Optoelectronics
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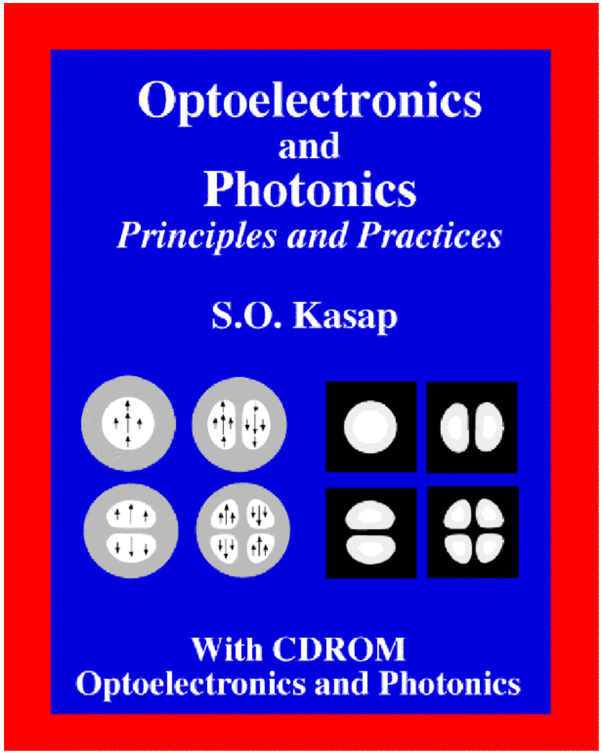
Chapter 2: Dielectric Waveguides

- **2.1 Symmetric Planar Dielectric Slab Waveguide**
  - A. Waveguide Condition
  - B. Single and Multimode Waveguides
  - C. TE and TM Modes
- **2.2 Modal and Waveguide Dispersion in the Planar Waveguide**
  - A. Waveguide Dispersion Diagram
  - B. Intermodal dispersion
- **Chapter 2 Homework Problems: 1-7**



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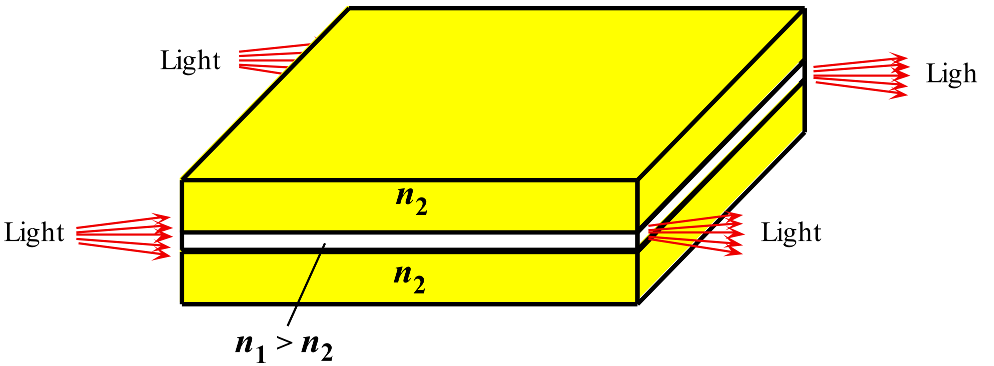
Slab Waveguides

- Consider a dielectric slab of thickness 2a and refractive index n_1
- Let the dielectric be sandwiched between two semi-infinite regions of index n_2
- Note that $n_2 < n_1$
- The high refractive index is called the core
- The low refractive index is called the cladding



A planar dielectric waveguide has a central rectangular region of higher refractive index n_1 than the surrounding region which has a refractive index n_2. It is assumed that the waveguide is infinitely wide and the central region is of thickness 2 a. It is illuminated at one end by a monochromatic light source.

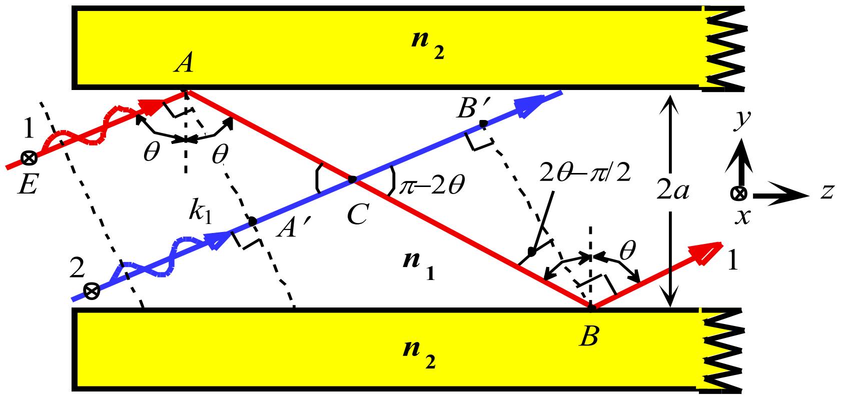
- Only a very thin light beam with a diameter much less than the slab thickness, 2a, will make it into the dielectric slab to reflect off of the cladding.
- The remaining light used to illuminate the structure is “lost”
- Also note that for ease of calculation we will use light that enters the slab waveguide from another medium of index n_1.
- Mode coupling is required to assess the amount of light entering the waveguide from a generic medium of n that will reflect and transmit off the surface of n1 at the front of the slab

Wave Propagation in Slab Waveguides

- If TIR occurs, then light entering the waveguide easily propagates along in a zigzag fashion
- The zigzag pattern generated by reflection propagates in phase leading to constructive interference within the waveguide
- Light entering the waveguide or reflecting out of phase generates destructive interference and cancels out the propagation amplitude of the EM field.
- Let us suppose that $k1 = kn_1 = 2\pi n_1/\lambda$
- For constructive interference, the phase difference between the two points A and C in the diagram below must be multiples of 2π
- For constructive interference:

$$k_1[2d\cos\theta] - 2\phi = 2\pi m$$

- Only certain angles of θ and ϕ satisfy this equation for a given integer multiple, m (mode number)
- However, ϕ depends on θ and the polarization state of the incident waves
- Therefore for each m, there will be only 1 allowable θ_m and ϕ_m



Two arbitrary waves 1 and 2 that are initially in phase must remain in
after reflections. Otherwise the two will interfere destructively and ca
other.

Waveguide Condition

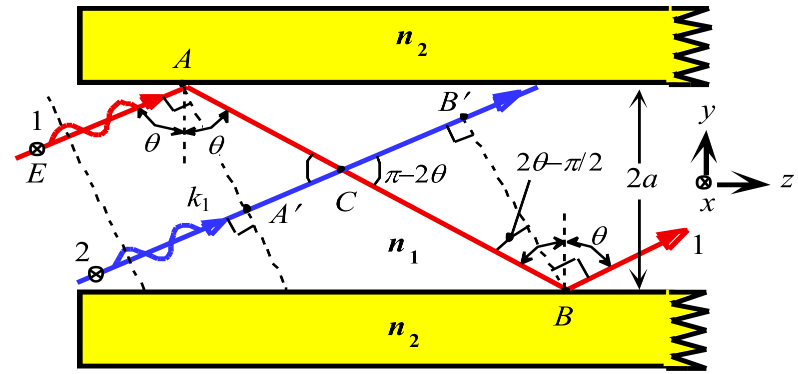
- If we divide the equation for constructive interference by 2 and rewrite, then we have the waveguide condition:

$$\frac{2\pi n_1(2a)}{\lambda}\cos\theta_m - \phi_m = \pi m$$

- Where the constructed phase of the wave packet, ϕm is a function of the incidence angle, θm
- This condition is generic for different waveguide shapes, incidence angles, and incident wavelengths.
- Remember: both rays must initially start in phase with one another and remain so after reflection or they will destructively interfere and prevent propagation
- Field in the waveguide

$$E = 2E_m(y)\cos(\omega t - (k_1 \sin\theta_m)z)$$

- $E_m(y)$ is the mode of propagation



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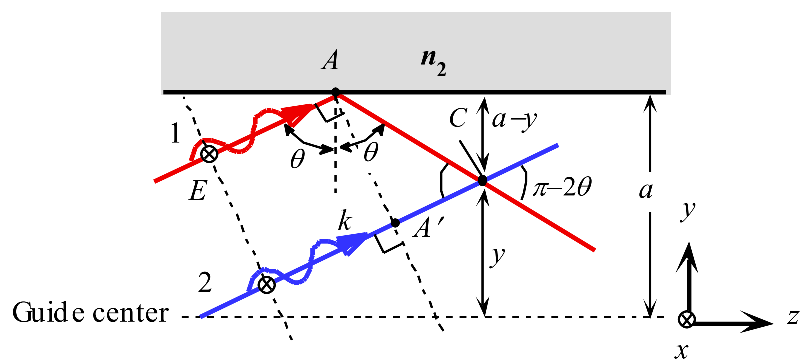
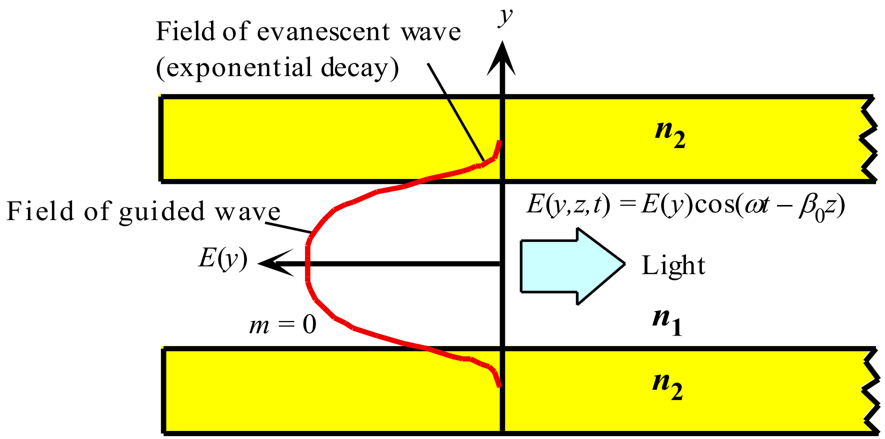
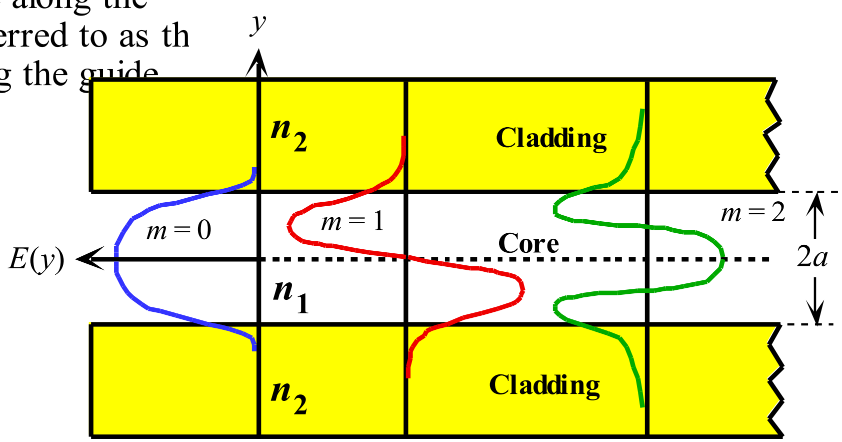


Interference of waves such as 1 and 2 leads to a standing wave pattern along the *y* direction which propagates along *z*.

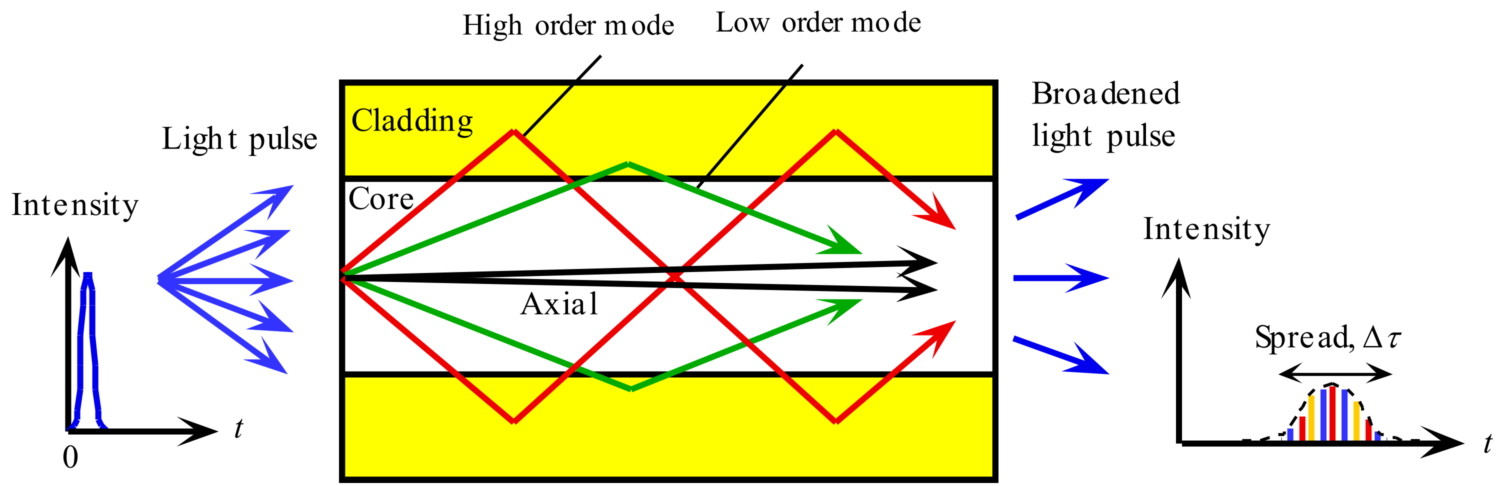
Allowed modes in the waveguide



The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest θ. It is often referred to as th glazing incidence ray. It has the highest phase velocity along the guide.



The electric field patterns of the first three modes ($m = 0, 1, 2$) traveling wave along the guide. Notice different extents of field penetration into the cladding.



Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at differer group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

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Single and Multimode Waveguides

- By imposing both TIR and the waveguide condition on the solution for waveguide propagation, we find that only a certain number of modes are allowed in the waveguide
- From

$$\frac{2\pi n_1(2a)}{\lambda}\cos\theta_m - \phi_m = \pi m$$

  we can find an expression for $\sin(\theta_m)$
- Applying the TIR condition,

$$\sin\theta_m > \sin\theta_c$$

- The mode number, m, must satisfy

$$m \le \frac{(2V-\phi)}{\pi}$$

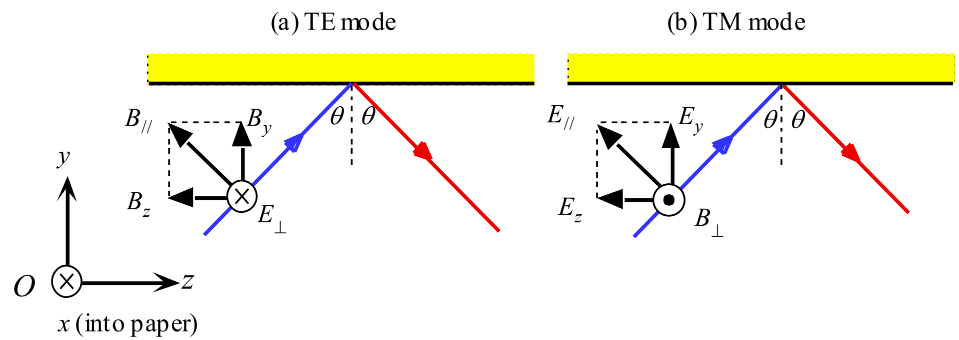
- The V-number, V, also called the normalized thickness or normalized frequency is defined by

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

- Note: the term thickness is more common for planer waveguides
- The 2a in the term refers to the waveguide geometry, and thus will change with the shape of the waveguide
- Question how does one get V such that only a single mode of propagation exists?
  - At grazing incidence $\theta_m = 90^o$ and $\phi_m = \pi$
  - Solving for V as a function of m
  - At V $<\pi/2$ only the m=0 mode propagates
  - At V $= \pi/2$ gives the free space cut-off wavelength. Above this wavelength, only the single mode propagation exists

TE and TM Modes

- All discussion up to now have assumed a propagating wave
- However we have two types of propagating waves that generate different phase changes upon reflection and refraction
- So let us now consider TE modes perpendicular to the cross section of the slab: $E_{\perp}=E_x$
- And TM modes parallel to the cross section of the slab: $E_{||} = E_y + E_z$
  - It is interesting that Ez exist along the direction of propagation. It is apparent that Ez is a propagating longitudinal electric field. In free space this is IMPOSSIBLE for such a field to exists, however in a waveguide the interference allows such a phenomenon
  - Note that the same occurs for B in the TE mode
- Because the phase change that accompanies TIR depends on polarization yet is negligible for n_1-n_2<<1, the waveguide condition and the cut-off condition can be taken to be identical for both TE and TM



Possible modes can be classified in terms of (a) transelectric field (TE and (b) transmagnetic field (TM). Plane of incidence is the paper.

Waveguide Modes

- Planer waveguide: 2a=20um, N1 = 1.455, N2 = 1.440, λ = 900nm ($9\text{x}10^{-7}$m)
- Using waveguide equation and TIR for the TE mode:

$$\tan\left(\frac{1}{2}\phi_m\right) = \frac{\sqrt{\sin^2\theta_m - \left(n_2/n_1\right)^2}}{\cos\theta_m}$$

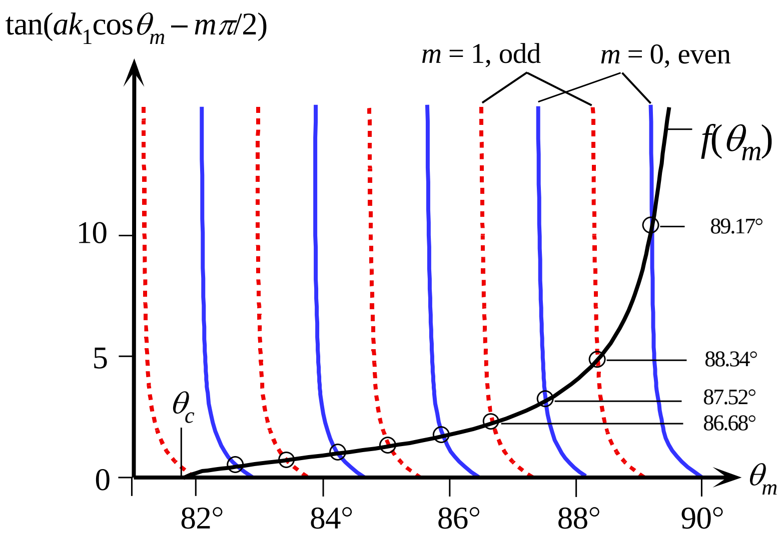
- Using a graphical solution, find the angles for all of the modes.
- Consider:

$$k_1[2a\cos\theta_m] - \phi_m = \pi m$$

$$\tan(ak_1\cos\theta_m - \pi m/2) = \frac{\sqrt{\sin^2\theta_m - \left(n_2/n_1\right)^2}}{\cos\theta_m} = f(\theta)$$

- The left hand side reproduces itself for m = 0,2,4,... and becomes a cot function for odd m

Note: $$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$



Modes in a planar dielectric waveguide can be determined by plotting the LHS and the RHS of eq. (11).

Skin depth of wave into n_2

$$\alpha_m = \frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}$$

$$\delta_{m=0} = 1/\alpha_{m=0} = 6.91\times10^{-7}\,m$$

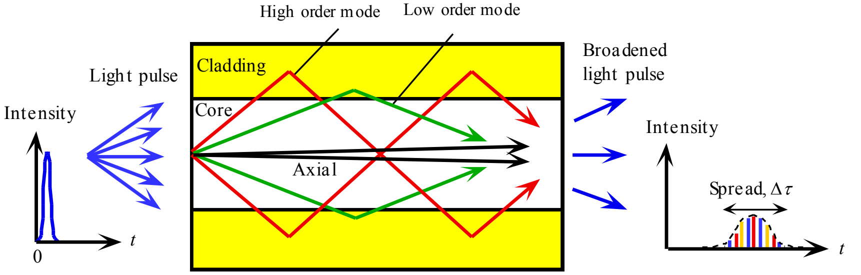
$$\delta_{m=9} = 1/\alpha_{m=9} = 38.3\times10^{-7}\,m$$

Dispersion Diagram

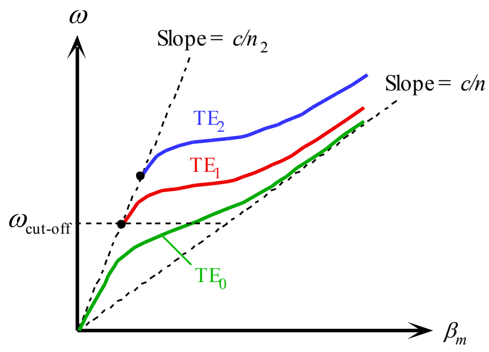
- Propagating modes in a waveguide are determined by the waveguide condition
- Each choice of m = 0… m_{max} results in one distinct and only one possible propagation constant, $\beta = k_1 \sin\theta_m$
- Also each mode propagates along a different propagation constant even if the initial wave is a monochromatic plane wave
- The top left figure gives the impression that lower order modes travel faster in the waveguide
- This is not exactly the case. Group velocity defines the speed of the wave packet and is dependent on mode

$$v_g = \frac{\partial\omega}{\partial\beta}$$

- Furthermore higher modes penetrate more into the cladding where the refractive index is smaller yielding higher velocities over short time intervals



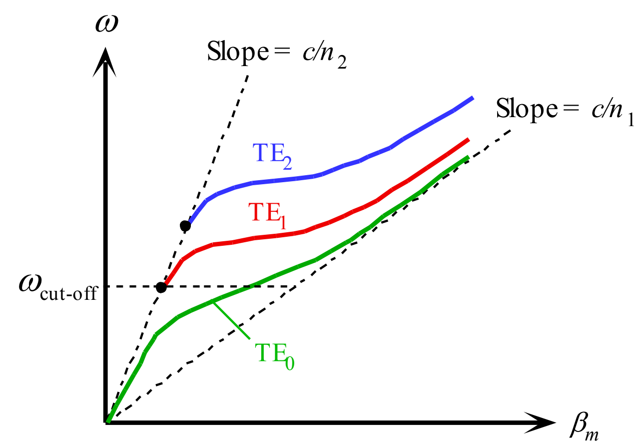
Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at differen group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.



Schematic dispersion diagram, ω vs. β for the slab waveguide for various TE_m modes. $\omega_{cut-off}$ corresponds to $V = \pi/2$. The group velocity v_g at any ω is the slope of the ω vs. β curve at that frequency.

Dispersion Diagram

- So group velocity of a give mode is a function of the light frequency and the waveguide properties
- So even if the refractive index was frequency independent, then the group velocity would still depend on the waveguide structure
- For a given waveguide we can use the refractive indices, and the slab thicknesses to calculate the propagation constant for each frequency to obtain a plot of ω vs. β
- This plot is called the dispersion diagram
- The slope of the plot is the group velocity
- All allowable modes generate group velocities between the ideal slopes defined by the speed of light divided by the refractive indices of the waveguide
- The cut-off frequency corresponds to single mode propagation



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Modal Dispersion

- In multimode operation, the lowest mode has the slowest group velocity
- The highest group velocity is in the highest mode where a good portion of the field is carried by the cladding
- Modes therefore take different times to travel through the waveguide. This is called modal dispersion
- The modal dispersion is the time required for an input plane wave to completely exit the waveguide

$$\Delta\tau = \frac{L}{v_g}$$ L = length of the waveguide

- The lowest order mode has a group velocity v_{gmin}=c/n1.
- The highest order mode has a group velocity almost equal to =c/n2
- Thus in the most simple terms, the modal dispersion is

$$\Delta\tau \approx \frac{n_1 - n_2}{c}$$