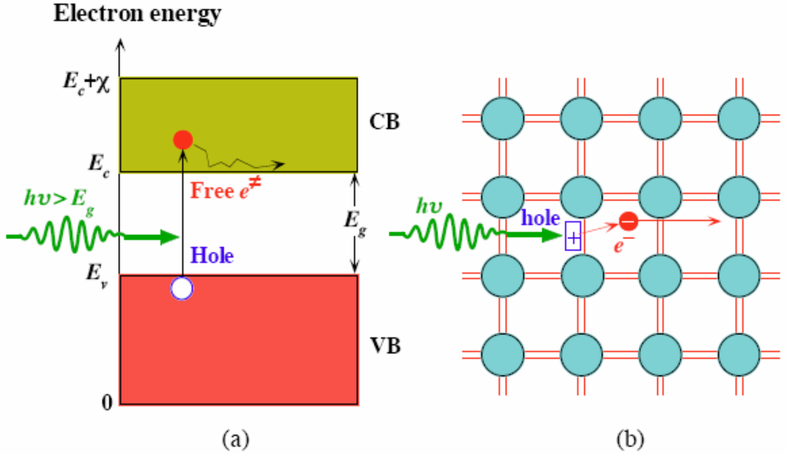


Fig. 5.3: (a) A photon with an energy greater than E_g can excite an electron from the VB to the CB. (b) When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.

Bandgap Basics

- The application of excess energy (light, thermal, electrical) or the addition of extra electrons into the system results in conduction by moving electrons into the conduction band
- In thermal equilibrium electrons can be excited into the conduction band leaving a hole in the valance band
- Holes and electrons propagate in throughout the material via quantum mechanical tunneling from site to site randomly
- The application of a driving potential forces electrons and holes to migrate in opposite directions based on charge density
- The effective mass of holes ,m_h^*, and electrons m_e^* is a quantum mechanical quantity relating the inertial resistance to acceleration of each under a driving force due to electric fields within the periodic structure

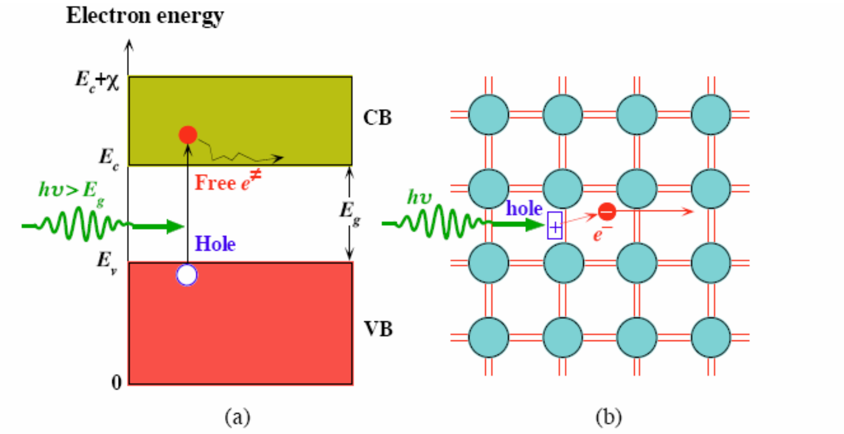
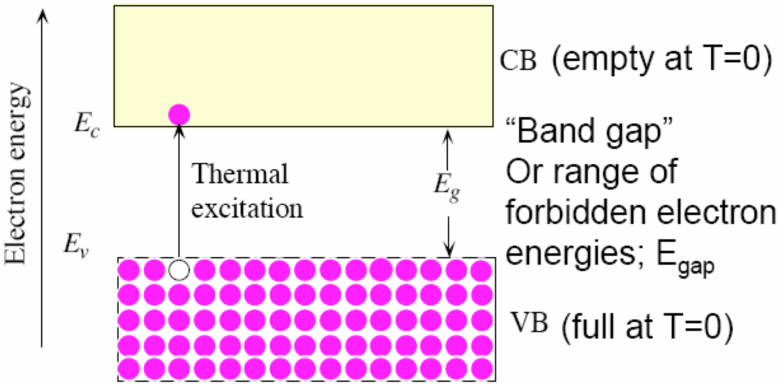


Fig. 5.3: (a) A photon with an energy greater than E_g can excite an electron from the VB to the CB. (b) When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.



Thermal Considerations and Recombination

- The presence of a finite band gap requires that at T=0K, there is no electrical conduction within the material
- As temperature increases, more and more free energy present in the semiconductor allows for the population of conduction bands with electrons.
  - Due to atomic vibrations that increase with temperature allowing for excitation of conduction band energy states
  - Production of electrons in the conduction band due to increased free energy generates an equal number of holes in the valance band
  - This is referred to as thermal generation
- When a wondering electron crosses a site within the lattice where a hole is present, the electron releases its free energy and binds to the atoms valence band. This process is called recombination
- Electron concentration, n, within the conduction band
- Hole concentration, p, within the valance band

http://jas.eng.buffalo.edu/education/semicon/recombination/indirect.html

Semiconductor Statistics: Density of States

- Many important properties of semiconductors are described by considering electrons in the conduction band and holes in the valance band.
- Density of States (DOS), g(E), represents the number of electronics states in a band per unit energy per unit volume of the crystal
- We use quantum mechanics (QM) to calculate the DOS by considering how many electron wave functions there are within a given energy range per unit volume
- According to QM

$$E = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{h^2\mathbf{n}^2}{8mL^2}$$

$$\mathbf{n} = \sqrt{\frac{8\mathrm{mL}^2 n^2}{h^2}}$$

$$g(E) = (8\pi 2^{1/2}(\frac{m_e}{h^2})^{3/2} E^{1/2}$$

where E = E – E_c for electrons in the conduction band

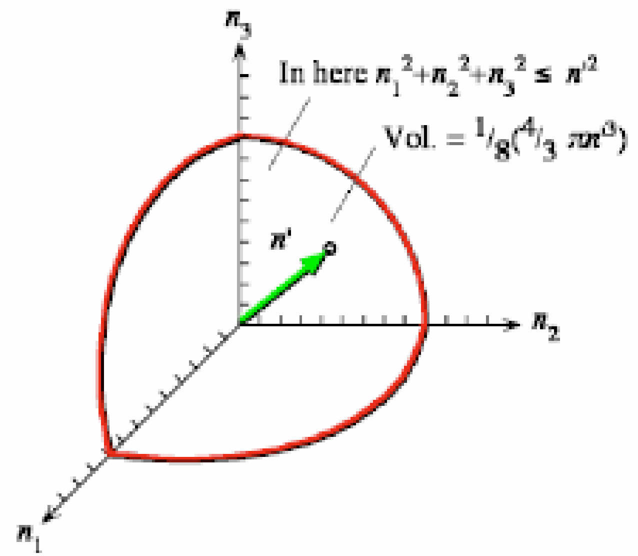


Fig. 4.22: In three dimensions, the volume defined by a sphere of radius n' and the positive axes n_1, n_2 and n_3, is all the possible combinations of poisitive n_1, n_2 and n_3, values which satisfy $n_1^2+n_2^2+n_3^2 \le n'^2$.

DOS Continued

Due to spin

Positive integers only

Volume of sphere

$$\text{from\_}E = \frac{h^2}{8mL^2} n^2$$

$$\#\,states = \boxed{\;} \times \boxed{\;} \times \boxed{\frac{4}{3}\pi n^3} = \frac{2\pi}{6}\left(\frac{8mL^2E}{h^2}\right)^{3/2} = \frac{2\pi}{6}\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{3/2}$$

$$\frac{d}{dE}(\#\,states) = \frac{d}{dE}\,\frac{2\pi}{6}\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{3/2}$$

$$\longrightarrow\longrightarrow = \frac{2\pi}{6}\left(\frac{8mL^2}{h^2}\right)^{3/2} \frac{3}{2} E^{1/2} dE$$

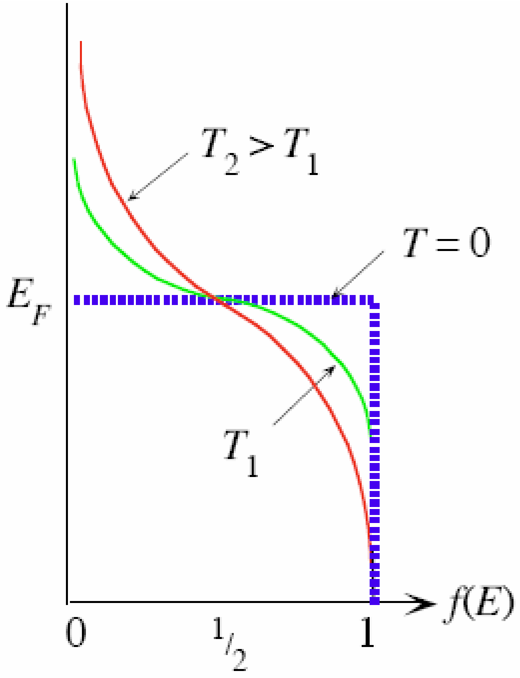
$$g(E) = \frac{4\pi L^3 (2m)^{3/2}}{h^3} E^{1/2} dE = (constants) \times E^{1/2} dE$$

Semiconductor Statistics: Fermi Dirac Function

- The Fermi Dirac Function, f(E), is the probability of finding an electron in a quantum state with energy E. This function is a fundamental property of a collection of interacting electrons in thermal equilibrium

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

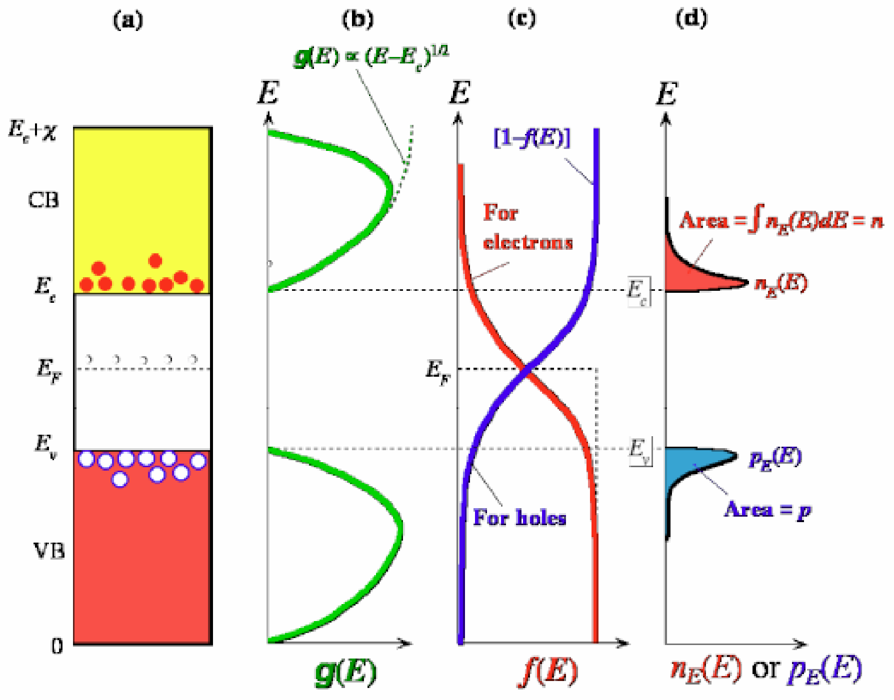
- Where k_b is the Boltzmann constant, T is the temperature in Kelvin, E_f is the Fermi energy
- Fermi Energy = energy required to fill all states at T=0K
- The Fermi energy is the chemical potential (or Gibbs free energy) per electron in the material
- Changes in the Fermi energy across the material represent the electrical work input or output per electron
- In the equilibrium state of a semiconductor with no light or applied voltage, the change in Fermi energy, $\Delta E_f = 0$, AND E_f must be uniform throughout the system
- Note: The probability of a finding a hole is 1-F(E)



http://jas.eng.buffalo.edu/education/semicon/fermi/functionAndStates/functionAndStates.html

Conduction Band Concentrations in a Semiconductor

- Carrier concentration applet for variable temperature
  - http://jas.eng.buffalo.edu/education/semicon/fermi/levelAndDOS/index.html
- Note: DOS, g(E), remains constant. The carrier concentration varies as a function of F(E)



At large energies where $E-E_F >> kT$

(note: kT=0.025eV at 293K)

$$F(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1} \cong \exp\left(\frac{-(E-E_F)}{kT}\right)$$

Electron concentration in the conduction band

$$n = \int_{E_C}^{E_C+\chi} n_e(E)dE = \int_{E_C}^{E_C+\chi} g_{CB}(E)f(E)dE$$

$$n = \frac{\pi 8\sqrt{2}}{h^3} m_e^{*3/2} \int_{E_C}^{\infty} (E-E_C)^{1/2} \exp -\left[\frac{(E-E_F)}{kT}\right] dE$$

$$n = N_c \exp\left[-\frac{(E_c-E_F)}{kT}\right]$$

where $$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

is the effective density of states at the conduction band edge

Valance Band Solution

At energies where $E_F - E >> kT$ (i.e. E is below E_F)

$$F(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1} \cong \frac{1}{0+1} \cong 1$$

Large negative exponential

So, the probability of occupancy far below E_F is 100%.

Note that the probability of non-occupancy is 1-F(E) (or the probability of a state being empty)

Important note: The only assumptions specific to these derivations for n and p is that the Fermi energy is only a few k_BT away from the band edges

hole concentration in the valence band

$$p = \int_0^{E_V} p_e(E)dE = \int_0^{E_V} g_{VB}(E)[1 - f(E)]dE$$

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$$

where $N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$

is the effective density of states at the valance band edge

Intrinsic Semiconductor

- Intrinsic semiconductors are pure crystals where n = p
- It can be shown that in an intrinsic semiconductor that the Fermi level, E_{fi}, is above E_v and located in the bandgap at

$$E_{fi} = E_v + \frac{1}{2}E_g - \frac{1}{2}k_B T \ln\left(\frac{N_c}{N_v}\right)$$

- Typically N_c and N_v values are comparable and both occur in the logarithmic term so that Efi is approximately in the middle of the bandgap as shown in previous slides
- The product of n and p in an intrinsic semiconductor provides the mass action law

$$np = N_c N_v e^{\left(-\frac{E_g}{k_B T}\right)} = n_i^2$$

- Where $E_g = E_c - E_v$ s the bandgap energy, n_i^2 is the constant that depends on temperature and material properties, and not the Fermi energy.
- Thermal velocity of electrons in an intrinsic semiconductor at room temperature

$$E = \left\langle \frac{m_e^* v^2}{2} \right\rangle = \frac{3}{2}k_B T$$

$$\sqrt{\left\langle v^2 \right\rangle} \approx 10^5 \frac{m}{s}$$

Extrinsic Semiconductors

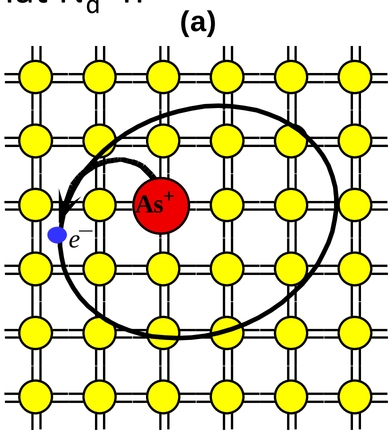
- Semiconductors with small amounts of impurities
- These impurities increase/decrease the probability of obtaining an electron in the conduction band
- N-type semiconductors
  - extrinsic semiconductors with excess electrons
  - Arsenic added to silicon to which have one more valence (available electron) than silicon
  - Arsenic is called a donor b/c it donates electrons to the system
  - For $N_d >> n_i$, at room temperature, the electron concentration inside the conduction band will be nearly equal to N_d such that $N_d=n$
  - Number of holes,

    $p = n_i^2/N_d$
  - Conductivity, σ, depends on drift mobilities,μ, of electrons and holes

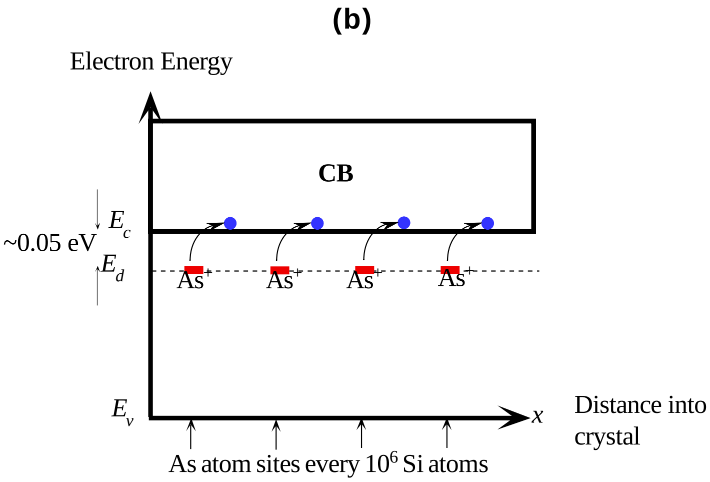
$$\sigma = en\mu_e + ep\mu_h$$

$$\sigma = eN_d\mu_e + e\frac{n_i^2}{N_d}\mu_h$$

$$\sigma \approx eN_d\mu_e$$



(a) The four valence electrons of As allow it to bond just like Si but the fifth electron is left orbiting the As site. The energy required to release to free fifth-electron into the CB is very small.



(b) Energy band diagram for an *n*-type Si doped with 1 ppm As. There are donor energy levels just below E_c around As^+ sites.

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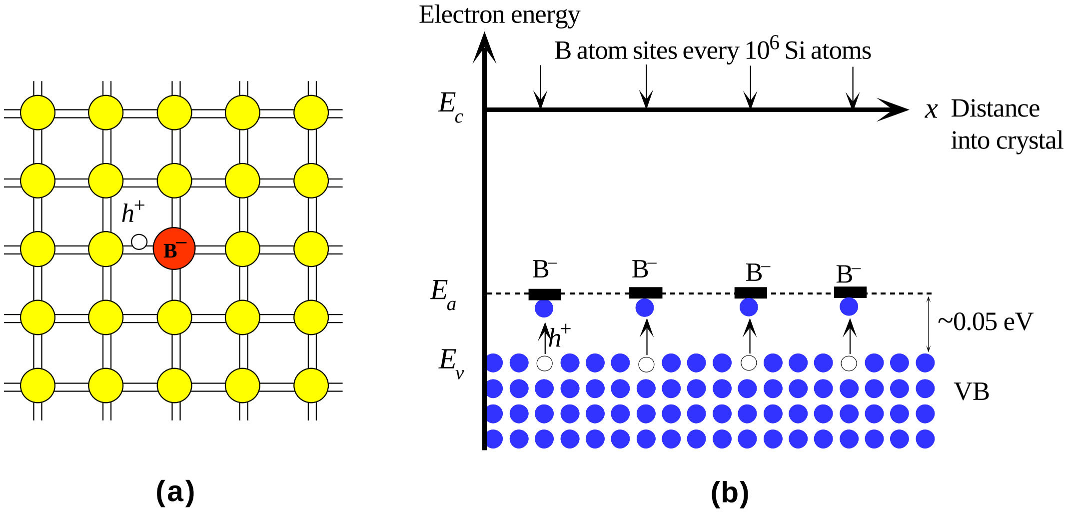
Extrinsic Semiconductors

- Semiconductors with small amounts of impurities
- These impurities increase/decrease the probability of obtaining an electron in the conduction band
- P-type semiconductor
  - Extrinsic with less electrons
  - Adding Boron (+3) metal which has one fewer electron and yields an increased hole per doped atom
  - Boron is called an acceptor
  - For $N_a >> n_i$, at room temperature, the hole concentration inside the valence band will be nearly equal to N_a such that $N_a = p$
  - Electron carrier concentration is determined by the mass action law as $n = n_i^2/N_a$
  - This value is much smaller than p and thus the conductivity is given by $\sigma = eN_a\mu_h$



(a) **(b)**

(a) Boron doped Si crystal. B has only three valence electrons. When it substitutes for a Si atom one of its bonds has an electron missing and therefore a hole. (b) Energy band diagram for a *p*-type Si doped with 1 ppm B. There are acceptor energy levels just above E_v around B$^-$ sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

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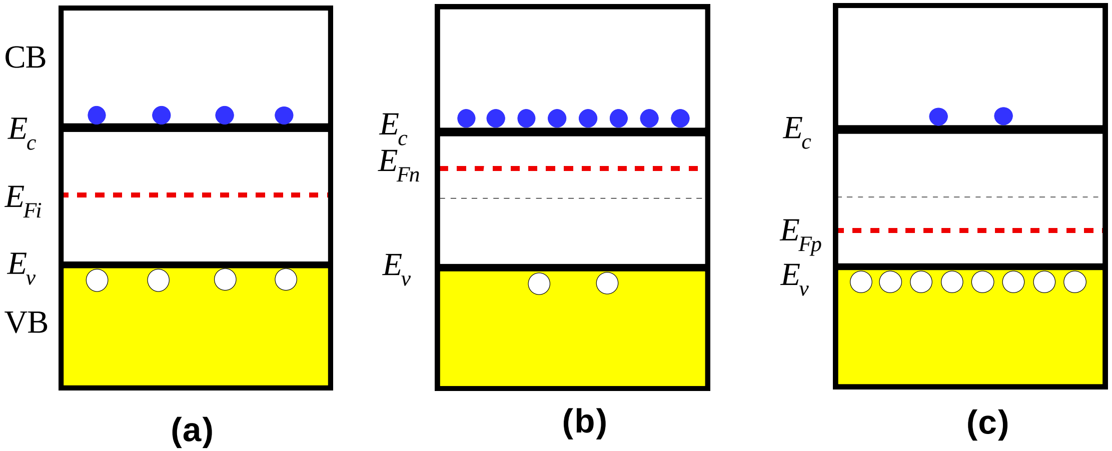
Simplified Band Diagrams for Semiconductors

- Notice in the chart below that the Fermi level changes as a function of doping
- Notice also that carrier concentration (holes or electrons) also changes as a function of doping
- N-type: majority carriers are electrons and minority carriers are holes
- P-type: majority carriers are holes and minority carriers are electrons
- Mass action law still valid: $n_{no}p_{no}=n_i^2$ where no is the doped equilibrium carrier concentration



Energy band diagrams for (a) intrinsic (b) *n*-type and (c) *p*-type semiconductors. In all cases, $np = n_i^2$. Note that donor and acceptor energy levels are not shown.

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http://jas.eng.buffalo.edu/education/semicon/fermi/heavyVSmoderate/index.html

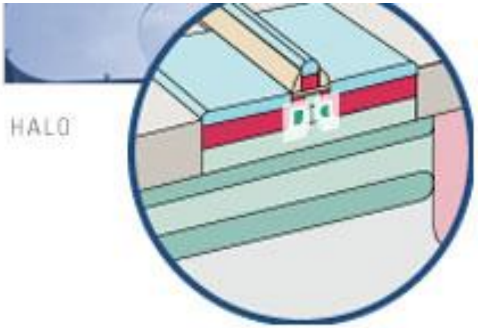
Compensation Doping

- Doping of a semiconductor with both donors and acceptors to control properties
- Provides precise control of carrier concentrations

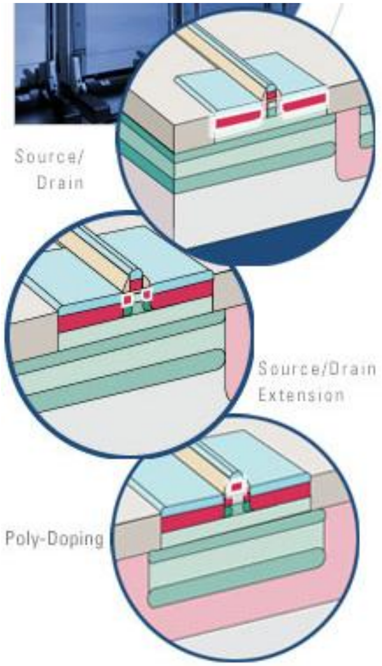
$$n = N_d - N_a$$

- Utilized extensively where p and n type doping of different regions meet.

Medium Dose Applications



Buried channel doping
0.5 keV to 750 keV
35-65 nm ULSI



High Dose Applications

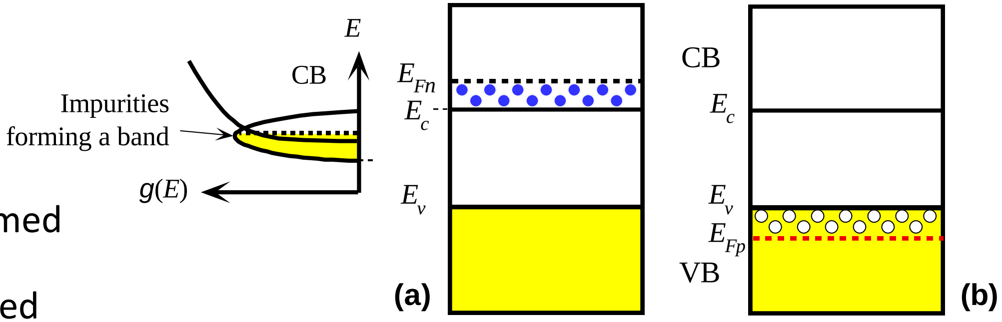
0.2 keV to 80 keV

Degenerate and Nondegenerate Semiconductors

- In nondegenerate semiconductors the number of states in the carrier band far exceeds the number of electrons
- Thus the probability of two electrons trying to occupy the same allowed state is virtually zero
- This means that the Pauli exclusion principle can be neglected and the DOS is represented only by Boltzmann statistics
- In this case,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

  is only valid when n << N_c
- Semiconductors where n<< N_c and p << N_v are termed nondegenerate
- When a semiconductors has been excessively doped with donors
  - then n may be $10^{19} – 10^{20}$ /cm^3 n > N_p or P>Nv
  - N is then comparable to N_c and the Pauli exclusion principle comes into play
  - In this case Fermi Dirac statistics are required
  - Degenerate semiconductor



(a) Degenerate *n*-type semiconductor. Large number of donors form a band that overlaps the CB. (b) Degenerate *p*-type semiconductor.

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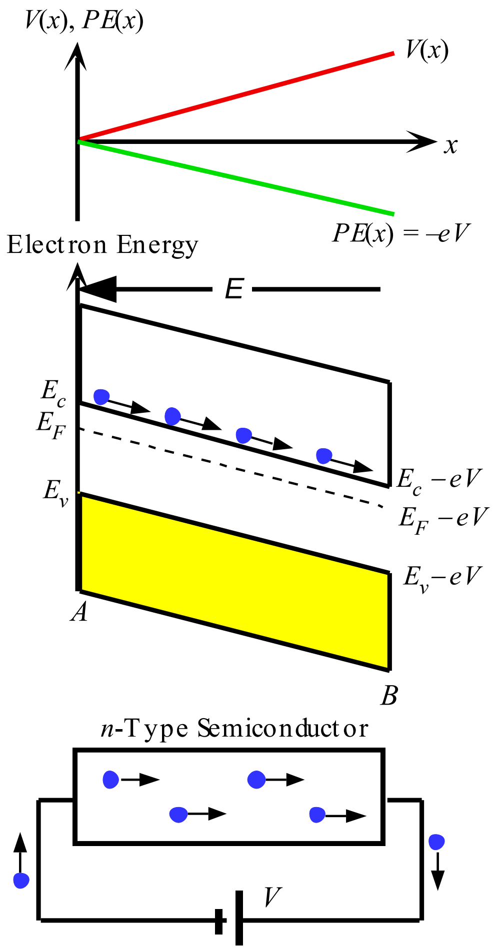
Energy Band Diagram in an Applied Field



Energy band diagram of an *n*-type semiconductor connected to a voltage supply of *V* volts. The whole energy diagram tilts because the electron now has an electrostatic potential energy as well

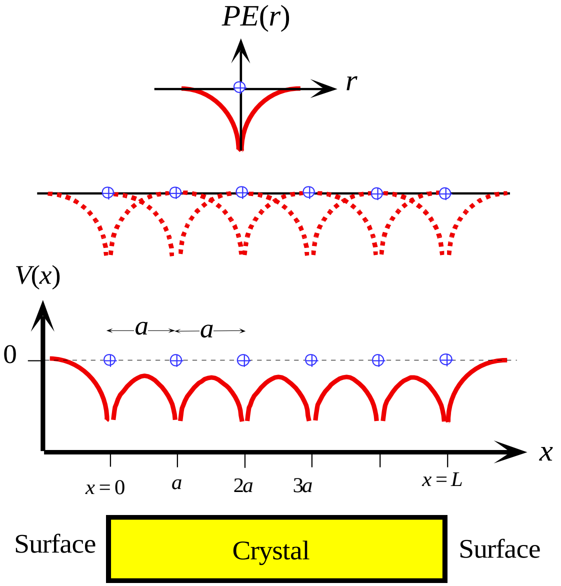
Potential Theory: A More Precise Band Diagram

- The time independent Schrödinger equation for a given potential function is written as

$$\nabla\Psi + \frac{2m_e}{\hbar^2}\left[E - V(x)\right] = 0$$ with a general solution of $$\Psi(x) = Ce^{ikx} + De^{-ikx}$$

If the potential energy, V, is periodic in nature as that shown below, then one can write it as

$$V(x) = V(x + ma) \qquad m = 1,2,3,\ldots$$



PE of the electron around an isolated atom

When *N* atoms are arranged to form the crystal then there is an overlap of individual electron *PE* functions.

PE of the electron, *V*(*x*), inside the crystal is periodic with a period *a*.

The electron potential energy (*PE*), *V*(*x*), inside the crystal is periodic with the same periodicity as that of the crystal, *a*. Far away outside the crystal, by choice, *V* = 0 (the electron is free and *PE* = 0).

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E-K Bandgap Diagram (Bloch Wavefunction)

- The solution is called a Bloch Wavefunction

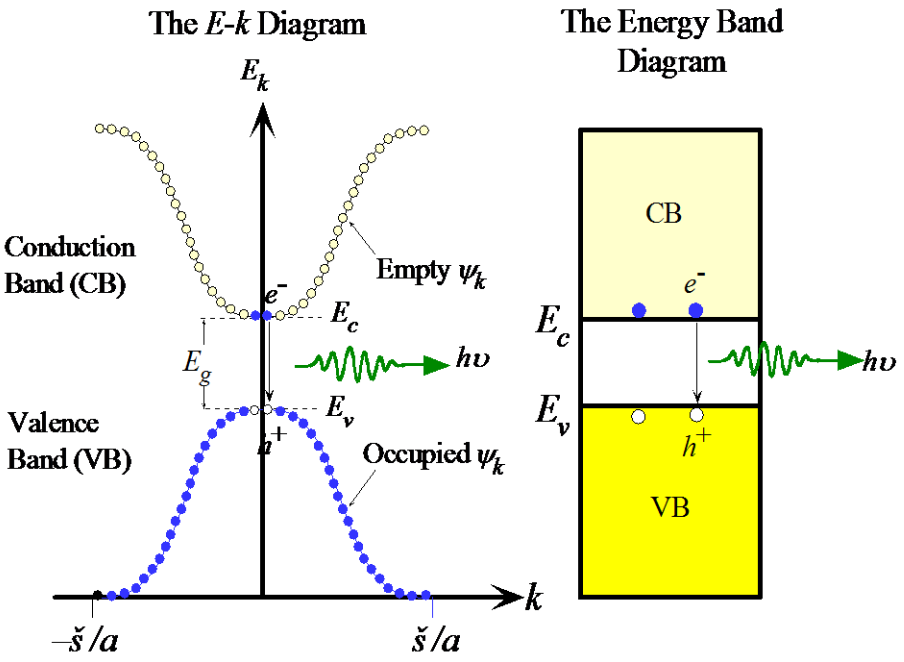
$$\Psi_k(x) = U(x)_k e^{jkx}$$

  - where U(x) is a periodic function that depends on V(x). The two share the same periodicity
- The wavevector, k, in this solution acts like a quantum number and has values from –π/a to π /a

$$k^2 = \frac{2mE}{\hbar^2}$$

- Momentum, p, in the crystal is ħk
- External forces:

$$F = qE = \frac{dp}{dt} = \frac{d(\hbar k)}{dt}$$



The *E-k* diagram of a direct bandgap semiconductor such as GaAs. The *E-k* curve consists of many discrete points with each point corresponding to a possible state, wavefunction $\psi_k(x)$, that is allowed to exist in the crystal. The points are so close that we normally draw the *E-k* relationship as a continuous curve. In the energy range E_v to E_c there are no points ($\psi_k(x)$ solutions).

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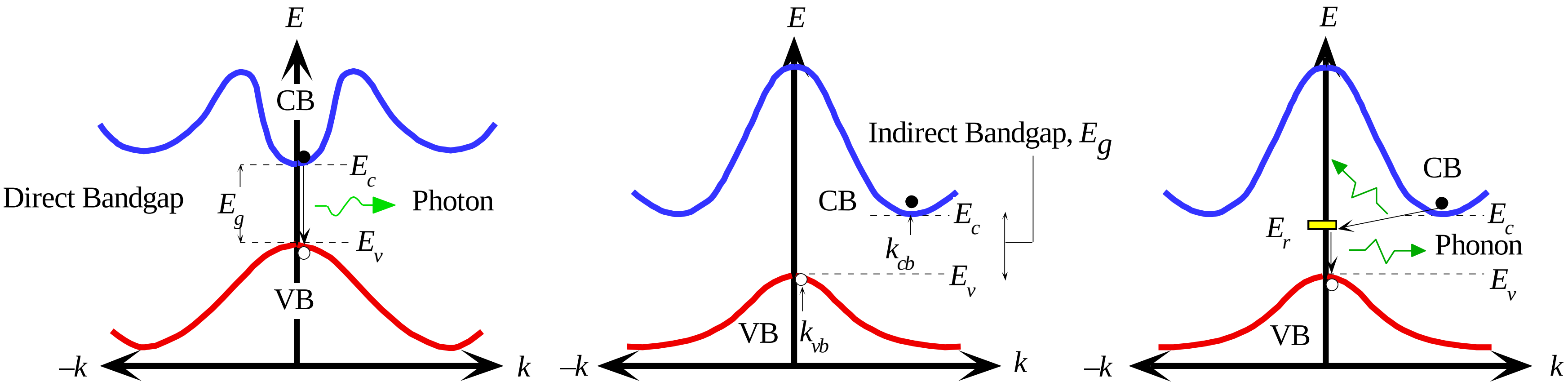
Direct vs. Indirect Bandgap

- Direct Bandgap
  - Base of the conduction band is matched to the max height of the valence band
  - Recombination through the emission of a photon (Light!!!!!!)
- Indirect Bandgap
  - direct recombination would require a momentum change (not allowed)
  - Recombination centers (lattice defects) are required to recombine CB to VB bands
  - The result is a phonon emission (lattice vibration) that propagates across the lattice



(a) GaAs (b) Si (c) Si with a recombination center

(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor. (c) Recombination of an electron and a hole in Si involves a recombination center .

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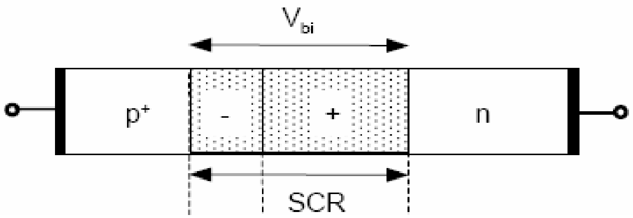
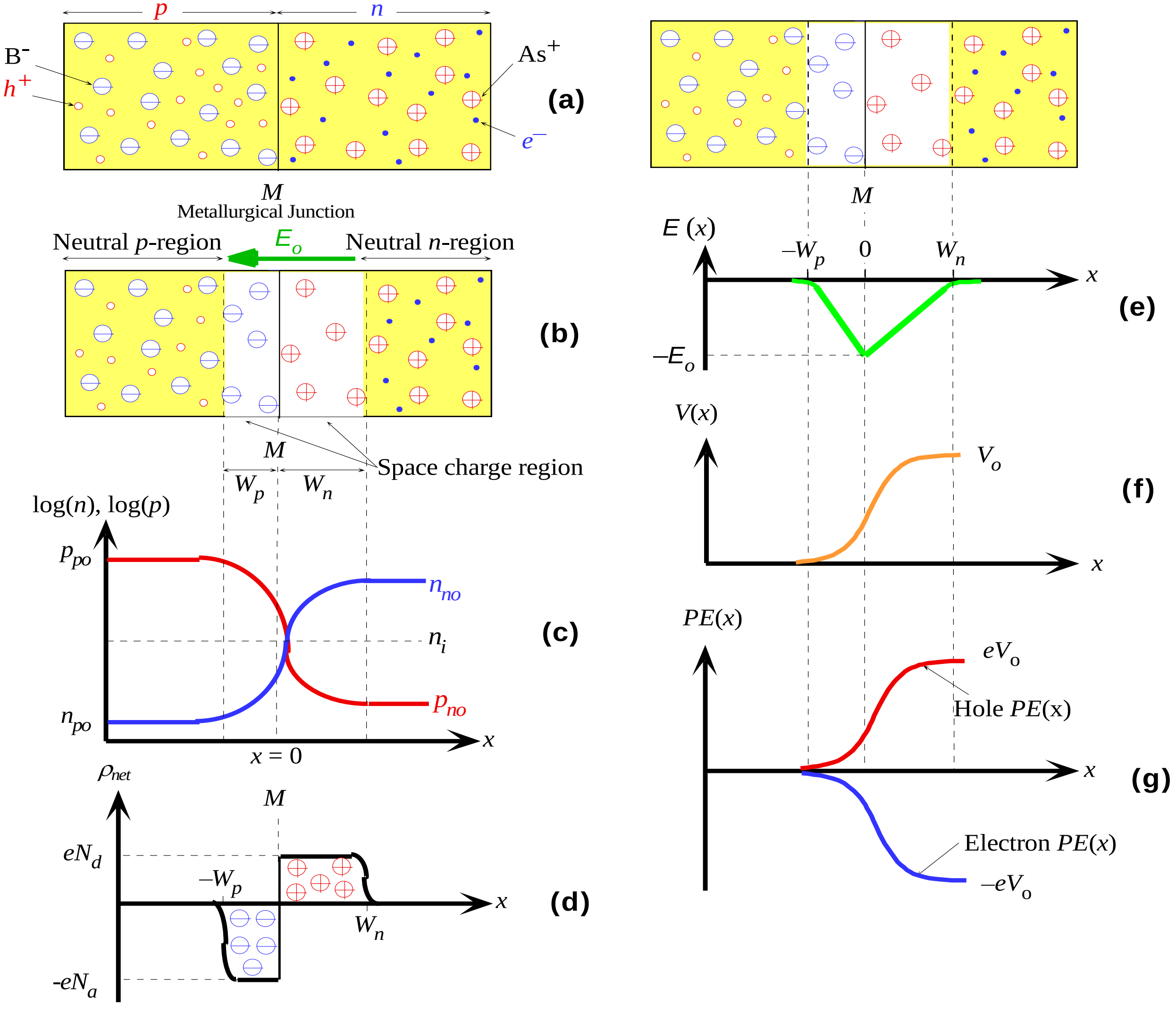
pn Junction Principles



- Consider what happens when one side of a semiconductor such as Si is doped n-type and the other is d p-type
- The abrupt discontinuity between the two sides is called a metallurgical junction, M
- The M region contains a depletion region of carriers of width W = Wp + Wn where Wn is the space charge region of the n-doped Si and vice versa
- An electric field is generated in the M region in order to minimize the free energy at the boundary and satisfy the mass action law $n_i^2 = pn$
- The result is a bias voltage generated across the junction



Properties of the *pn* junction.

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Example: Fermi Level In Semiconductors

- n-type Si is doped with 10^{16} antimony (Sb) atoms/cm^3. Note antimony is group V → n dopes
- Calculate the Fermi energy with respect to the intrinsic Fermi energy of Silicon

Intrinsic Carriers $n_i = 1.45 \times 10^{10} / cm^3$ $\quad n_i = N_c \exp\left[-\left(E_c - E_{Fi}\right)/k_B T\right]$

Doped Carriers $N_d = 10^{16} / cm^3$ $\quad N_d = N_c \exp\left[-\left(E_c - E_{Fn}\right)/k_B T\right]$

$N_d >> n_i$

$n = N_d$

$$E_{Fn} - E_{Fi} = k_B T \ln\left(N_d / n_i\right) = \left(0.0259 eV\right) \ln\left(\frac{10^{16}}{1.45 \times 10^{10}}\right) = 0.348 eV$$

- If the n-type Si is further doped with 2x10^{17} boron(B) atoms/cm^3 Boron is group III → p dopes
- Calculate the position of the Fermi energy with respect to the intrinsic Si Fermi energy at room temperature (300K) and hence with respect to the n-type case for antimony doping

$N_a = 2 \times 10^{17} / cm^3$ $\quad p = n_i = N_c \exp\left[-\left(E_{Fi} - E_v\right)/k_B T\right]$ Intrinsic Carriers

$N_d = 10^{16} / cm^3$ $\quad p = N_c \exp\left[-\left(E_{Fp} - E_v\right)/k_B T\right] = N_a - N_d$ Doped Carriers

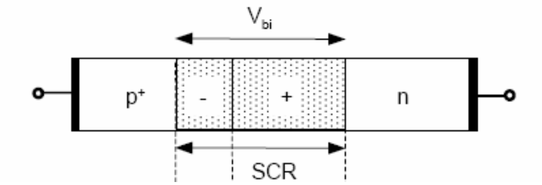
$N_a > N_d$ $\quad p / n_i = \exp\left[-\left(E_{Fp} - E_{Fi}\right)/k_B T\right]$

$p = N_a - N_d$

$p = 1.9 \times 10^{17} / cm^3$

$$E_{Fp} - E_{Fi} = -k_B T \ln\left(p / n_i\right) = -\left(0.0259 eV\right) \ln\left(\frac{1.9 \times 10^{17} / cm^3}{1.45 \times 10^{10}}\right) = -0.424 eV$$

Governing Equations for pn Junctions



- Conservation of charge $N_a W_p = N_d W_n$
- The potential established across the boundary on the n-side is derived by integrating the Electric field established by the change in charge density across the boundary

$$\rho = \begin{cases} -qN_A & for \quad -x_p < x < 0 \\ qN_D & for \quad 0 < x < x_n \end{cases}$$

Charge Density

$$F = \begin{cases} -\dfrac{qN_A}{\varepsilon_S} \cdot (x_p + x) & for \quad -x_p < x < 0 \\ -\dfrac{qN_D}{\varepsilon_S} \cdot (x_n - x) & for \quad 0 < x < x_n \end{cases}$$

Force, $F = qE = q\int \frac{\rho}{\varepsilon} dx$

$$\varphi = \begin{cases} -\varphi(-x_p) + \dfrac{qN_A (x + x_p)^2}{2\varepsilon_S} & for \quad -x_p < x < 0 \\ \varphi(x_n) - \dfrac{qN_D (x - x_n)^2}{2\varepsilon_S} & for \quad 0 < x < x_n \end{cases}$$

Potential $V = -\int E dx$

- The Maximum value of the electric field and built in potential generated at the edge of the n-side of the M region are

$$E_o = -\frac{eN_d W_n}{\varepsilon} = -\frac{eN_a W_p}{\varepsilon}$$

$$V_o = -\frac{1}{2} E_o W_o = \frac{eN_d N_a W_o^2}{2\varepsilon (N_a + N_d)}$$

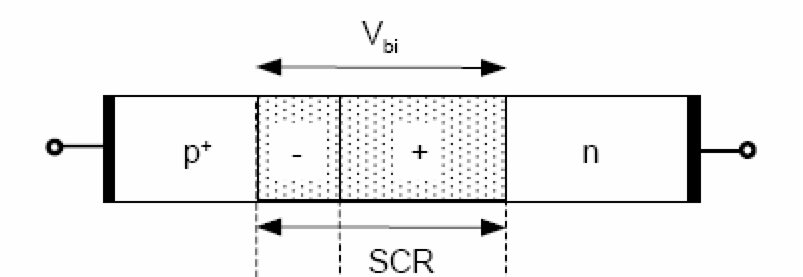
Governing Equations for pn Junctions

- One can relate Vo to doping parameters using the ratios of the carriers n2 and n1

$$\frac{n_2}{n_1} = \exp\left[\frac{-(E_2 - E_1\,\_)}{k_B T}\right] \rightarrow \quad \frac{n_{no}}{n_{po}} = \exp\left[\frac{-eV_o}{k_B T}\right] \text{ and } \quad \frac{p_{no}}{p_{po}} = \exp\left[\frac{-eV_o}{k_B T}\right]$$

$$\rightarrow \quad \frac{N_d}{n_i} = \exp\left[\frac{-eV_o}{k_B T}\right] \qquad \frac{N_a}{n_i} = \exp\left[\frac{-eV_o}{k_B T}\right]$$

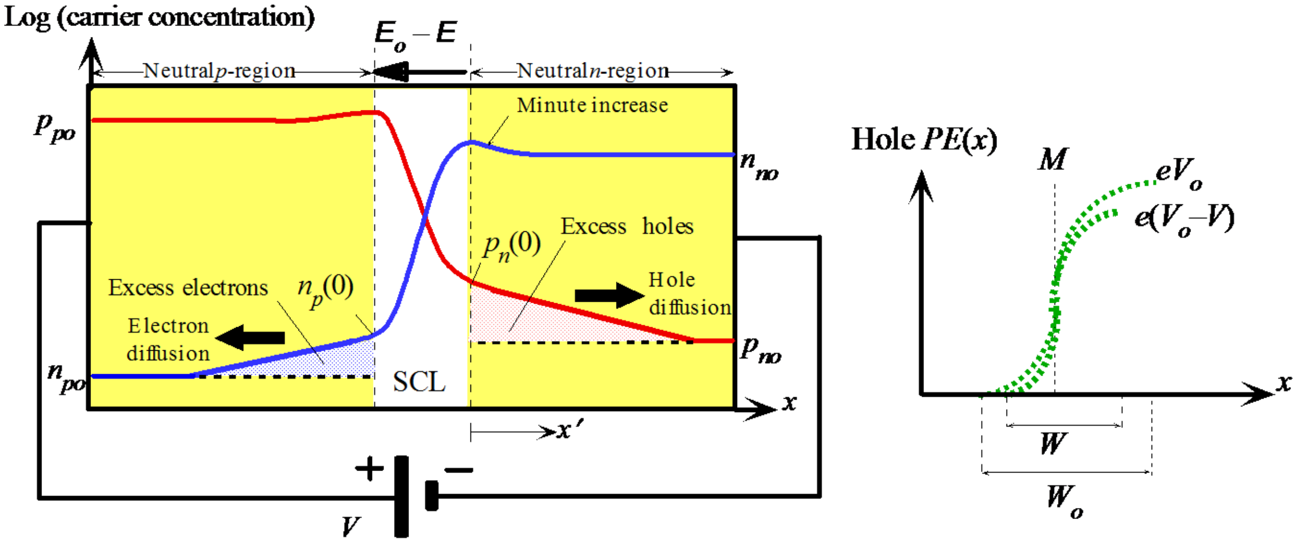
$$\rightarrow \quad V_o = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$



Forward Bias on pn Junctions

- Applied Bias V-V$_o$ yields –(V$_o$-V) in exponential term
- Resultant equations for carrier concentrations are referred to as the Law of the Junction

$$n_p(0) = n_{po}\exp\left[\frac{-e(V_o - V)}{k_BT}\right] = \frac{n_i^2}{N_a}\exp\left[\frac{eV}{k_BT}\right] \qquad p_n(0) = p_{no}\exp\left[\frac{-e(V_o - V)}{k_BT}\right] = \frac{n_i^2}{N_d}\exp\left[\frac{eV}{k_BT}\right]$$



(a) **(b)**

Forward biased *pn* junction and the injection of minority carriers (a) Carrier concentration profiles across the device under forward bias. (b). The hole potential energy with and without an applied bias. *W* is the width of the SCL with forward bias

Hole Diffusion

- The increased length of carrier regions under an applied forward bias lead to excess minority carrier concentrations and a hole diffusion length, L

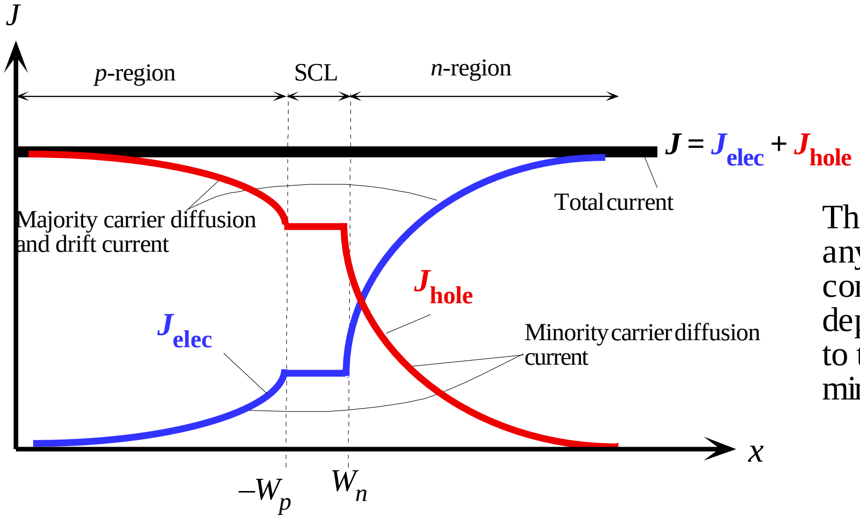
$$\Delta p_n(x') = p_n(x') - p_{no} = \Delta p_n(0)\exp\left(\frac{-x'}{L_h}\right) \qquad L_h = \sqrt{(D_h \tau_h)}$$

where D is the diffusion coefficient of holes in the lattice and τ is the hole recombination lifetime

- Current density due to carrier diffusion is

$$J_{d,hole} = -eD_h \frac{d\Delta p_n(x')}{dx} = \frac{-eD_h}{L_h}\Delta p_n(0)\exp\left(\frac{-x'}{L_h}\right) = \frac{eD_h n_i^2}{L_h N_d}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

$$J_{d,elec} = \frac{eD_e n_i^2}{L_e N_a}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$



The total current anywhere in the device is constant. Just outside the depletion region it is due to the diffusion of minority carriers.

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- Yields the Shockley equation

(reverse saturation current density)

$$J_{so} = \frac{eD_h n_i^2}{L_h N_d} + \frac{eD_e n_i^2}{L_e N_a}$$

Recombination Times

$$J_{recom} = \frac{eABC}{\tau_e} + \frac{eBCD}{\tau_h}$$

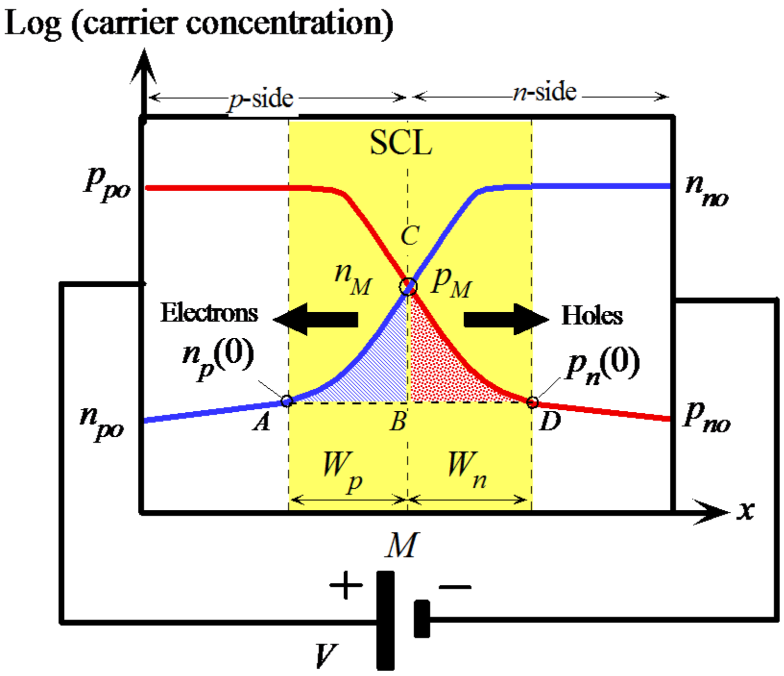
$$J_{recom} \approx \frac{e(W_p n_M)}{2\tau_e} + \frac{e(W_n p_M)}{2\tau_h}$$

using $M = \frac{1}{2}e(V_o - V)$

$$p_M = n_i \exp\left[\frac{-eV}{2k_B T}\right]$$

$$J_{recom} = \frac{en_i}{2}\left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h}\right)\exp\left(\frac{eV}{2k_B T}\right)$$

$$I = \frac{en_i}{2}\left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h}\right)A\left[\exp\left(\frac{eV}{\eta k_B T}\right) - 1\right]$$



Forward biased *pn* junction and the injection of carriers and their recombination in the SCL.

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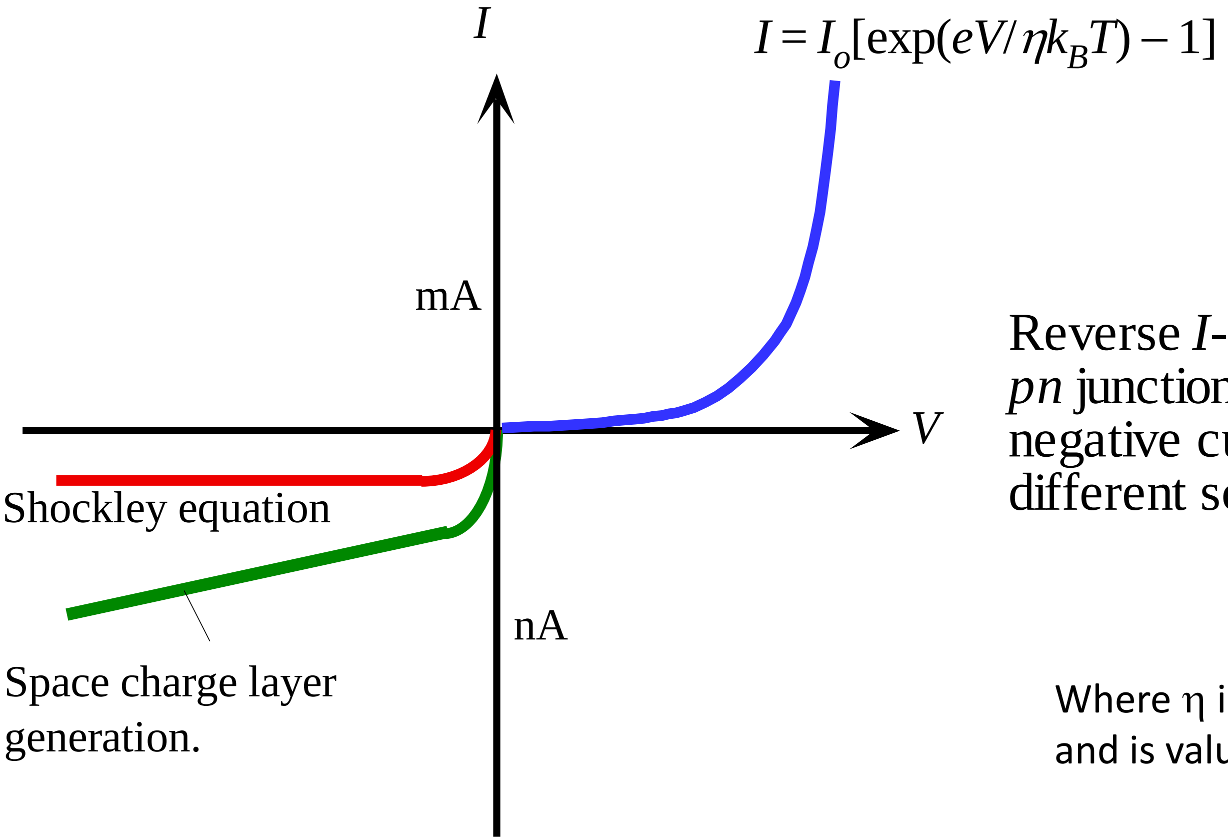
Where η is the diode ideality factor and is valued between 1 and 2

Current in pn Junctions



Reverse *I*-*V* characteristics of a *pn* junction (the positive and negative current axes have different scales)

Where η is the diode ideality factor and is valued between 1 and 2

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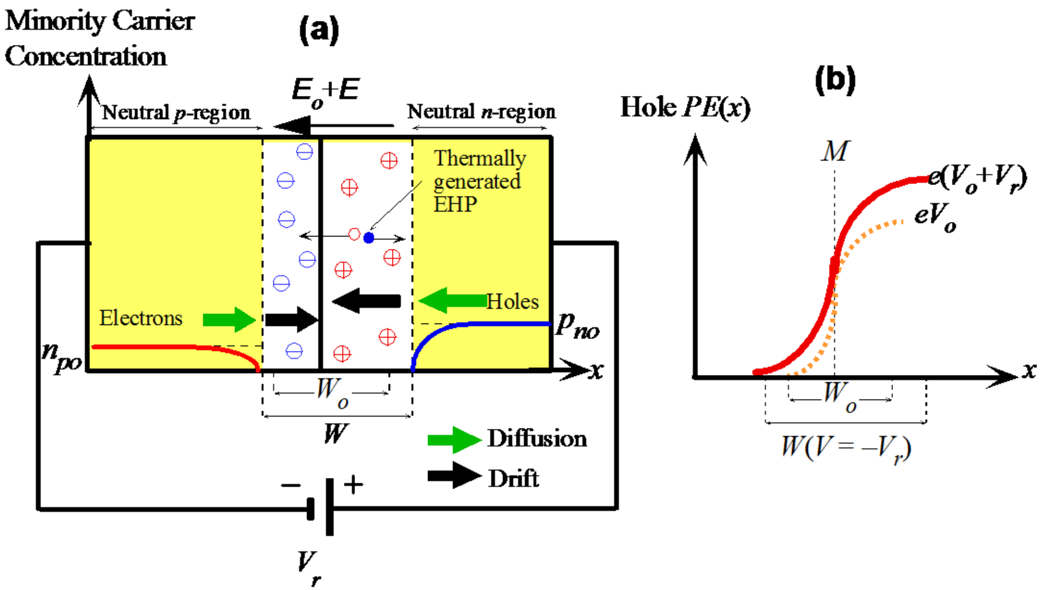
Reverse Bias pn Junctions

- $V = V_o + V_r$
- Increased bias leads to thermal generation of electrons and holes
- Mean thermal generation time, τ_g
- Reverse current density due to thermal generation of CB electrons $$J_{gen} = \frac{eWn_i}{\tau_g}$$
- Total reverse current density

$$J_{gen} = \frac{eD_h n_i^2}{L_h N_d} + \frac{eD_e n_i^2}{L_e N_a} + \frac{eWn_i}{\tau_g}$$

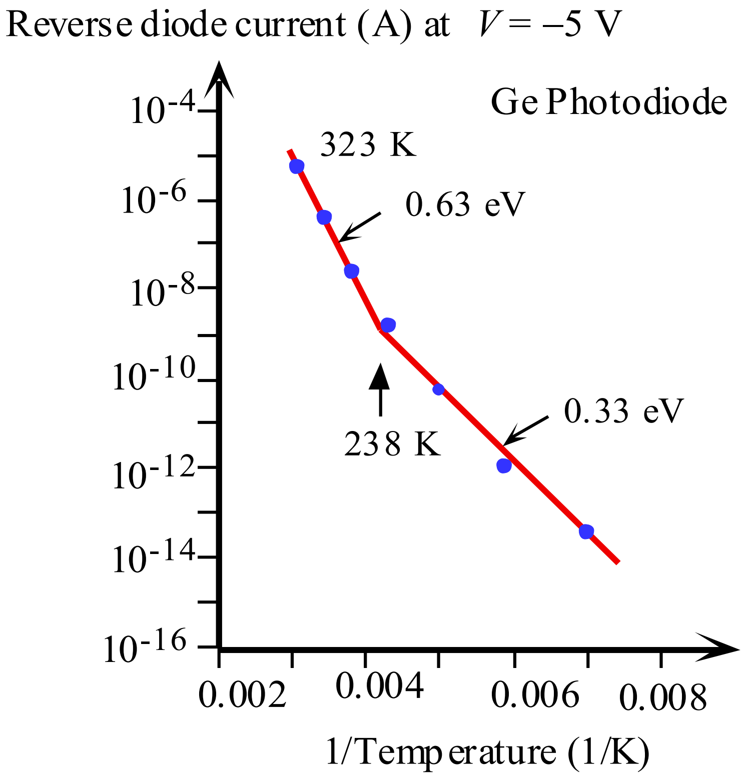
where $$n_i \propto \exp\left[\frac{-E_g}{2k_B T}\right]$$



Reverse biased *pn* junction. (a) Minority carrier profiles and the origin of the reverse current. (b) Hole *PE* across the junction under reverse bias

Reverse current in Ge pn junction

Ge is a direct bandgap semiconductor, thus the pn junction emits a photon and is referred to as a photodiode



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Reverse diode current in a Ge*pn* junction as a function of temperature in a ln(I_{rev}) vs. 1/T plot. Above 238 K, I_{rev} is controlled by n_i^2 and below 238 K it is controlled by n_i. The vertical axis is a logarithmic scale with actual current values. (From D. Scansen and S.O. Kasap, *Cnd. J. Physics*. **70**, 1070-1075, 1992.)

$$J_{gen} = \left(\frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a} \right) n_i^2 + \frac{eW}{\tau_g} n_i$$

Bias driven

Driven by thermal generation

Depletion Layer Capacitance

- Charge on each side of the diode

$$Q = eN_d W_n A$$
$$-Q = eN_a W_p A$$

- Width of the Depletion layer

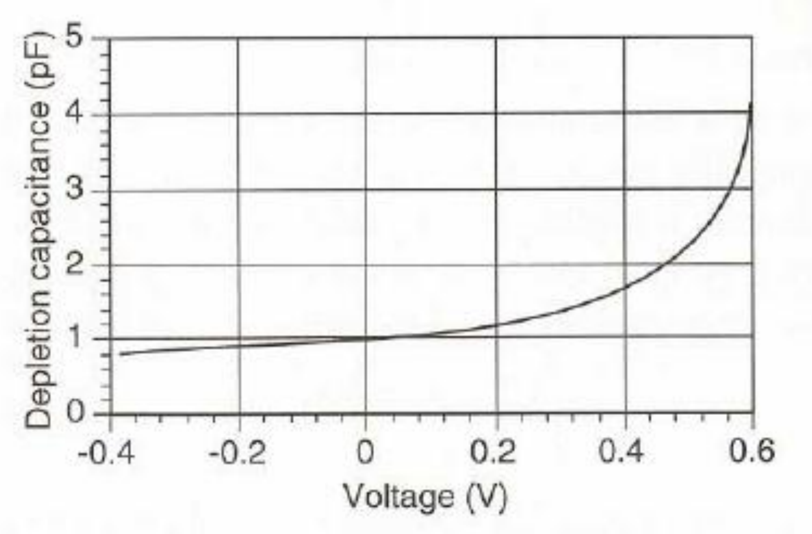
$$W = \sqrt{\frac{2\varepsilon (N_d + N_a)(V_o - V)}{eN_d N_a}}$$

- Depletion Layer Capacitance

$$Cdep = \left|\frac{dQ}{dV}\right| = \frac{\varepsilon A}{W} = A\sqrt{\frac{\varepsilon e N_d N_a}{2(N_d + N_a)(V_o - V)}}$$

$$C_j = A\sqrt{\frac{\varepsilon e N_d N_a}{2(N_d + N_a)(V_o)}}$$ Junction capacitance

$$C_{dep} = \frac{C_j}{(1 - V/V_o)^m}; m \approx 1/2$$



Recombination Lifetime

- Instantaneous minority carrier concentration $n_p = n_{po} + \Delta n_p$
- Instantaneous majority carrier concentration $p_p = p_{po} + \Delta n_p$
- Thermal generation rate, $G_{thermal}$
- Net change of holes in the semiconductor is

$$\frac{\partial \Delta n_p}{\partial t} = -B n_p p_p + G_{thermal}$$

$$\frac{\partial \Delta n_p}{\partial t} = -B\left(n_p p_p + n_{po} p_{po}\right)$$

- Where B is the direct recombination coefficient
- Excess minority carrier recombination lifetime, τ_e is defined by

$$\frac{\partial \Delta n_p}{\partial t} = -\frac{\Delta n_p}{\tau_e}$$

- Weak injections $\Delta n_p << p_{po}$

$$n_p \approx \Delta n_p$$

$$p_p \approx p_{po} \approx N_a$$

$$\frac{\partial \Delta n_p}{\partial t} = -B N_a \Delta n_p$$

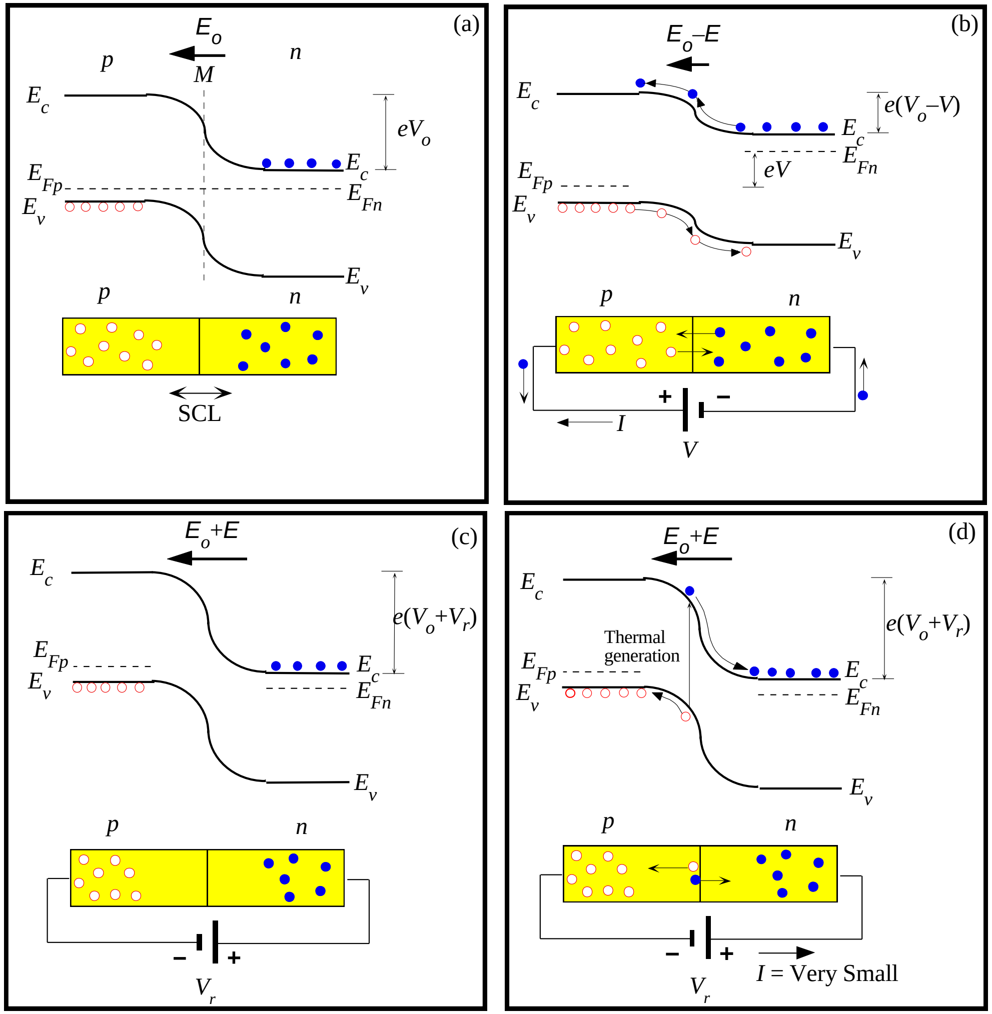
$$\tau_e = 1/BN_a$$

- Strong injections $\Delta n_p >> p_{po}$

$$\frac{\partial \Delta n_p}{\partial t} = B \Delta n_p \Delta p_p = B\left(\Delta n_p\right)^2$$

- LEDs modulated under high carrier injection have variable minority carrier concentrations which lead to distortion of the modulated light output

Pn Junction Band Diagram



Energy band diagrams for a *pn* junction under (a) open circuit, (b) forward bias and (c) reverse bias conditions. (d) Thermal generation of electron hole pairs in the depletion region results in a small reverse current.

Example: Direct Bandgap pn Junction

Symmetrical GaAs pn junction with a cross sectional area A= 1mm2. Compare the diode current due to minority carrier diffusion with the recombination current.

$$N_a(p_{side}) = N_d(n_{side}) = 10^{23}/m^3$$

$$B = 7.21\times10^{-16} m^3/s$$

$$ni = 1.8\times10^{-12}/m^3$$

$$\varepsilon_r = 13.2$$

$$\mu_h(n_{side}) = 250\frac{cm^2}{Vs}$$

$$\mu_e(p_{side}) = 5000\frac{cm^2}{Vs}$$

$$D_h = \mu_h k_B T/e$$

$$D_e = \mu_e k_B T/e$$

$$V_{applied} = 1V$$

$$T = 300K$$

$$\tau_{mean} \approx 1\times10^{-9} s$$

$$\tau_h = \tau_e = \frac{1}{BN_a} = 1.39\times10^{-8} s$$

due to time changes in carrier concentration

$$D_h = \mu_h k_B T/e = 6.46\times10^{-4} m^2/s$$

$$D_e = \mu_e k_B T/e = 1.29\times10^{-2} m^2/s$$

$$L_h = \sqrt{D_h\tau_h} = 3\times10^{-6} m$$

$$L_e = \sqrt{D_e\tau_e} = 1.34\times10^{-5} m$$

$$I_{so} = A\left(\frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_a}\right)en_i^2 = 6.13\times10^{-21} A$$

Reverse saturation current

$$I_{diff} = I_{so}\exp\left(\frac{eV}{k_B T}\right) = 3.9\times10^{-4} A$$

Forward diffusion current in pn junction

$$V_o = \frac{k_B T}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = 1.28V$$

Built in bias voltage in pn junction

$$W = \sqrt{\frac{e\varepsilon(N_a + N_d)(V_o - V)}{\varepsilon N_a N_d}} = 9\times10^{-8} m$$

(cont.)

$$W = \sqrt{\frac{e\varepsilon\left(N_a + N_d\right)\left(V_o - V\right)}{\varepsilon N_a N_d}} = 9\times10^{-8}\,m$$

$W_p = W_n = \tfrac{1}{2} W$ where $\tau_h = \tau_e \approx 1\times10^{-9}\,s$

For a symmetric diode, Wp = Wn
In equilibrium with stated mean carrier recombination time

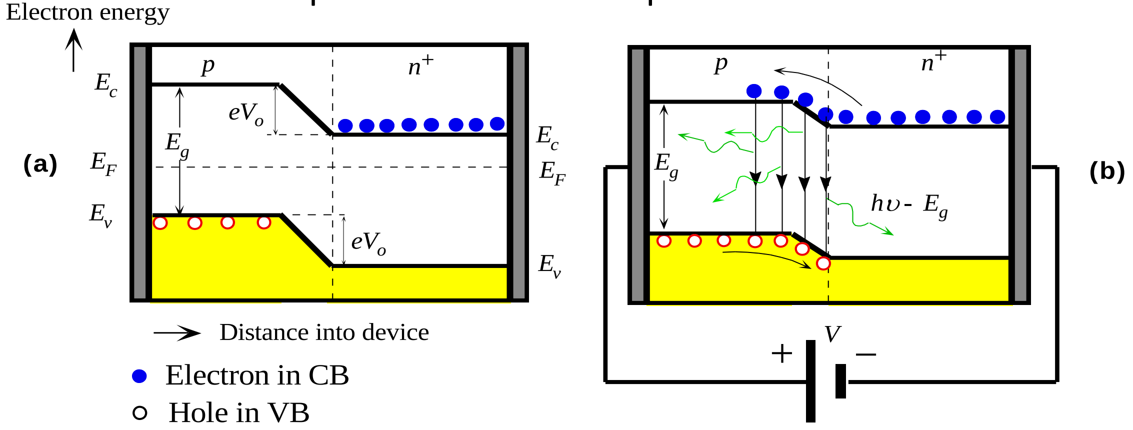
$$I_{ro} = \frac{Aen_i}{2}\left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h}\right) = 1.3\times10^{-12}\,A$$

$$I_{recom} \approx I_{ro}\exp\left(\frac{eV}{2k_B T}\right) = 3.3\times10^{-4}\,A$$

Intrinsic recombination value is nearly equal to diffusion current for doped pn junction

Principles of Light Emitting Diodes

- LEDs are pn junctions usually made from direct bandgap semiconductors. Ex GaAs
- Direct electron hole pair (EHP) recombination results in emission of a photon
- Photon energy is approximately equal to the bandgap energy $E_g \approx h\nu$
- Application of a forward bias drops the depletion region allowing more electrons into the p side of the device and increasing the probability of recombination in the depletion region
- The recombination zone is called the active region and is the volume in which photons are generated
- Light emission from EHP recombination as a result of minority carrier injection as shown here is called injection electroluminescence
- The statistical nature of this process requires that the p side be sufficiently narrow to prevent reabsorption of the emitted photons



(a) The energy band diagram of a p-n^+ (heavily n-type doped) junction without any bias. Built-in potential V_o prevents electrons from diffusing from n^+ to p side. (b) The applied bias reduces V_o and thereby allows electrons to diffuse, be injected, into the p-side. Recombination around the junction and within the diffusion length of the electrons in the p-side leads to photon emission.

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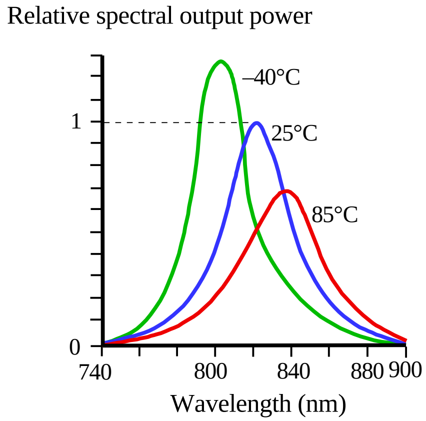

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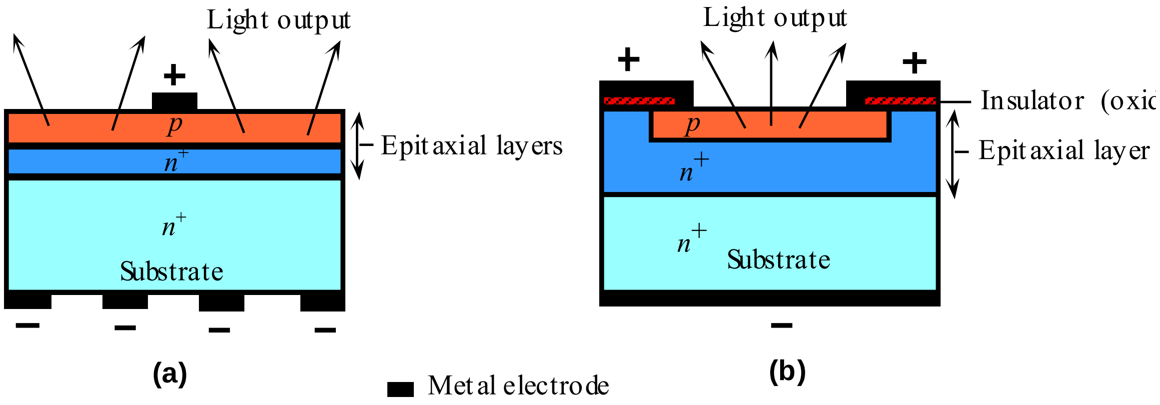
The output spectrum from AlGaAs LED. Values normalized to peak emission at 25°C.

LED Device Structures

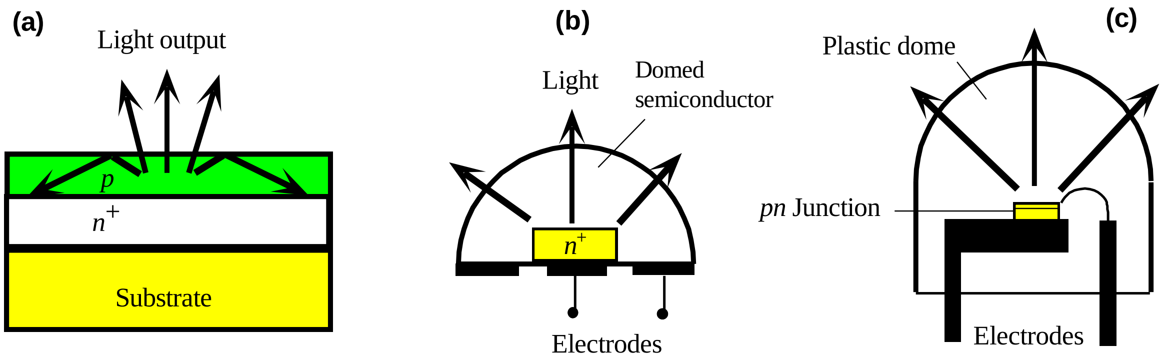
- LEDS are typically formed by epitaxially growing doped semiconductors layers on suitable substrate. The substrate is then essentially a mechanical support for the device
- However if the epi film and the substrate have mismatched lattice sizes then the lattice strain on the LED leads to crystalline defects that cause indirect recombination of EHPs and a loss of electroluminescence (photon emission). Thus the substrate is usually the same material as the epi layers
- To insure that recombination occurs on the p side, the n side is very heavily doped. Photons emitted toward the n side become absorbed or reflected back at the substrate interface.
- The use of segmented metal electrodes on the back promotes reflections



A schematic illustration of typical planar surface emitting LED devices. (a) *p*-layer grown epitaxially on an n^+ substrate. (b) First n^+ is epitaxially grown and then *p* region is formed by dopant diffusion into the epitaxial layer.

Optimizing Light Output vs. TIR

- Not all light reaching the semiconductor air interface escape the surface due to TIR
- For example the critical angle for TIR in GaAs-air is only 16°
- Thus engineers attempt to shape the surface of the semiconductor into a dome or hemisphere so that the light rays strike the surface at angles less than θ_c.
- The main drawback is the additional processing required to achieve these devices
- The common method is to seal a plastic dome to the LED surface that moderates the index change ($n_{GaAs} > n_{plastic} > n_{air}$)and increases the critical angle for TIR



(a) Some light suffers total internal reflection and cannot escape. (b) Internal reflections can be reduced and hence more light can be collected by shaping the semiconductor into dome so that the angles of incidence at the semiconductor-air surface are smaller than the critical angle. (b) An economic method of allowing more light to escape from the LED is to encapsulate it in a transparent plastic dome.

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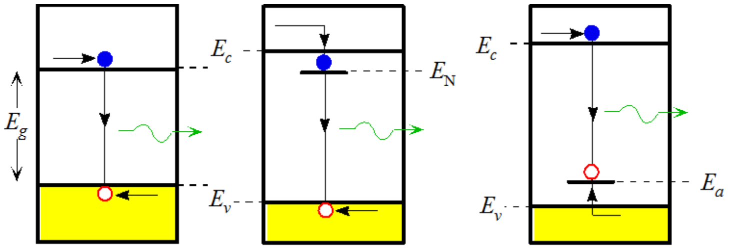
LED Materials

- Various direct bandgap semiconductor pn junctions can be used to make LEDs that emit in the red and infrared range
- III-V ternary alloys based on GaAs and GaP allow light in the visible spectrum
- Doping of Ga materials with different As, P, and Al ratios maintains the lattice constant while allowing for precise control of the bandgap (photon energy emitted)
- GaAsP with As concentrations greater than 0.55% are direct bandgap semiconductors
- GaAsP with As concentrations less than 0.55% are indirect bandgap semiconductors
- However, adding isoelectronic impurities such as N (same grp V as P) into the semiconductor to substitute for P atoms
  - Provides a trap for indirect ECP recombination and generates direct bandgap emission between the trap and the hole.
  - Reduces light efficiency and alters wavelength

$$GaAs_{1-y}P_y$$

$$y = 0.45$$

$$\lambda = 870nm$$



(a) $GaAs_{1-y}P_y$ $y < 0.45$ (b) N doped GaP (c) Al doped SiC

(a) Photon emission in a direct bandgap semiconductor. (b). GaP is an indirect bandgap semiconductor. When doped with nitrogen there is an electron trap at E_N. Direct recombination between a trapped electron at E_N and a hole emits a photon. (c) In Al doped SiC, EHP recombination is through an acceptor level like E_a.

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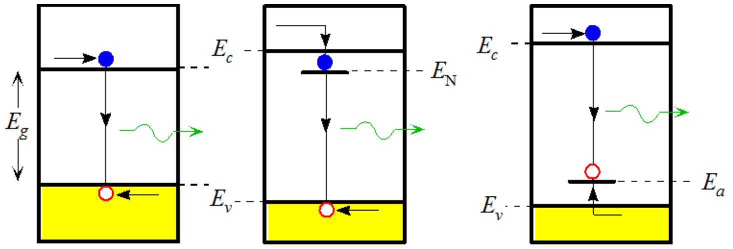
LED Materials (cont.)

- Blue LED materials
- GaN is a direct bandgap with Eg = 3.4 eV
- InGaN alloy has Eg = 2.7 eV (blue)
- Less efficient is Al doped SiC (indirect)
  - Aluminum captures holes and in a similar manner to N in GaAsPN materials and reduces the effective direct emission energy and efficiency of the device
- II-VI ZnSe semiconductors provide a direct bandgap blue emission

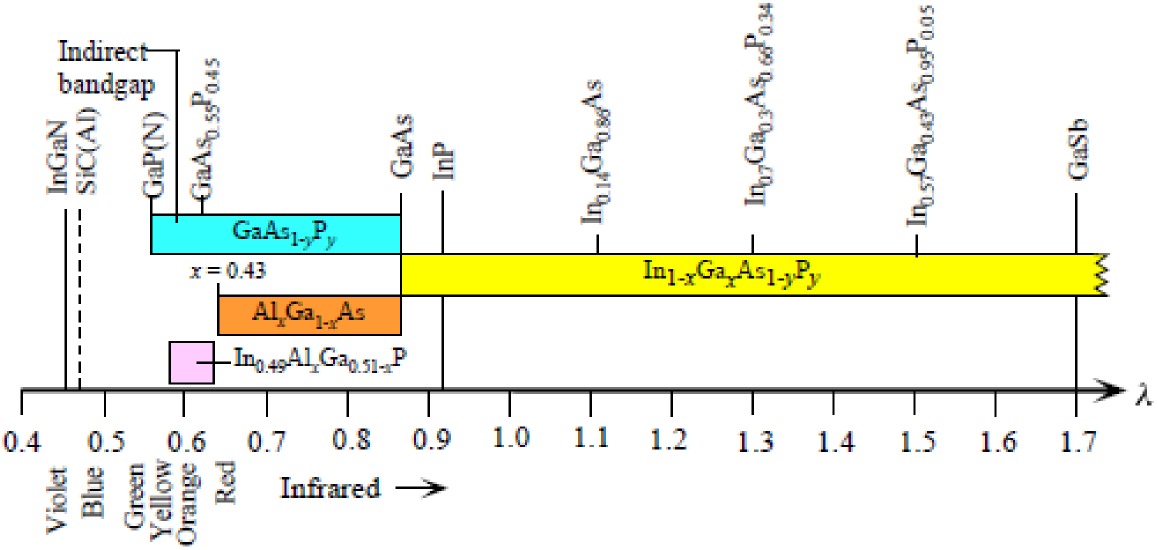


(a) $GaAs_{1-y}P_y$ $y < 0.45$ (b) N doped GaP (c) Al doped SiC

(a) Photon emission in a direct bandgap semiconductor. (b). GaP is an indirect bandgap semiconductor. When doped with nitrogen there is an electron trap at E_N. Direct recombination between a trapped electron at E_N and a hole emits a photon. (c) In Al doped SiC, EHP recombination is through an acceptor level like E_a.

LED Materials (cont.)

- Red and Infrared
- Three to four element alloys.
- $Al_{1-x}Ga_xAs$ with x<0.43 gives 870 nm
- Composition variances provide 650 – 870 nm
- $In_{1-x}Ga_xAl_{1-y}P_y$ can be varied to span 870 nm (GaAs) to 3.5 um (InAs)



Free space wavelength coverage by different LED materials from the visible spectrum to the infrared including wavelengths used in optical communications. Hatched region and dashed lines are indirect E_g materials.

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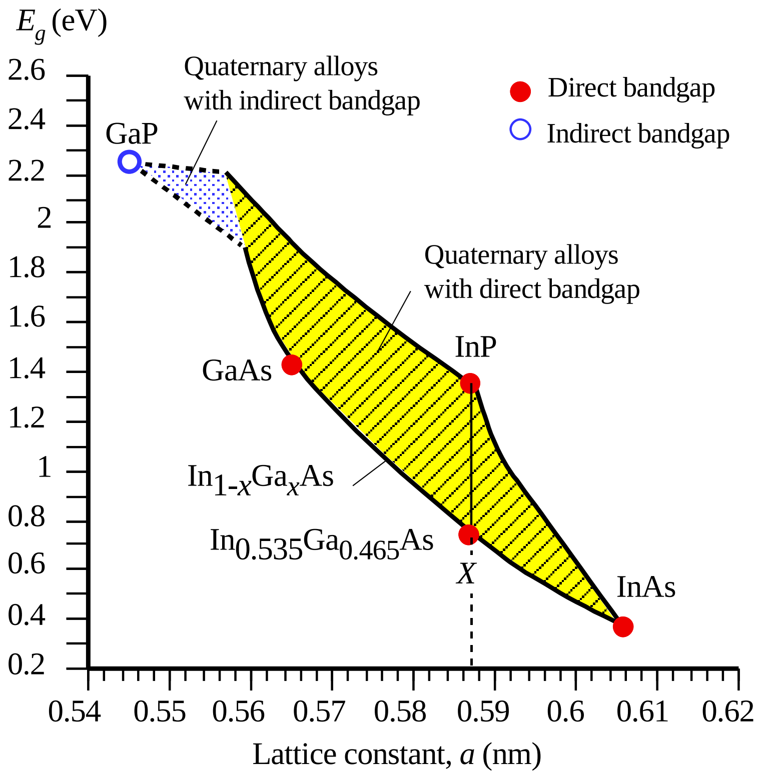
LED Materials (cont.)



Bandgap energy E_g and lattice constant a for various III-V alloys of GaP, GaAs, InP and InAs. A line represents a ternary alloy formed with compounds from the end points of the line. Solid lines are for direct bandgap alloys whereas dashed lines for indirect bandgap alloys. Regions between lines represent quaternary alloys. The line from X to InP represents quaternary alloys $In_{1-x}Ga_xAs_{1-y}P_y$ made from $In_{0.535}Ga_{0.465}As$ and InP which are lattice matched to InP.

Light Emitting Materials and Efficiency

- External efficiency
  - Quantifies the efficiency of conversion from electrical energy into emitted external optical energy

$$\eta_{external} = \frac{P_{out}(Optical)}{IV} \times 100\%$$

  - Typically less than 20% for direct bandgap semiconductors
  - Less than 1% for indirect bandgap semiconductors
- Efficiency has been increased by altering the shape, periodicity, and material interfaces within a device

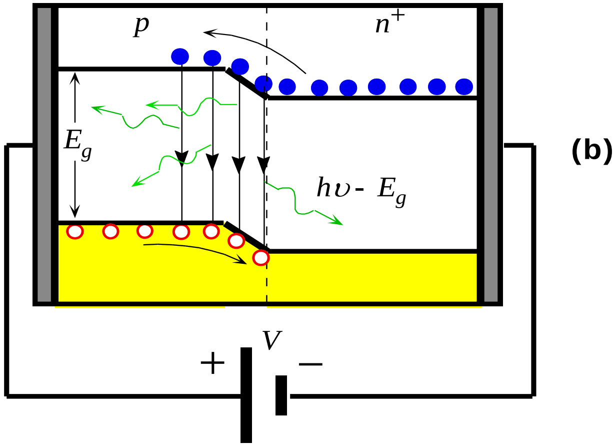
- **White light (wikipedia)**
- There are two primary ways of producing high intensity white-light using LEDs. One is to use individual LEDs that emit three primary colors[36] – red, green, and blue, and then mix all the colors to produce white light. The other is to use a phosphor material to convert monochromatic light from a blue or UV LED to broad-spectrum white light, much in the same way a fluorescent light bulb works.

	Color	Wavelength [nm]	Voltage [V]	Semiconductor Material
	Infrared	$\lambda > 760$	$\Delta V < 1.9$	Gallium arsenide (GaAs) Aluminium gallium arsenide (AlGaAs)
🟥	Red	$610 < \lambda < 760$	$1.63 < \Delta V < 2.03$	Aluminium gallium arsenide (AlGaAs) Gallium arsenide phosphide (GaAsP) Aluminium gallium indium phosphide (AlGaInP) Gallium(III) phosphide (GaP)
🟧	Orange	$590 < \lambda < 610$	$2.03 < \Delta V < 2.10$	Gallium arsenide phosphide (GaAsP) Aluminium gallium indium phosphide (AlGaInP) Gallium(III) phosphide (GaP)
🟨	Yellow	$570 < \lambda < 590$	$2.10 < \Delta V < 2.18$	Gallium arsenide phosphide (GaAsP) Aluminium gallium indium phosphide (AlGaInP) Gallium(III) phosphide (GaP)
🟩	Green	$500 < \lambda < 570$	1.9[29] $< \Delta V < 4.0$	Indium gallium nitride (InGaN) / Gallium(III) nitride (GaN) Gallium(III) phosphide (GaP) Aluminium gallium indium phosphide (AlGaInP) Aluminium gallium phosphide (AlGaP)
🟦	Blue	$450 < \lambda < 500$	$2.48 < \Delta V < 3.7$	Zinc selenide (ZnSe) Indium gallium nitride (InGaN) Silicon carbide (SiC) as substrate Silicon (Si) as substrate — (under development)
🟪	Violet	$400 < \lambda < 450$	$2.76 < \Delta V < 4.0$	Indium gallium nitride (InGaN)
🟪	Purple	multiple types	$2.48 < \Delta V < 3.7$	Dual blue/red LEDs, blue with red phosphor, or white with purple plastic
	Ultraviolet	$\lambda < 400$	$3.1 < \Delta V < 4.4$	diamond (C) Aluminium nitride (AlN) Aluminium gallium nitride (AlGaN) Aluminium gallium indium nitride (AlGaInN) — (down to 210 nm[30])
	White	Broad spectrum	$\Delta V = 3.5$	Blue/UV diode with yellow phosphor

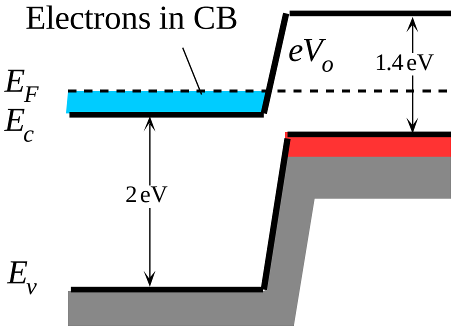
http://en.wikipedia.org/wiki/Light-emitting_diode

Homojunction vs. heterojunction LEDS

- pn junctions between two materials doped components of the same material (and thus the same bandgap) are called homojunctions
- Require narrow p type wells to channel photons out of the device prior to absorption
- Narrow channels lead to indirect recombination of electrons that reach defects located at the top surface of the p-type material, thereby reducing efficiency
- Junctions formed by two different bandgap semiconductor materials are called heterojunctions
  - Heterostructure devices (HD) are devices between two different bandgap semiconductors such as AlGaAs and GaAs



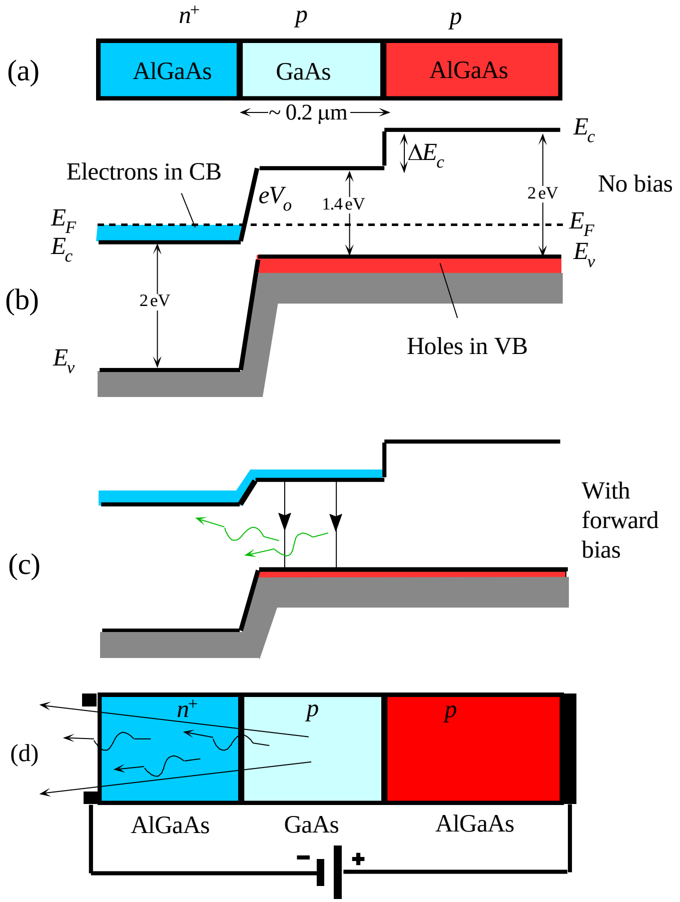
homojunction



heterojunction

Heterojunction High Intensity LEDS

- The refractive index, n, depends directly on the bandgap.
  - wide bandgap semiconductors have lower refractive indices.
  - we can engineer the dielectric waveguide within the device and channel the photons out from the recombination region
- Adding a double heterostructure (DH) to LEDs reduces radiationless recombination
  - Introduces a higher bandgap behind the pn junction that localizes the optical generation region.
  - The additiona p type region is known as a confining layer
  - Since the bandgap of AlGaAs is larger than that of GaAs emitted photons cannot get reabsorbed in the AlGaAs regions
  - Metal reflects light from the back side of the confining layer improving efficiency
  - n+ layer is used as topside of the device to reduce lattice defects in the active region and improve device efficiency
- DH LEDs are more efficient than homojunction LEDs



(a) A double heterostructure diode has two junctions which are between two different bandgap semiconductors (GaAs and AlGaAs)

(b) A simplified energy band diagram with exaggerated features. E_F must be uniform.

(c) Forward biased simplified energy band diagram.

(d) Forward biased LED. Schematic illustration of photons escaping reabsorption in the AlGaAs layer and being emitted from the device.

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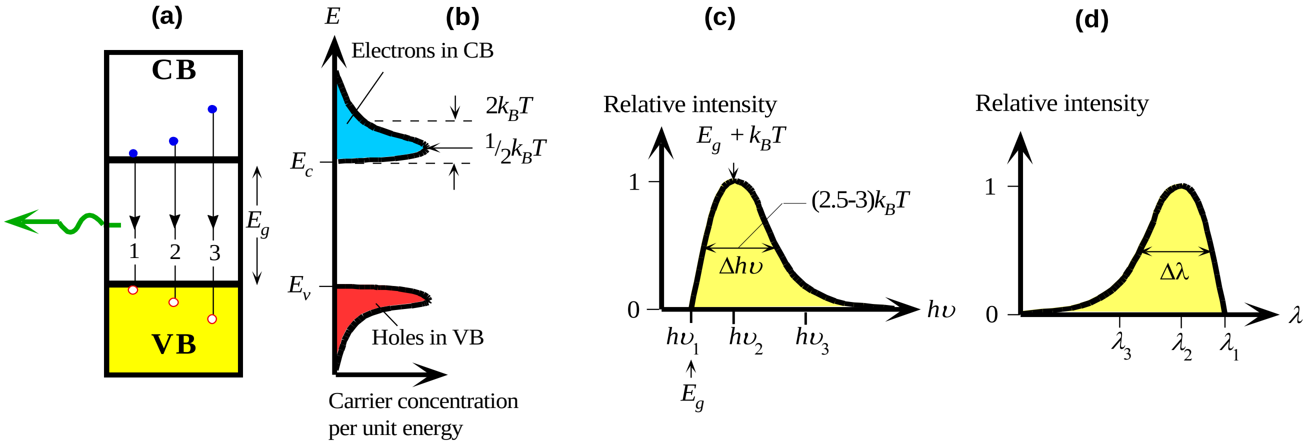
LED Device Characteristics



(a) Energy band diagram with possible recombination paths. (b) Energy distribution of electrons in the CB and holes in the VB. The highest electron concentration is $(1/2)k_BT$ above E_c. (c) The relative light intensity as a function of photon energy based on (b). (d) Relative intensity as a function of wavelength in the output spectrum based on (b) and (c).

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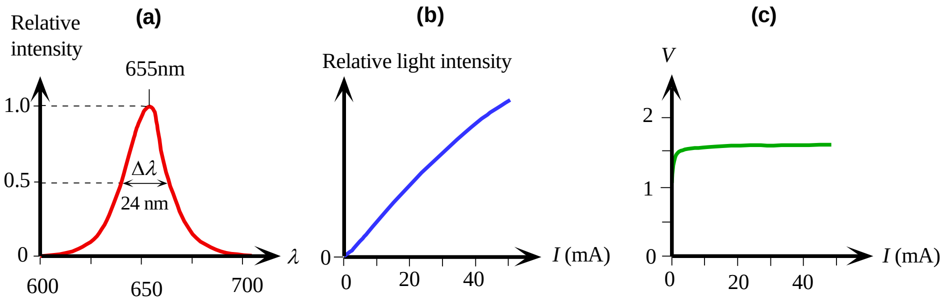
- CB conduction as a function of energy is asymmetrical with a peak at $1/2k_BT$ above E_c
- The energy spread of electrons is typically $2k_BT$
- Similar observation is made in the VB.
- Highest energy photon emissions have small probability
- Highest intensity comes from largest carrier concentration
- Intensity falls off again with carrier concentration near the CB band edge

LED Device Characteristics

- Spread of available carrier recombination probabilities generates a spread in optical wavelength emitted
- Linewidth of the spectral output is typically between 2.5 and 3.5k_BT
- Notice in figure a that the relative intensity does not match the probabilistic intensity plotted on the previous slide
- This is due to the fact that as heavily doped n type semiconductors used to create efficiency in active p-type regions create a donor band that overlaps the conduction band and lowers the effective output wavelength



(a) A typical output spectrum (relative intensity vs wavelength) from a red GaAsP LED. (b) Typical output light power vs. forward current. (c) Typical I-V characteristics of a red LED. The turn-on voltage is around 1.5V.

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- Turn on voltage is achieved at low operating currents and remains flat as current is increased
- Below the turn on voltage, no light is emitted
- The number of populated electrons in the p-type region CB increases and thus the relative light intensity also increases with increasing current

Example: LED Output Spectrum

- Width of the relative light intensity vs. photon energy spectrum of an LED is typically $3k_BT$. What is the linewidth, $\Delta\lambda_{1/2}$ in the output spectrum in terms of wavelength?

Emitted light is related to photon energy by

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_{ph}}$$

Differentiating, wavelength w.r.t photon energy:

$$\frac{d\lambda}{dE_{ph}} = -\frac{hc}{E_{ph}^2}$$

Small changes in the differential :

$$\Delta\lambda \approx \frac{hc}{E_{ph}^2}\Delta E_{ph}$$

Given the energy width of the output spectrum:

$$\Delta E_{ph} = \Delta(h\nu) \approx 3k_BT$$

Then substituting in terms of wavelength:

$$\Delta\lambda \approx \frac{hc}{E_{ph}^2}(3k_BT) = \frac{hc}{\left(\frac{hc}{\lambda}\right)^2}(3k_BT) = \lambda^2\left(\frac{3k_BT}{hc}\right)$$

That at various LED wavelengths one receives:

$$\lambda = 870nm \qquad \Delta\lambda = 47nm$$

$$\lambda = 1300nm \qquad \Delta\lambda = 105nm$$

$$\lambda = 1550nm \qquad \Delta\lambda = 149nm$$

Example: LED Output Wavelength Variation

- Consider a GaAs LED
- GaAs bandgap at 300K is 1.42 eV
- Derivative of the bandgap is

$$\frac{dE_g}{dT} = -4.5\times10^{-4}\frac{eV}{K}$$

- What is the change in emitted wavelength if the temperature is 10°C?

$$E_g = \frac{hc}{e\lambda} - \cancelto{}{\frac{k_B T}{e}}$$

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_g} = \frac{(6.626\times10^{-34})(3\times10^{8})}{(1.42\times1.6\times10^{-19})} = \frac{1.24\times10^{-6}\,eV*m}{1.42eV} = 875nm$$

$$\frac{d\lambda}{dT} = -\frac{hc}{E_g^2}\left(\frac{dE_g}{dT}\right) = -\frac{(6.626\times10^{-34})(3\times10^{8})}{(1.42\times1.6\times10^{-19})^2}(-4.5\times10^{-4})(1.6\times10^{-19})\frac{m}{K}$$

$$\frac{d\lambda}{dT} \approx 2.77\times10^{-10}\frac{m}{K} = 0.277\frac{nm}{K}$$

$$\Delta\lambda = \frac{d\lambda}{dT}\Delta T \approx 0.277\frac{nm}{K}\times10K \approx 2.8nm$$

- Since E_g decreases with temperature, the wavelength increases with temperature. This calculated change is within 10% of typical values for GaAs LEDs quoted in the literature

Example: InGaAsP on InP Substrate

- Ternary alloy $In_{1-x}Ga_xAs_yP_{1-y}$ is grown on an InP crystal substrate for LED applications
- The device requires sufficiently small lattice strain due to mismatch between the two crystals at the interface to allow for electroluminescence.
- Strain reduction in tern requires a value for y =2.2x
- The bandgap energy for the alloy in eV is given by the empirical relationship

$$E_g \approx 1.35 - 0.72y + 1.12y^2$$
$$0 \le x \le 0.47$$

- Calculate the composition of InGaAsP ternary alloys for peak emission at a wavelength of 1.3 um

$$E_g = \frac{hc}{e\lambda} - \frac{k_B T}{e} \qquad \text{For} \qquad \lambda = 1300nm; T = 300K$$

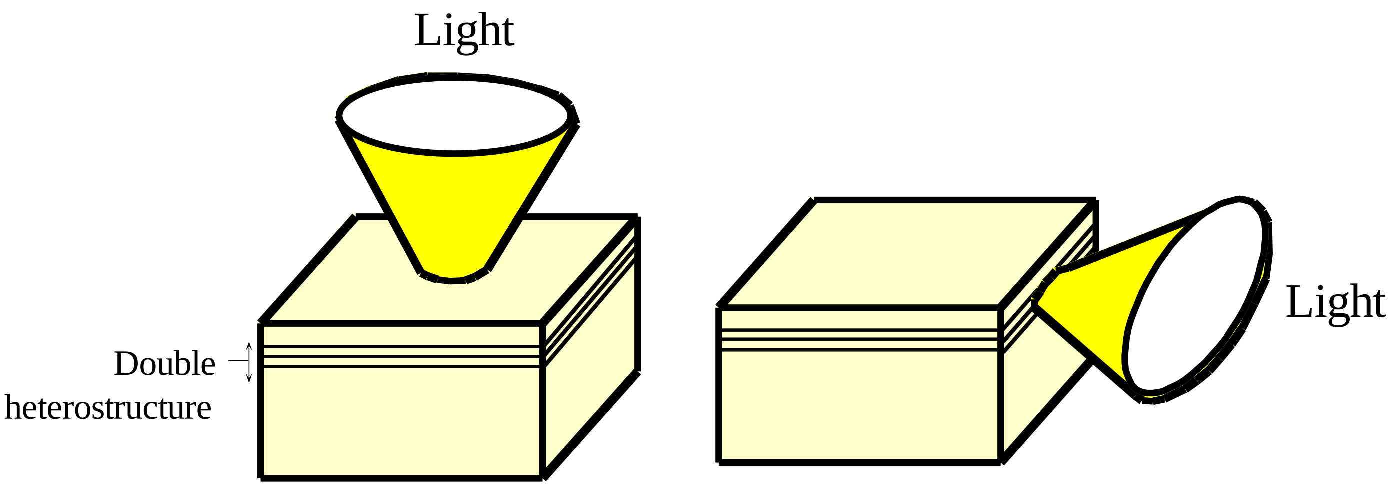
$$E_g = \frac{hc}{e\lambda} - \frac{k_B T}{e} = \frac{(6.626\times10^{-34})(3\times10^{8})}{(1.6\times10^{-19})(1.3\times10^{-6})} - 0.0259eV = 0.928eV$$

$$E_g = 0.928 = 1.35 - 0.72y + 1.12y^2$$
$$y = 0.66$$
$$x = 0.66/2.2 = 0.3$$

- Requires $In_{0.7}Ga_{0.3}As_{0.66}P_{0.34}$

LEDs for Optical Fiber Communication

- Two types of LEDs fabricated today
- Surface emitting LED (SLED)
- Edge emitting LED (ELED)



(a) Surface emitting LED (b) Edge emitting LED

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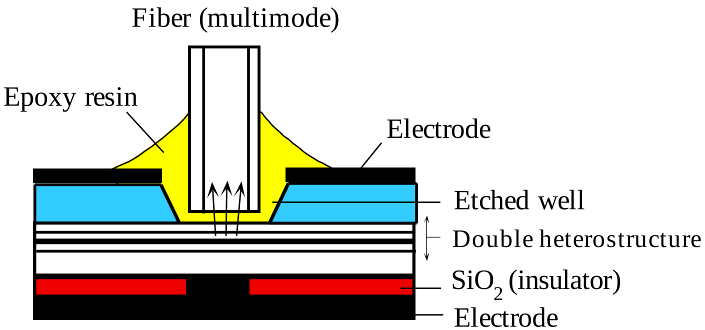
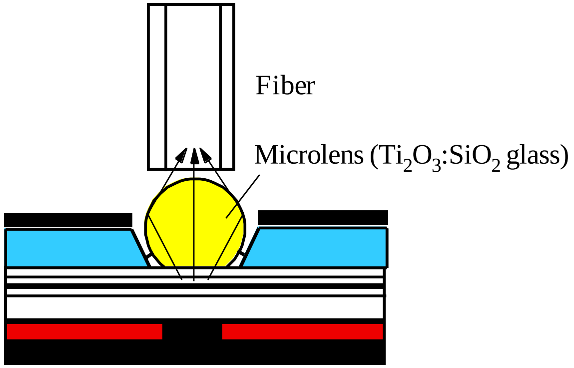
Surface Emitting LEDs

- Etch a well into a passive layer over a DH device and couple the flat end of a fiber as close as possible to the active region of the emitter as possible
- Epoxy bonding the blunt end of a fiber to the DH surface produces a Burrus device. Named after its origionator
- Remember that the epoxy is chosen in such a means as to reduce TIR of photons exiting the DH device
- Alternatively, truncated spherical microlenses with n =19.-2 can also be used to focus emitted light and guide it into a fiber. Lens is bonded to both fiber and LED with index matching glue



(a)

Light is coupled from a surface emitting LED into a multimode fiber using an index matching epoxy. The fiber is bonded to the LED structure.

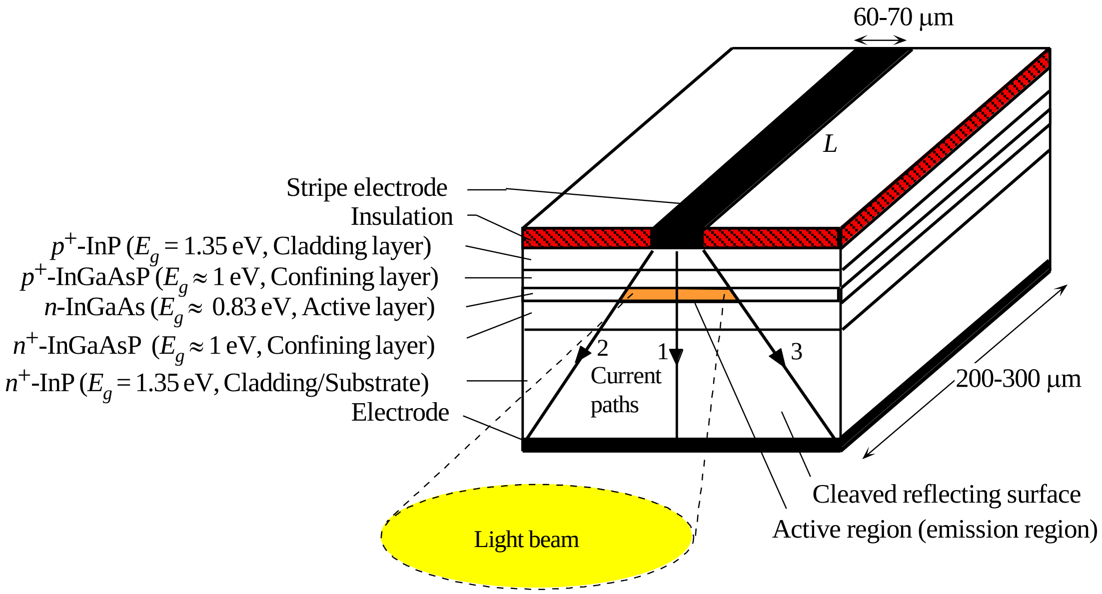


(b)

A microlens focuses diverging light from a surface emitting LED into a multimode optical fiber.

Edge Emitting LEDs

- ELEDs provide greater intensity and a more collimated beam
- Light is guided through the edge of a crystal using a slab created by three stacked bandgap materials surrounding one small one
- Recall that the larger the bandgap, the lower the dielectric
  - Thus the active region comprised of InGaAs with a bandgap of 0.83 eV has n1 <n2
  - The cladding regions hare InGaAsP with a bandgap of 1eV
- i.e. the active photon generation region (p side of the pn junction) is used as the transmission slab
- Recombination of injected carriers occurs is confined to the slab by confining layers with wider bandgaps
- Light spreads along the waveguide by field generated between the voltage biased regions top and bottom
- Diode is typically diced and polished to create a smooth transmission edge and then coupled to a graded index lens that is bound to the end of an optical fiber



Schematic illustration of the the structure of a double heterojunction stripe contact edge emitting LED

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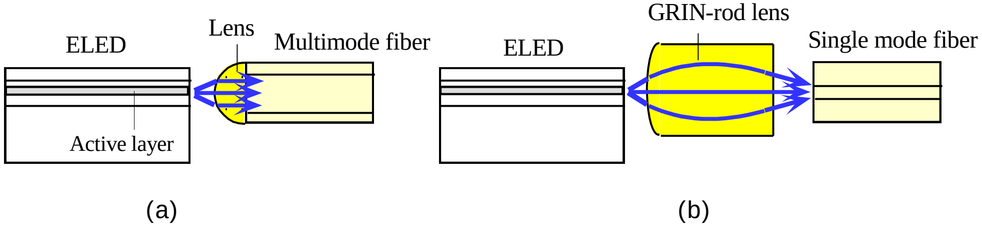

Light from an edge emitting LED is coupled into a fiber typically by using a lens or a GRIN rod lens.

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