Optoelectronics
EE/OPE 451, OPT 444
Fall 20XX Section 1:

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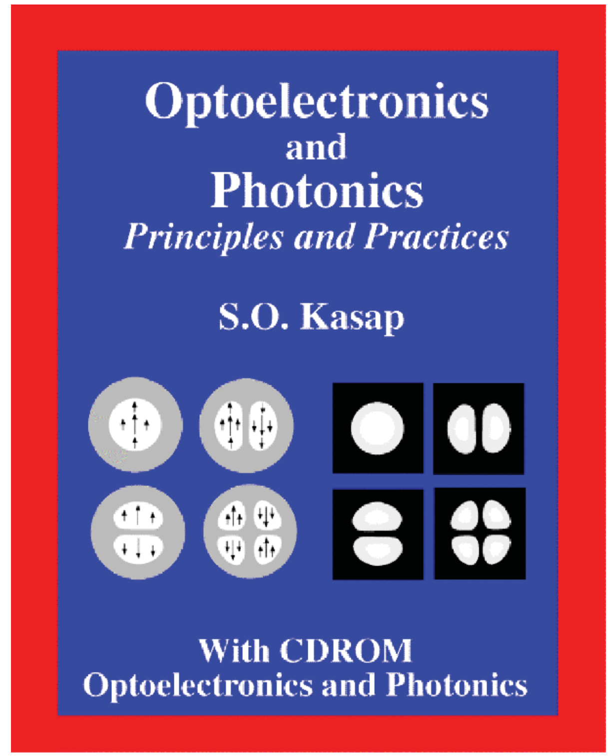
Course Textbook and Topics Covered

- Ch. 1: Wave Nature of Light
- Ch. 2: Dielectric Waveguides
  - Sections 2.1 and 2.2 only
- Ch. 3: Semiconductor Science and Light Emitting Diodes
- Ch. 4: Stimulated Emission Devices
  - Sections 4.9- 4.14 only
- Ch. 5: Photodetectors
- Ch. 7: Photovoltaic Devices
- Ch. 8: Polarization and Modulation of Light



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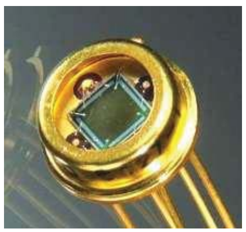
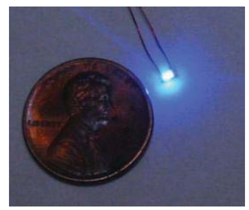
http://photonics.usask.ca/

Introduction to Optoelectronics



www.udt.com

- Definition of Optoelectronics
  - Sub field of photonics in which voltage driven devices are used to create, detect, or modulate optical signals using quantum mechanical effects of light on semiconductors materials
  - Examples of optoelectronic devices
    - Photodiode
    - LED
    - DFB LASER
    - VSCEL
    - Semiconductor Photomultipliers
    - Integrated Optical Circuit



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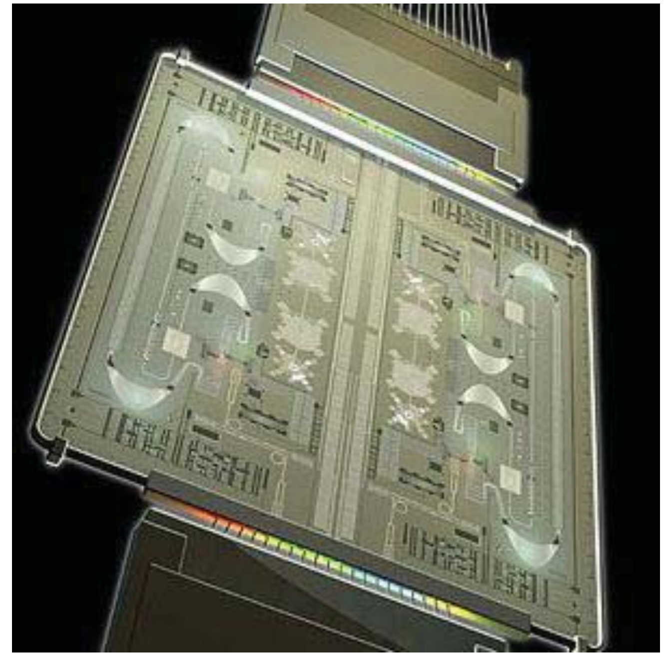
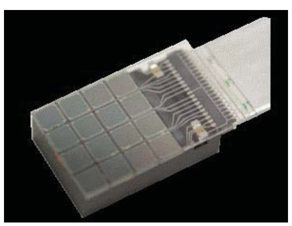


240 Optical components on a chip – Infinera Wavelength Multiplexer2007



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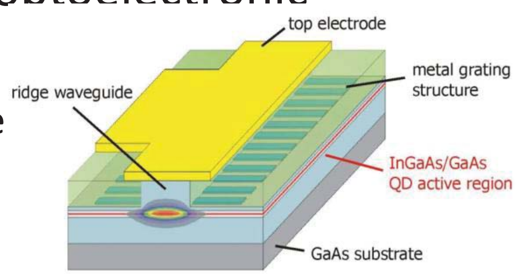

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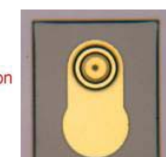

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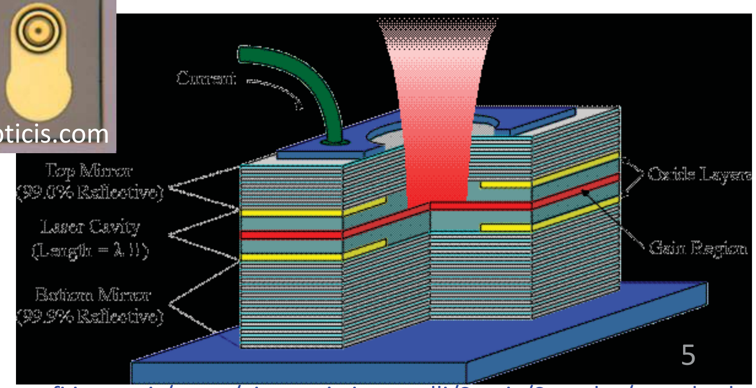
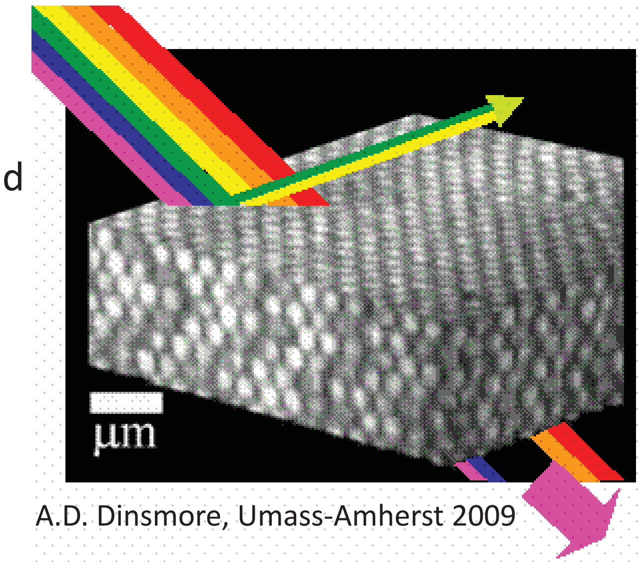


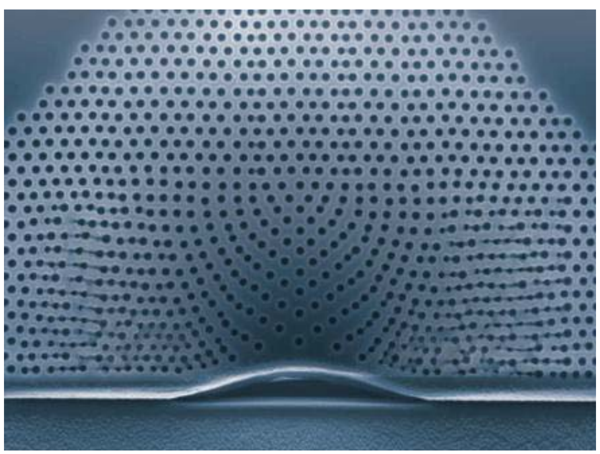
http://www.fi.isc.cnr.it/users/giovanni.giacomelli/Semic/Samples/samples.html

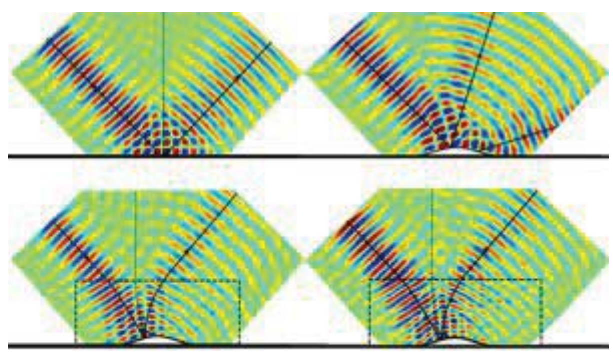
What is Photonics?

- Broader topic than Optoelectronics alone
- Study of wave/particle duality devices in optics.
  - Study of optical devices that utilize photons instead of the classical electromagnetic wave solution.
  - emision, detection, modulation, signal processing, transmission and amplification of light based on QM and Solid State principles
- State of the art is the development of light modulation through periodic structure

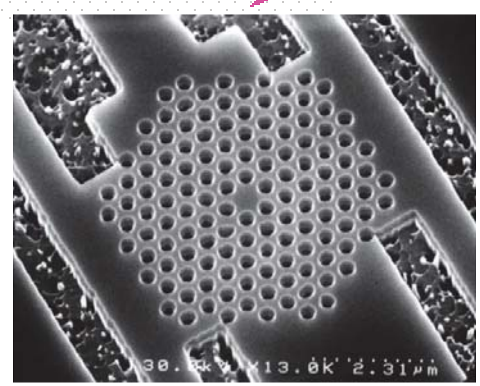


A.D. Dinsmore, Umass-Amherst 2009





L.H. Gabrielli, Nature Photonics 3, 461-463 (2009).

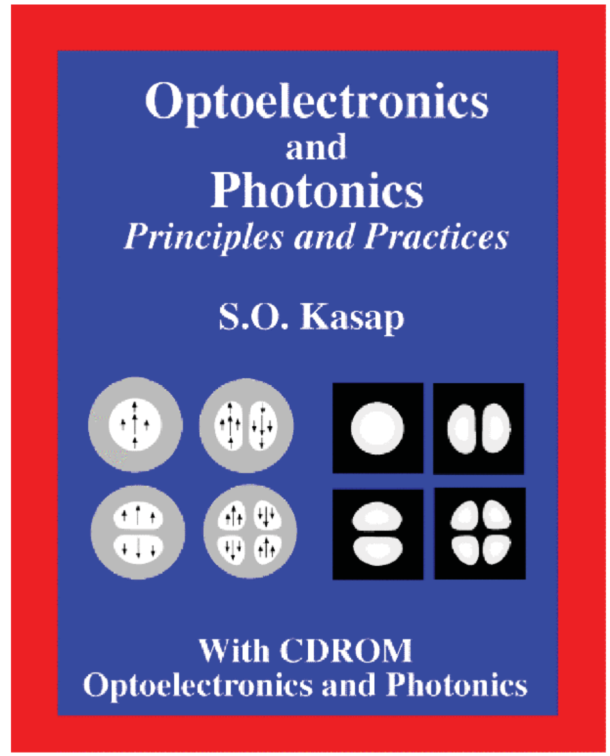


J. Obrien , USC Photonics Group 2009

Ch. 1: Wave Nature of Light

- **1.1 Light Waves in a Homogeneous Medium**
  - A. Plane Electromagnetic Wave
  - B. Maxwell's Wave Equation and Diverging Waves
- **1.2 Refractive Index**
- **1.3 Group Velocity and Group Index**
- **1.4 Magnetic Field, Irradiance and Poynting Vector**
- **1.5 Snell's Law and Total Internal Reflection (TIR)**
- **1.6 Fresnel's Equations**
  - A. Amplitude Reflection and Transmission Coefficients
  - B. Intensity, Reflectance and Transmittance
- **1.7 Multiple Interference and Optical Resonators**
- **1.8 Goos-Hänchen Shift and Optical Tunneling**
- **1.9 Temporal and Spatial Coherence**
- **1.10 Diffraction Principles**
  - A. Fraunhofer Diffraction
  - B. Diffraction grating

- **Chapter 1 Homework Problems: 1,2, 4-17**



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Wave Nature of Light

- Plane Electromagnetic Wave
  - Treated as time varying electric E_x and magnetic, H_y, fields
  - E and H are always perpendicular to each other
  - Propagate through space in the *z* direction
  - Simplest representation is a sinusoidal wave (or a Monochromatic plane wave)

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$

Where E_x =electric field at position z at time t,

E_o =amplitude of the electric field

k = wave number ($k=2\pi/\lambda$)

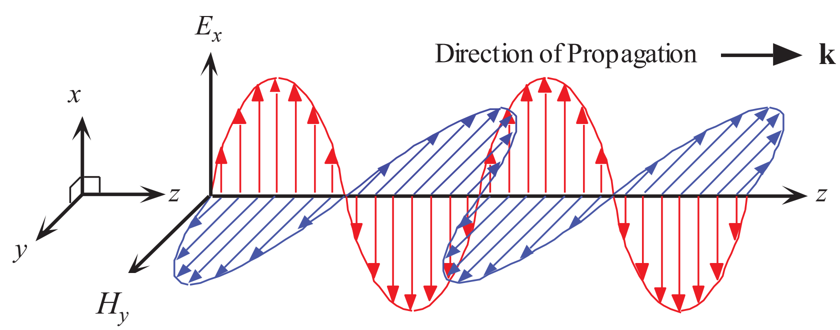
λ = wavelength

ω = angular frequency

ϕ_o = phase constant

$(\omega t - kz + \phi_o)$ = ϕ = phase of the wave

  - A planer surface over which the phase of the wave is constant is called a wavefront

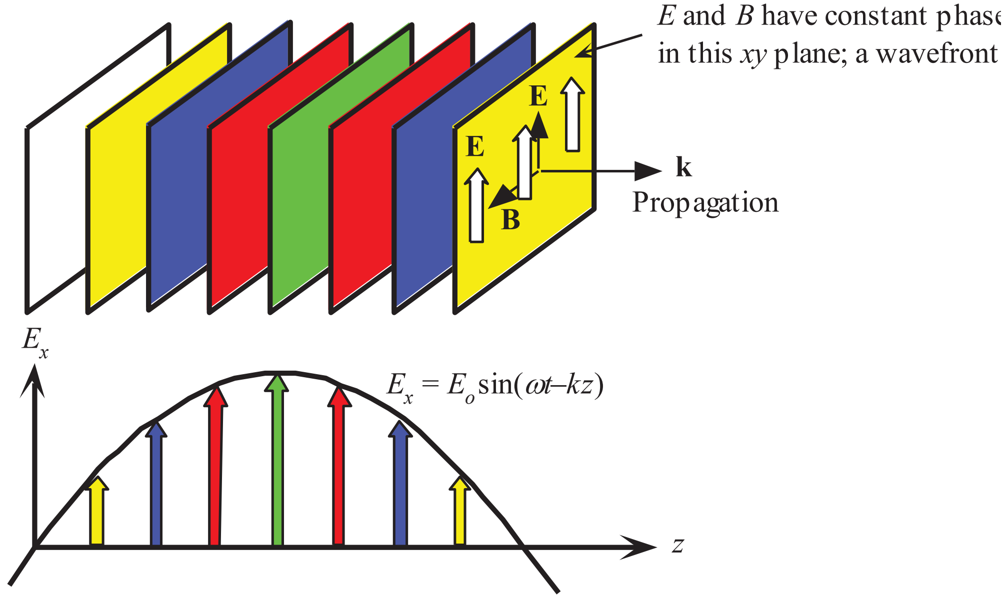


An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, *z*.

Wave Fronts

– A planer surface over which the phase of the wave is constant is called a wavefront

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$



A plane EM wave travelling along z, has the same E_x (or B_y) at any point in a given xy plane. All electric field vectors in a given xy plane are therefore in phase. The xy planes are of infinite extent in the x and y directions.

Optical Field

- Use of *E* fields to describe light
  - We know from Electrodynamics that a time varying *H* field results in time varying *E* fields and vise versa
  - Thus all oscillating *E* fields have a mutually oscillating *H* field perpendicular to both the *E* field and the direction of propagation
  - However, one uses the *E* field rather than the *H* field to describe the system
    - It is the E field that displaces electrons in molecules and ions in the crystals *at optical frequencies* and thereby gives rise to the polarization of matter
    - Note that the fields are indeed symmetrically linked, but it is the E field that is most often used to characterize the system

Optional Plane Wave Representations

- 1-D solution

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$
$$E_x = E(z,t) = E_o \cos(\omega t - kz + \phi_o)$$

Since $\cos(\phi) = \mathrm{Re}[e^{i\phi}]$

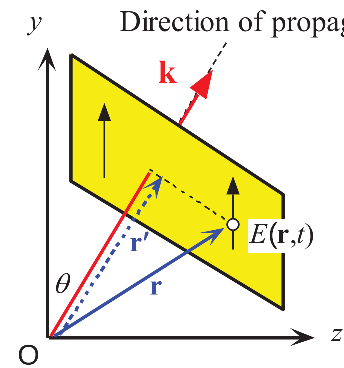
$$E(z,t) = \mathrm{Re}[E_o e^{j\phi_o} e^{j(\omega t - kz)}]$$
$$E(z,t) = \mathrm{Re}[E_c e^{j(\omega t - kz)}]$$

- General solution

$$E = E(r,t) = E_o \cos(\omega t - kz + \phi_o)$$
$$E(\vec{r},t) = \mathrm{Re}[E_o e^{j\phi_o} e^{j(\omega t - \vec{k}\cdot\vec{r})}]$$
$$E(\vec{r},t) = \mathrm{Re}[E_c e^{j(\omega t - \vec{k}\cdot\vec{r})}]$$

Where $\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$

k = wave vector whose magnitude is $2\pi/\lambda$



A travelling plane EM wave along a direction **k**

Phase Velocity

- For a plane wave, the relationship between time and space for any give phase, ϕ, is constant

$$(\omega t - kz + \phi_o) = \phi = \text{constant}$$

- During any time interval, δt, this constant phase (and hence maximum value of *E*) moves a distance δr.
- The phase velocity of the wave is

$$v = \frac{\delta r}{\delta t} = \frac{dr}{dt} = \frac{\omega}{|k|} = \frac{2\pi v}{2\pi / \lambda} = v\lambda$$

- The phase difference, $\Delta\phi$ at any given time between two points on a wave that are separated by a distance Δz is

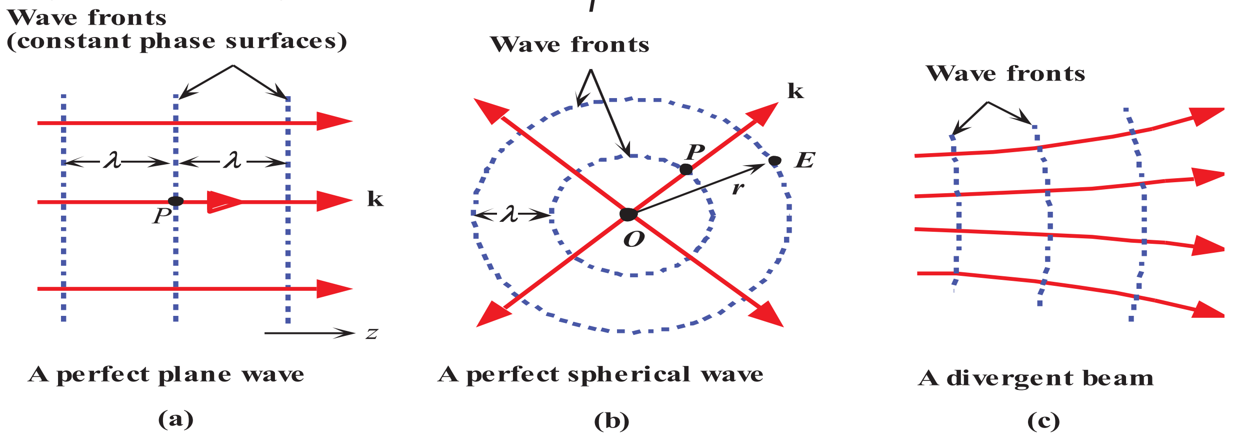
$$\Delta\phi = k\Delta z = \frac{2\pi\Delta z}{\lambda}$$ Since ωt is the same for each point

- The fields are said to be in phase if he phase difference is zero if $\Delta\phi = 0$ or 2π multiples of $k\Delta z$ with regards to the initial value.

Maxwell’s Wave Eqn. and Diverging Waves

$$\nabla^2 E = \varepsilon\mu \frac{\partial^2 E}{\partial t^2} = \varepsilon_r \varepsilon_o \mu_o \frac{\partial^2 E}{\partial t^2}$$

$$E = \frac{A}{r}\cos(\omega t - kr)$$



Examples of possible EM waves

- Plane waves emanate from surface of relatively infinite size
  - Wavefronts are planes
- A spherical wave emanates from a EM point source whose amplitude decays with distance
  - Wavefronts are spheres centered at the point source
- A divergent beam emanates from a defined surface
  - The optical divergence refers to the angular separation of the wave vectors on a given wavefront

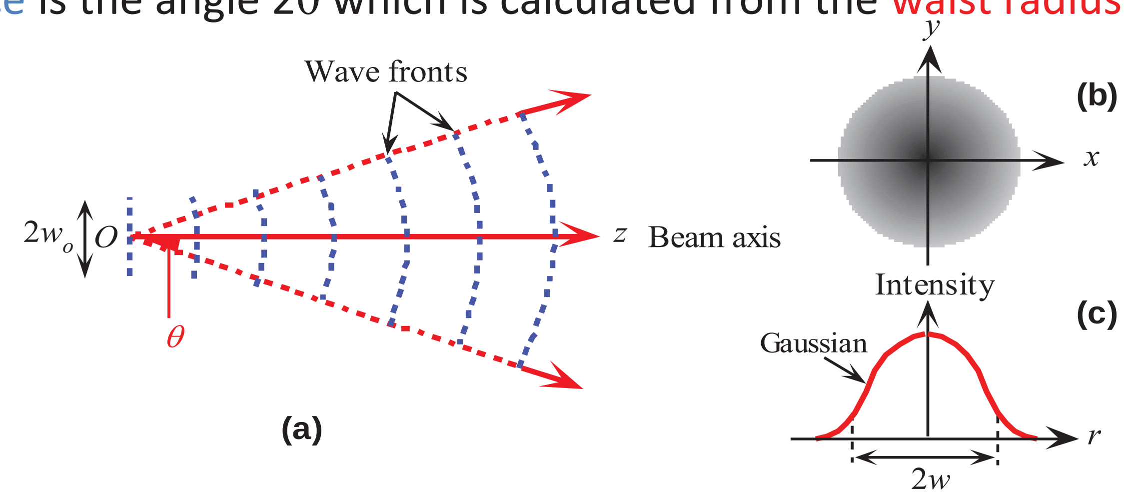
Beam Divergence

- Consider a Gaussian laser emitting from a slab of finite radius (or waist radius) $2w_o$
- We define the initial waist of the beam as w_o
- As the beam moves far enough from the surface such that source no longer looks like an infinite plane, then the wavefronts begin to diverge at a constant angle
- The half angle of the divergence is θ
- The beam diameter, $2w$, at any distance z from the origin is defined such that the cross sectional area of the beam (πw^2) contains 85% of the total beam power.
- The beam divergence is the angle 2θ which is calculated from the waist radius

Beam divergence

$$2\theta = \frac{4\lambda}{\pi(2w_o)}$$

In radians!!!!!!



(a) Wavefronts of a Gaussian light beam. (b) Light intensity across beam cross section. (c) Light irradiance (intensity) vs. radial distance r from beam axis (z).

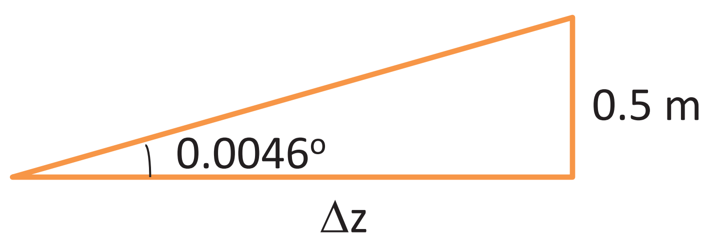
Example 1

- Consider a HeNe laser beam at 633 nm with a spot size of 10 mm. Assuming a Gaussian beam, what is the divergence of the beam?

Beam divergence

$$2\theta = \frac{4\lambda}{\pi(2w_o)} = \frac{4(633\times10^{-9}m)}{\pi(10\times10^{-3}m)} = 8.06\times10^{-5}rad = 0.0046^o$$

- At what distance is the spot size of the diverged beam equal to 1 m?



$$\Delta z = \tan^{-1}(0.0046^o)\times\frac{1m}{2} \cong 6227m$$

Refractive Index

- Assume an EM wave traveling in a dielectric medium with permittivity $\varepsilon = \varepsilon_r \varepsilon_o$
- EM propagation is equal to the propagation of the polarization in the medium
- During propagation, the induced molecular dipoles become coupled and the polarization decays the propagation of the EM wave
- ie THE WAVE SLOWS DOWN and the velocity of the wave depends directly on the permittivity and permeability of the material it is traveling through
- For an EM wave traveling through a nonmagnetic dielectric, the phase velocity of the wave is:

$$v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_o \mu_o}}$$

From the wave eqn.

$$\nabla^2 E = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = \varepsilon\mu \frac{\partial^2 E}{\partial t^2} = \varepsilon_r \varepsilon_o \mu_o \frac{\partial^2 E}{\partial t^2}$$

- In a vacuum, v = c = speed of light = 3 x 10^8 m/s where $c = \dfrac{1}{\sqrt{\varepsilon_o \mu_o}}$
- The refractive index, n, is the relative ratio of c/v $= \sqrt{\varepsilon_r}$

Optical Constants in a medium

- Index of refraction, $n = c/v$
- Wave vector, $k_{medium}=nk$
- Wavelength, $\lambda_{medium}=n\ \lambda$
- In noncrystalline materials such as glasses and liquids, the material structure is statistically the same in al directions, and thus n does not depend on direction. The refractive index is then said to be isotropic
- In crystals the atomic arrangements between atoms often demonstrate different permittivities in different directions. Such materials are said to be anisotropic
- In general the propagation of an EM field in a solid will depend on the permittivity of the solid along the k direction.
- Anisotropic permittivities that introduce a relative phase shift along the direction of propagation have complex terms in the off diagonals terms of the permittivity matrix and will be the discussion of various device concepts described in Ch. 7

Frequency Dependent Permittivity

- Materials do not often demonstrate a single degree of polarization along any one direction across the entire frequency range.
- In fact the frequency dependence of permittivity is what gives rise to properties such as absorption within a solid and allows one to see objects in "color".
- Most materials of optical interest have absorption bands in which the permittivity, and thus the refractive index, changes drastically. Shifting of these constants by doping the material, (or adding large magnetic fields) has allowed for the development of bandgap semiconductors with specific optical properties for optical generation and detection.
- Consider the simplest expression used to calculate the permittivity

$$\varepsilon_r = 1 + N\alpha / \varepsilon_o$$

Where N is the number of polarizable molecules per unit volume, and α is the polarizability per molecule.

If I can inject or remove the relative N value in a solid, then one can change the permittivity of that solid and therefore its electronic and optical properties.

If the solid is a stack of semiconductor materials with different N values that respond optically when biased, then one can create an optoelectronic device!!!

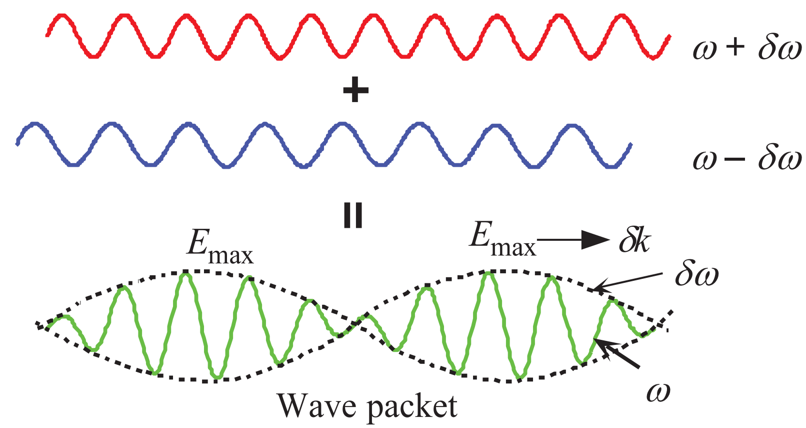
If the polarizability, α, is frequency dependent (and it is), then our optoelectronic device will work over a particular frequency range which can be engineered for the spectral band of interest!!!!

Group Velocity

- First and foremost: THERE ARE NO PERFECT MONOCHROMATIC WAVES in practice
- There are always bundles of waves with slightly different frequencies and wave vectors
- Assume the waves travel with slightly different frequencies, **ω+δω** and **ω - δω**
- The wave vectors are therefore represented by **K + δk** and **k - δk**
- The combined transform generates a wave packet oscillating at a mean beat frequency ω that is amplitude modulated by a slowly time varying field at δω
- The maximum amplitude moves with a wave vector δk
- The velocity of the packet is called the group velocity and is defined as

$$v_g = \frac{d\omega}{d\kappa}$$

- The group velocity defines the speed at which the energy is propagated since it defines the speed of the envelope of the amplitude variation

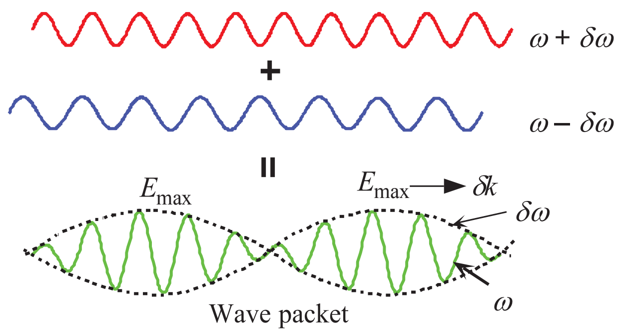


Two slightly different wavelength waves travelling in the same direction result in a wave packet that has an amplitude variation which travels at the group velocity.

For video of wave packets with and without $v = v_g$:

http://newton.ex.ac.uk/teaching/resources/au/phy1106/animationpages/

Example: Group Velocity



Two slightly different wavelength waves travelling in the same direction result in a wave packet that has an amplitude variation which travels at the group velocity.

- Resulting wave is:

$$E_x(z,t) = E_o \cos[(\omega - \delta\omega)t - (k - \delta k)z] + E_o \cos[(\omega + \delta\omega)t - (k + \delta k)z]$$

- Using the Trig identity:

$$\cos A + \cos B = 2\cos[1/2(A - B)]\cos[1/2(A + B)]$$

- We get:

$$E_x(z,t) = 2E_o \cos[(\delta\omega)t - (\delta k)z]\cos[\omega t - kz]$$

- Maximum field occurs when:

$$[(\delta\omega)t - (\delta k)z] = 2m\pi$$

- Yields velocity:

$$\frac{dz}{dt} = \frac{d\omega}{d\kappa} = v_g$$

Group Index

- Suppose v depends on the λ or K

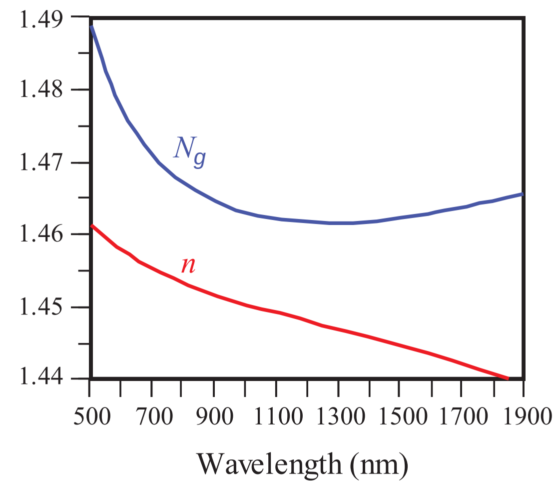
$$\omega = vk = \left(\frac{c}{n}\right)\left(\frac{2\pi}{\lambda}\right) \quad \text{where } n = n(\lambda)$$

- By definition, the group velocity is then

$$v_g = \frac{d\omega}{dk} = \frac{c}{n - \lambda \frac{dn}{d\lambda}} = \frac{c}{N_g}$$

- We define N_g as the group index of the medium.
- We now have a way to determine the effect of the medium on the group velocity at different wavelengths (frequency dependence!!!)
- The refractive index, n, and group index, N_g, depend on the permittivity of the material, ε_r
- We define a dispersive medium is a medium in which both the group and phase velocities depend on the wavelength.
- All materials are said to be dispersive over particular frequency ranges

Dispersive medium example: SiO_2



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

- N_g and n are frequency (wavelength) dependent
- Notice the minima for Ng at 1300 nm.
    - N_g is wavelength independent near 1300 nm
    - Light at 1300 nm travels through SiO_2 at the same group velocity without dispersion

Example: Effects of a Dispersive Medium

- Consider 1 um wavelength light propagating through SiO_2
- At this wavelength, N_g and n are both frequency dependent with no local minima
- Thus the medium is dispersive
- Now we must ask the question, are the group and phase velocities of the propagating wave packet the same?

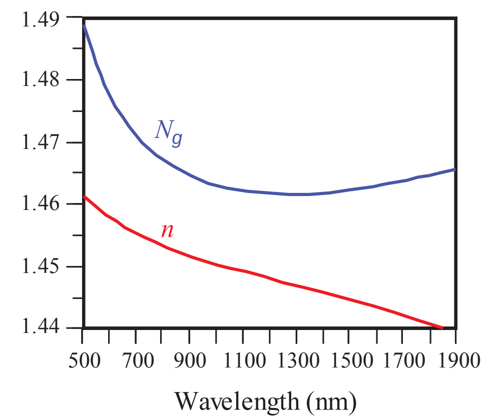
- Phase Velocity

$$v = \frac{dz}{dt} = \frac{c}{n} = \left(3x10^8 \frac{m}{s}\right)/1.450 = 2.069x10^8 \frac{m}{s}$$

- Group Velocity

$$v_g = \frac{d\omega}{dk} = \frac{c}{N_g} = \left(3x10^8 \frac{m}{s}\right)/1.463 = 2.051x10^8 \frac{m}{s}$$

- Answer: NO!!!!
  The group velocity is 0.9% slower than the phase velocity



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

Energy Flow in EM Waves

- Let us recall that there is indeed a B field in the EM wave.
- Recall from electrostatics that

$$E_x = vB_y = \frac{c}{n} B_y$$

- where $v = \left(\sqrt{\varepsilon_r \varepsilon_o \mu_o}\right)^{-1}$ Speed of light in the medium

$$n = \sqrt{\varepsilon_r}$$

- As the EM wave propagates along the direction k, there is an energy flow in that direction
  - Electrostatic energy density $\frac{1}{2}\varepsilon_r \varepsilon_o E_x^2$
  - Magnetostatic energy density $\frac{1}{2\mu_o} B_y^2 = \frac{\mu_o}{2} H_y^2$

  Where both these values are equal

- The Energy flow per unit time per unit area, S, is defined as the Poynting Vector

$$\vec{S} = \vec{E}_o \times \vec{H}_o = \frac{(Av\Delta t)(\varepsilon_r \varepsilon_o E_x^2)}{A\Delta t} = v(\varepsilon_r \varepsilon_o E_x^2) = v^2 \varepsilon_r \varepsilon_o E_x B_y$$

$$\vec{S} = v^2 \varepsilon_r \varepsilon_o \vec{E} \times \vec{B} = v^2 \varepsilon_r \varepsilon_o \mu_o \vec{E} \times \vec{H} = \frac{c^2}{n^2}\frac{n^2}{c^2}\vec{E} \times \vec{H} = \vec{E} \times \vec{H}$$

Irradiance

- Magnitude of the Pointing Vector is called the irradiance
- Note that because we are discussing sinusoidal waveforms, that the instantaneous irradiance of light propagating in phase is taken from the instantaneous amplitude of E and B respectively

$$S = v^2 \varepsilon_r \varepsilon_o E_x B_y$$

- Instantaneous irradiance can only be measured if the power meter responds more quickly than the electric field oscillations.
- As one might imagine, at optical frequencies, all practice measurements are made using the average irradiance.
- The average irradiance is

$$I = S_{avg} = \langle S \rangle = \frac{1}{2} v \varepsilon_r \varepsilon_o E_o^2$$

$$= \frac{1}{2} \frac{c}{n} \varepsilon_r \varepsilon_o E_o^2 = \frac{1}{2} c n \varepsilon_o E_o^2 = \left(1.33 \times 10^8 \frac{m}{s} \right) n E_o^2$$

$$= \frac{1}{2} v^2 \varepsilon_r \varepsilon_o E_o B_o = \frac{1}{2} v^2 \varepsilon_r \varepsilon_o \mu_o \left| \vec{E} \times \vec{H} \right|$$

$$= \frac{1}{2} \frac{c}{n^2}^2 \varepsilon_r \varepsilon_o \mu_o \left| \vec{E} \times \vec{H} \right| = \frac{1}{2} \frac{c}{n^2}^2 \frac{n^2}{c^2} \left| \vec{E} \times \vec{H} \right| = \frac{1}{2} \left| \vec{E} \times \vec{H} \right|$$

Example: Electric and Magnetic Fields in Light

- The intensity (irradiance) of the red laser from a He-Ne laser at a certain location was measured to about 1mW/cm^2.
- What are the magnitudes of the electric and magnetic fields?

$$E_o = \sqrt{\frac{2I}{cn\varepsilon_o}} = \sqrt{\frac{2(1\times10^{-3}\times10^{4}Wm^{-2})}{\left(1.33\times10^{8}\frac{m}{s}\right)(n=1)}} = 87\frac{V}{m}$$

$$B_o = E_o / c = 0.29\mu T$$

- What are the magnitudes if this beam is in a glass medium with refractive index 1.45?

$$E_o = \sqrt{\frac{2I}{cn\varepsilon_o}} = \sqrt{\frac{2(1\times10^{-3}\times10^{4}Wm^{-2})}{\left(1.33\times10^{8}\frac{m}{s}\right)(1.45)}} = 72\frac{V}{m}$$

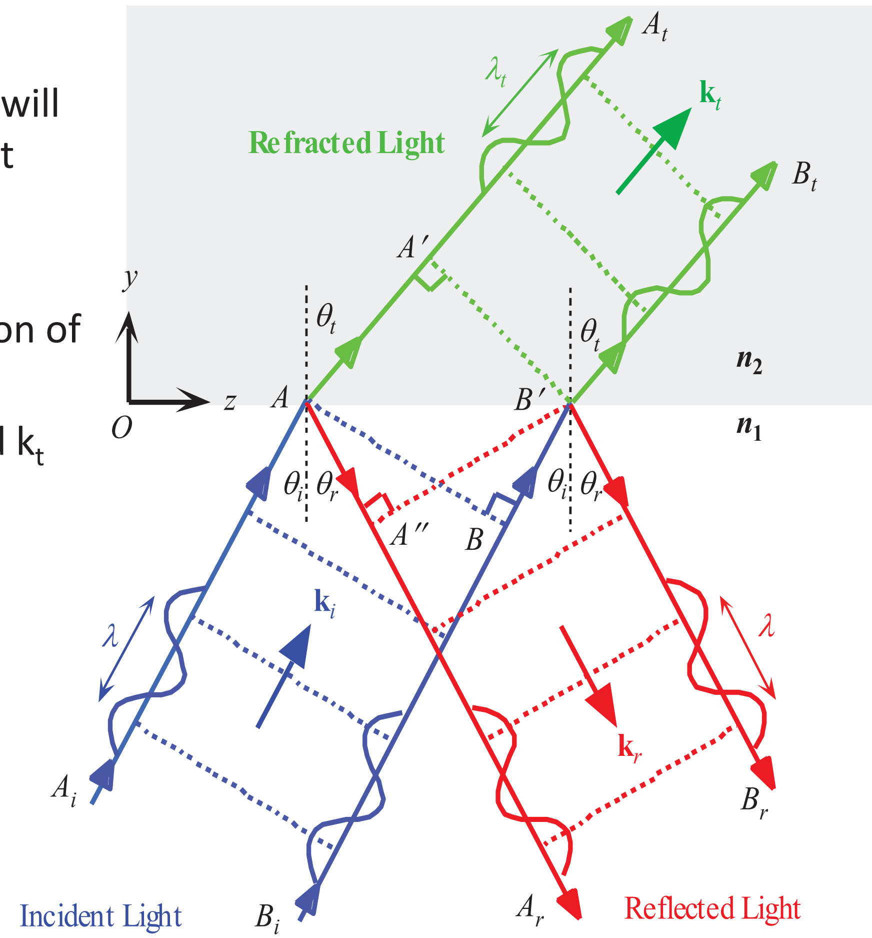
$$B_o = nE_o / c = 0.35\mu T$$

Note: the relative amplitude of E decreased, and B (H) increased and thus the polarized wave became more ellipsoidal

Snell's Law

- Neglect absorption and emission
- Light interfacing with a surface boundary will reflect back into the medium and transmit through the second medium
- Transmitted wave is called refracted light
- The angles θ_i, θ_r, and θ_t define the direction of the wave normal to the interface.
- The wave vectors are defined as k_i, k_r, and k_t
- At any interface, $\theta_I = \theta_r$
- Snell's Law States

$$\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$



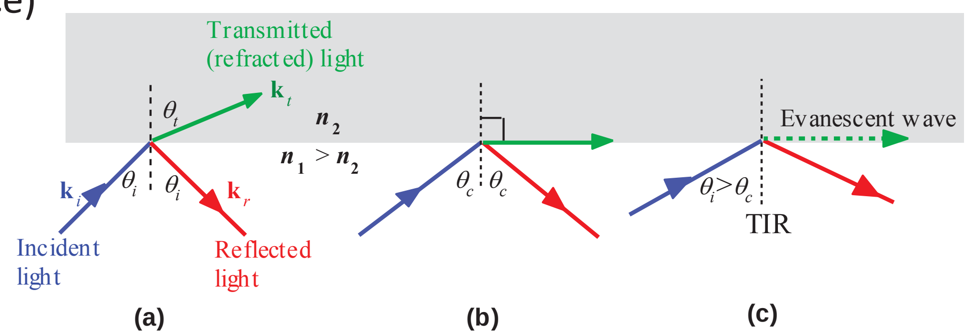
A light wave travelling in a medium with a greater refractive index ($n_1 > n_2$) suffers reflection and refraction at the boundary.

Total Internal Reflection

- If $n_1 > n_2$ then transmitted angle > incidence angle.
- When θ_t =90°, then the incidence angle is called the critical angle

$$\sin(\theta_c) = \frac{n_2}{n_1}$$

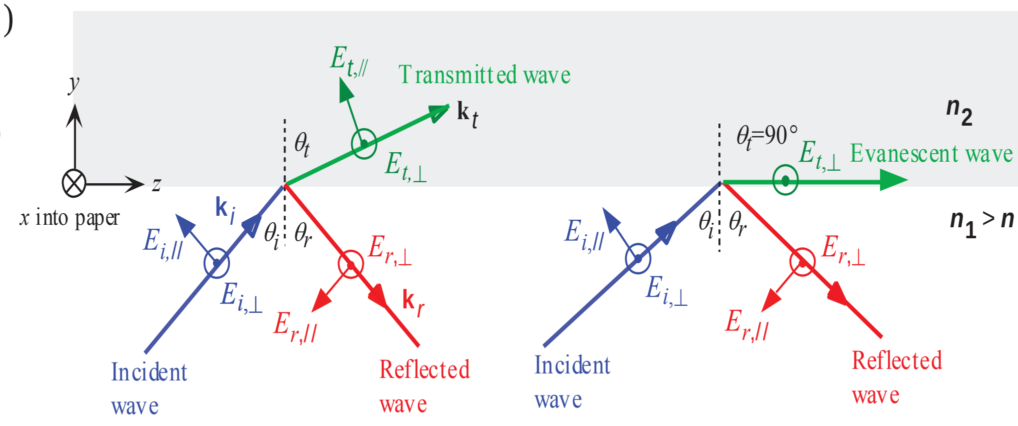
- When $\theta_i > \theta c$ there is
  - no transmitted wave in medium
  - Total internal reflection occurs
  - an evanescent wave propagates along the boundary (i.e. high loss electric field propagating along the surface)



Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to θ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$ and total internal reflection (TIR).

Fresnel's Equations (1)

- Amplitude Reflection and Transmission Coefficients
  - Transverse Electric Field (TE) waves if $E_{i\perp}$, $E_{r\perp}$, and $E_{t\perp}$
  - Transverse Magnetic Field (TM) waves if $E_{i//}$, $E_{r\,//}$, and $E_{t\,//}$
  - Incident wave $$E_i = E_{io} e^{j(\omega t - \vec{k}_i \cdot \vec{r})}$$
  - Reflected Wave $$E_r = E_{ro} e^{j(\omega t - \vec{k}_r \cdot \vec{r})}$$
  - Transmitted Wave $$E_t = E_{to} e^{j(\omega t - \vec{k}_t \cdot \vec{r})}$$
  - Boundary Conditions
    - $E_{tangential}$ (1) = $E_{tangential}$ (2)
    - $B_{tangential}$ (1) = $B_{tangential}$ (2)
  - Applying the boundary conditions to the equations above yields amplitudes of reflected and transmitted waves. These equations were first derived by Fresnel



(a) $\theta_i < \theta_c$ then some of the wave is transmitted into the less dense medium. Some of the wave is reflected.

(b) $\theta_i > \theta_c$ then the incident wave suffers total internal reflection. However, there is an evanescent wave at the surface of the medium.

Light wave travelling in a more dense medium strikes a less dense medium. The plane of incidence is the plane of the paper and is perpendicular to the flat interface between the two media. The electric field is normal to the direction of propagation . It can be resolved into perpendicular (⊥) and parallel (//) components

Fresnel's Equations (2)

- Define $n = n_2/n_1$ as the relative refractive index of the system
- Refection and transmission coefficients for $E_\perp$ are

$$r_\perp = \frac{E_{ro\perp}}{E_{io\perp}} = \frac{\cos\theta_i - \sqrt{n^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n^2 - \sin^2\theta_i}}$$

$$t_\perp = \frac{E_{to\perp}}{E_{io\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{n^2 - \sin^2\theta_i}}$$

These equations allow one to calculate the amplitude and phases of light propagating through different media

- Refection and transmission coefficients for E_{II} are

$$r_{II} = \frac{E_{roII}}{E_{ioII}} = \frac{\sqrt{n^2 - \sin^2\theta_i} - n^2\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i} + n^2\cos\theta_i}$$

$$t_{II} = \frac{E_{toII}}{E_{ioII}} = \frac{2n\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i} + n^2\cos\theta_i}$$

If we let E_{io} be real, then the phase angles of $r_\perp$ and $t_\perp$ correspond to the phase changes measured with respect to the incident wave

- These coefficients are related by the following two equations

$$r_{II} + nt_{II} = 1 \qquad r_\perp + 1 = t_\perp$$

Internal Reflection

- Light traveling from a more dense medium into a less dense one ($n_2 < n_1$)
- Critical angle

$$\sin(\theta_c) = \frac{n_2}{n_1} = \frac{1}{1.44}$$

$$\theta_c = 44^o$$

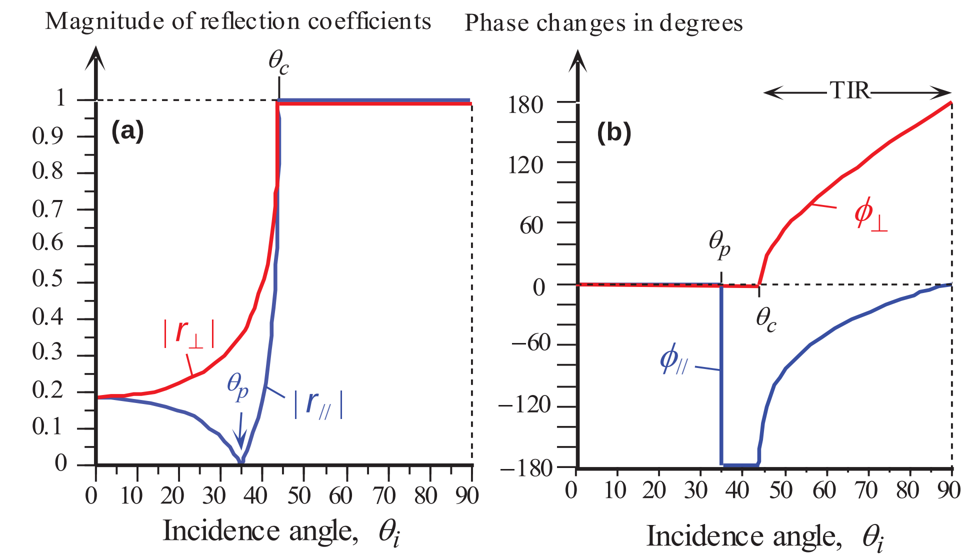
- For $n_2 < n_1$ at $\theta_i = 0$

$$r_{II} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2}$$

- This value is always positive, meaning that the reflective wave undergoes no phase change prior to $r_{//}$ going to zero
- Brewster's Angle = Polarization angle = θ_p is the angle at which $r_{//}$ becomes zero and TE/TM polarization begins to occur

$$\tan(\theta_p) = \frac{n_2}{n_1} = \frac{1}{1.44}$$

$$\theta_p = 35^o$$



Internal reflection: (a) Magnitude of the reflection coefficients $r_{//}$ and $r_\perp$ vs. angle of incidence θ_i for $n_1 = 1.44$ and $n_2 = 1.00$. The critical angle i 44°. (b) The corresponding phase changes $\phi_{//}$ and $\phi_\perp$ vs. incidence angle.

- Reflected waves at angles greater than θ_p are linearly polarized because they contain field oscillations that are contained within a well defined plane perpendicular to the plane of incidence AND the ~~plane~~ of propagation

Phase Change in TIR

- For $\theta_p < \theta_i < \theta_c$, Fresnel's eqn. gives $r_{//} < 0$. Predicts a phase shift of 180°
- For $\theta_i \geq \theta_c$, Fresnel's eqn. gives $r_{//}$ and $r_{\perp} = 1$ such that the reflected wave has the same amplitude as the incident wave and TIR occurs
- For $\theta_i > \theta_c$ we have $r_{\perp} = 1$, but the phase change, $\phi_{\perp}$ and $\phi_{//}$ are derived from

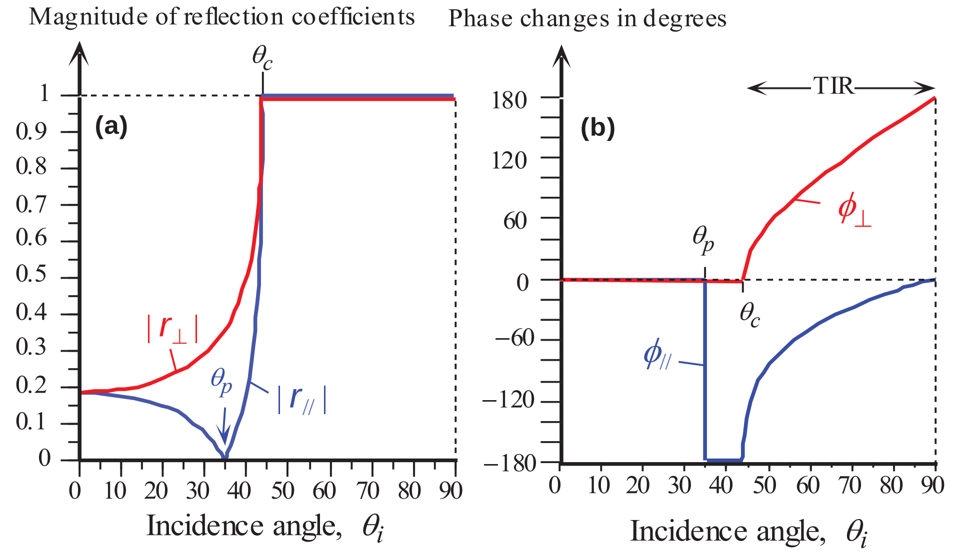
$$\tan\left(\frac{1}{2}\phi_{\perp}\right) = \frac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i}$$

$$\tan\left(\frac{1}{2}\phi_{\mathrm{II}} + \frac{\pi}{2}\right) = \frac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i}$$

Phase change in TIR

Transmitted light does **NOT** experience a phase shift

TIR = Total Internal Reflection



Internal reflection: (a) Magnitude of the reflection coefficients $r_{//}$ and $r_{\perp}$ vs. angle of incidence θ_i for $n_1 = 1.44$ and $n_2 = 1.00$. The critical angle i 44°. (b) The corresponding phase changes $\phi_{//}$ and $\phi_{\perp}$ vs. incidence angle

Evanescent Waves

- When $\theta_i \geq \theta_c$ there must still be an electric field in medium 2 or the boundary conditions will not be satisfied
- The field in medium 2 is an evanescent wave that travels along the boundary edge at the same speed as the incident wave and dissipates into the 2nd medium

$$E_{t\perp}(y,z,t) = e^{-\alpha_2 y} e^{j(\omega t - k_{iz}\cdot z)}$$

$$k_{iz} = k_i \sin\theta_i$$ evanescent wave vector

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$ attenuation coefficient

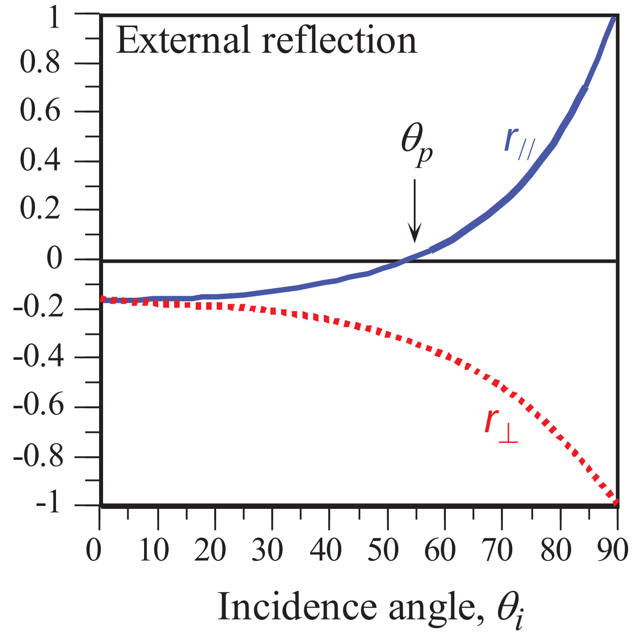
- The penetration depth of the electric field into medium 2 is

$$\delta = 1/\alpha_2 \quad \rightarrow \quad E_{t\perp} = e^{-1}$$

External Reflection

- Light traveling from a less dense medium into a more dense one ($n_2 > n_1$)
- At normal incidence, both Fresnel coefficients for $r_{//}$ and $r_{\perp}$ are negative
- External reflection for TM and TE at normal incidence generates a 180 degree phase shift. This phase shift is observed at all angles for TE waves and up to θ_p for TM waves
- Also, $r_{//}$ goes through zero at the Brewster angle, θ_p
- At θ_p the reflected wave is polarized in the $E_{\perp}$ component only. Thus Light incident at θ_p or higher in angle does not generate a phase shift in reflection for TM waves.

- Transmitted light in both internal and external reflection does **NOT** experience a phase shift



The reflection coefficients $r_{//}$ and $r_{\perp}$ vs. angle of incidence θ_i for $n_1 = 1.00$ and $n_2 = 1.44$.

Example: Evanescent Wave

- TIR from a boundary $n_1 > n_2$ generates an evanescent wave in medium 2 near the boundary
- Describe the evanescent wave characteristics and its penetration into medium 2

$$E_{t\perp} = t_\perp E_{io\perp} e^{j(\omega t - k_t \cdot r)}$$

$$k_t \cdot r = y k_t \cos\theta_t + z k_t \sin\theta_t$$

Apply Snell's Law at $\theta_c > \theta_i$

$$\sin\theta_t = \left(n_1 / n_2 \right) \sin\theta_i > 1$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \pm j A_2$$

$$E_{t\perp} = t_\perp E_{io\perp} e^{j(\omega t - z k_t \sin\theta_t + j y k_t A_2)}$$

$$E_{t\perp} = t_\perp E_{io\perp} e^{-y k_t A_2} e^{j(\omega t - z k_t \sin\theta_t)}$$

For TIR

$$k_t \sin\theta_t = k_i \sin\theta_i = k_z$$

$$E_{t\perp} = t_\perp E_{io\perp} e^{-y k_t A_2} e^{j(\omega t - k_z z)}$$

$$k_t A_2 = \alpha_2 = \frac{2\pi n_2}{\lambda} \sqrt{\left(\frac{n_1}{n_2} \right)^2 \sin^2\theta_i - 1}$$

$\delta = 1 / \alpha_2$ Penetration depth

Additionally, TIR allows us to calculate $t_\perp$

$$t_\perp = \frac{E_{to\perp}}{E_{io\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1} \right)^2 - \sin^2\theta_i}}$$

$t_\perp = t_{o\perp} e^{-j\Psi_\perp}$ Complex transmission value with imaginary phase constant Ψ

Example: Internal Reflection

- Reflection of light from a less dense medium
- Wave is traveling in a glass of index $n_1 = 1.450$
- Wave becomes incident on a less dense medium of index $n_2 = 1.430$
- What is the minimum incidence angle for TIR? $\sin(\theta_c) = \frac{n_2}{n_1} = \frac{1.430}{1.450}$ $\theta_c = 80.47^o$
- What is the phase change in the reflected wave at an incidence angle of 85 degrees?

$$\tan\left(\frac{\theta_\perp}{2}\right) = \frac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i} = \frac{\sqrt{\sin^2(85^o) - \left(\frac{1.43}{1.45}\right)^2}}{\cos(85^o)} = 1.61447 \qquad \theta_\perp = 116.45^o$$

$$\tan\left(\frac{\theta_\parallel + \pi}{2}\right) = \frac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i} = \frac{1}{n^2}\tan\left(\frac{\theta_\perp}{2}\right) \qquad \theta_\parallel = 62.1^o$$

- What is the penetration depth of the evanescent wave into medium 2 when the incidence angle is 85°?

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1} = 1.28\times10^6 / m$$

$$\delta = {}^1\!/_{\alpha_2} = 7.8\times10^{-7}\, m$$

Intensity, Reflectance, and Transmittance

- Relative (%) intensity of the reflected light traveling through the media – Reflectance

$$R_{\perp} = \frac{\left|E_{ro\perp}\right|^2}{\left|E_{io\perp}\right|^2} = \left|r_{\perp}\right|^2 \qquad R_{\parallel} = \frac{\left|E_{ro\parallel}\right|^2}{\left|E_{io\parallel}\right|^2} = \left|r_{\parallel}\right|^2$$

$$R = R_{\perp} = R_{\parallel} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

- Relative (%) intensity of the transmitted light traveling through the media – Transmittance

$$T_{\perp} = \frac{n_2\left|E_{to\perp}\right|^2}{n_1\left|E_{io\perp}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{\perp}\right|^2 \qquad T_{\parallel} = \frac{n_2\left|E_{to\parallel}\right|^2}{n_1\left|E_{io\parallel}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{\parallel}\right|^2$$

$$T = T_{\perp} = T_{\parallel} = \frac{4n_1 n_2}{\left(n_1 + n_2\right)^2}$$

- Sum of the transmittance and reflectance in any conserved system must equal 1

$$R + T = 1$$

Example: Internal and External Reflection

- Light propagates at normal incidence from air, n = 1, to glass with a refractive index of 1.5. What is the reflection coefficient and the reflectance w.r.t to the incident beam?

$$r_{II} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1-1.5}{1+1.5} = -0.2 \qquad R_{\parallel} = \left|r_{\parallel}\right|^2 = 0.04 \quad \text{or 4\%}$$

- Light propagates at normal incidence from glass, n = 1.5, to air with a refractive index of 1.0. What is the reflection coefficient and the reflectance w.r.t to the incident beam?

$$r_{II} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.5-1}{1.5+1} = 0.2 \qquad R_{\parallel} = \left|r_{\parallel}\right|^2 = 0.04 \quad \text{or 4\%}$$

- What is the polarization angle of the in the external reflection for the air to glass interface described by the first question above? How would one make a polaroid device (device that polarizes light based on the polarization angle)?

$$\tan(\theta_p) = \frac{n_2}{n_1} = 1.5$$

$$\theta_p = 56.3^o$$

At an incidence angle of 56.3° the reflected light will be polarized with an E⊥ to the plane of incidence. Transmitted light will be partially polarized. By using a stack of N glass plates, one can increase the polarization of the transmitted light by a factor of N

Reflectance at Different Angles of Incidence

- Light propagates at 30° incidence from air, n = 1, to glass with a refractive index of 1.5. What is the reflection coefficient and the reflectance w.r.t to the incident beam?

Snell's Law $$\frac{n_1}{n_2}\sin\theta_i = \sin\theta_t$$

θ_t = 19.5°

replace
$$n_1 = n_1\cos\theta_i$$
$$n_2 = n_2\cos\theta_t$$
$$\cos\theta_i = 0.866$$
$$\cos\theta_t = 0.943$$

$$r_\perp = \frac{E_{ro\perp}}{E_{io\perp}} = \frac{\cos\theta_i - \sqrt{n^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n^2 - \sin^2\theta_i}}$$

$$\frac{\cos\left(\frac{\pi}{6}\right) - \sqrt{\left(\frac{1.5}{1}\right)^2 - \left(\sin\left(\frac{\pi}{6}\right)\right)^2}}{\cos\left(\frac{\pi}{6}\right) + \sqrt{\left(\frac{1.5}{1}\right)^2 - \left(\sin\left(\frac{\pi}{6}\right)\right)^2}} = -0.24$$

$$r_{II} = r_\perp = \frac{n_1 - n_2}{n_1 + n_2} = \frac{0.866 - 1.414}{0.866 + 1.414} = -0.24 \quad R_\parallel = |r_\parallel|^2 = 0.058$$ or 5.8%

Example: Antireflection Coatings on Solar Cells

- When light is incident on a semiconductor it becomes partially reflected
- This is important because it the transmitted light traveling into the solar cell is absorbed and converted to electrical energy
- Assume the refractive index of Silicon is 3.5 between 700 - 800 nm
- Calculate the reflectance of the silicon surface in air

$$R = \left(\frac{1-3.5}{1+3.5}\right)^2 = 0.309 \text{ or } 30.9\%$$

- This means there is a 30.9% loss in efficiency even before the light enters the silicon solar cell
- If one coats the solar cell with a thin layer of electric material such as Si_3N_4 (silicon nitride) that has an intermediate refractive index of 1.9, then we can reduce the loss

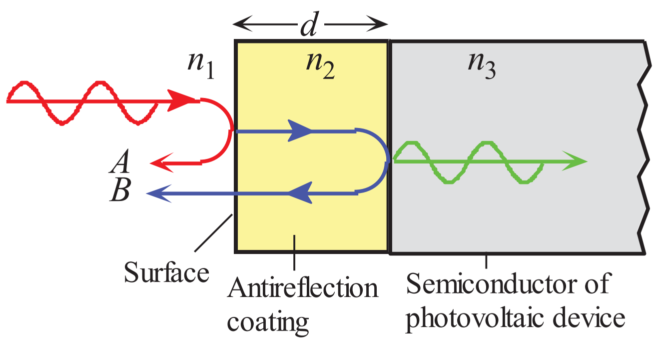


Illustration of how an antireflection coating reduces the reflected light intensity

Example: Antireflection Coatings on Solar Cells

- Note that in order for this concept to work the thickness of the antireflective layer must be matched to the wavelength of the light transmitted
  - We are dealing with external reflection, thus reflected light off of all normal interfaces is 180^o out of phase with incident light
  - Phase matching must be accomplished between the light reflecting from the air/coating interface and the light reflected from coating/silicon interface
  - The phase difference in the system is equivalent to $k_2(2d)$ where $k_2 = n_2 k = 2\pi n_2/\lambda$
  - Phase matching occurs when $k_2(2d) = m\pi$

$$\left(\frac{2\pi n_2}{\lambda}\right)2d = m\pi \rightarrow d = m\left(\frac{\lambda}{4n_2}\right)$$

  - Thus the thickness of the coating must be multiples of the quarter wavelength of light propagating through it.

  - Also, to obtain a good degree of destructive interference, the amplitudes of the A and B waves must be comparable. Thus we need $n_2 = \sqrt{n_1 n_3}$
  - This yields a reflection coefficient between the air and coating that is equal to that between the coating and the semiconductor. In our case n_2 should equal 1.87 which is close to that of Si_3N_4 at 1.9.

Example: Antireflection Coatings on Solar Cells

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$$R_A = \left(\frac{1-1.9}{1+1.9}\right)^2 = 0.096 \quad R_B = \left(\frac{1.9-3.5}{1.9+3.5}\right)^2 = 0.0877$$

- Thus about 10% of light is now reflected off of the coating surface and another 10% is reflected from the silicon. Of the second 10%, about 10% of that is reflected back from the air/nitride interface onto the silicon again. So the total gain optical gain acquired through use of the antireflective coating is about 10%.

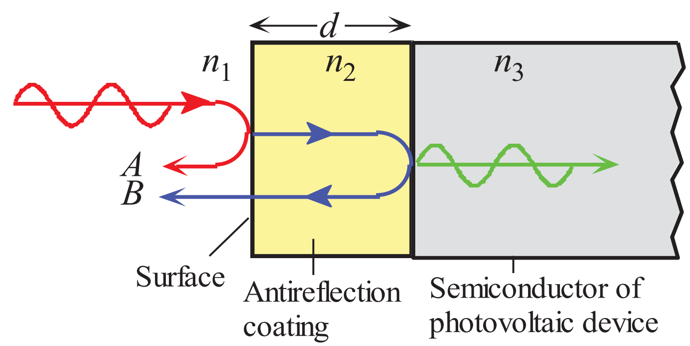


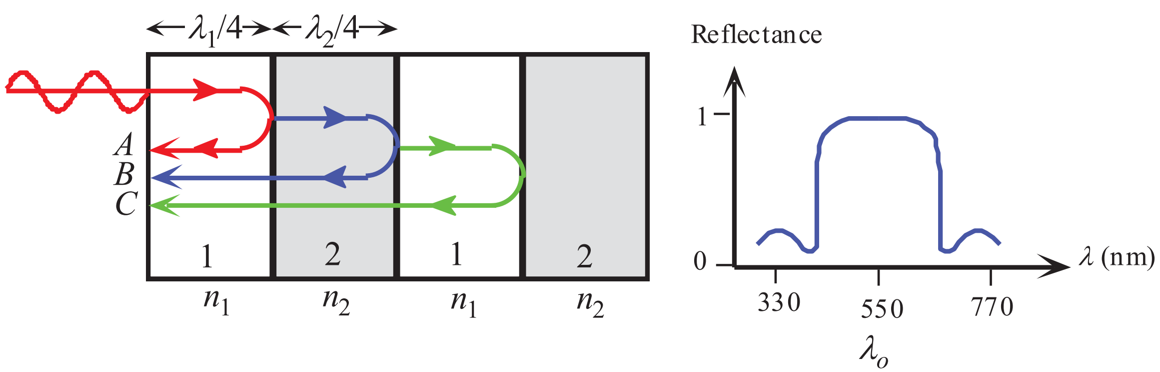
Illustration of how an antireflection coating reduces the reflected light intensity

Example: Dielectric Mirrors

- Dielectric mirror – stack of dielectrics with alternating refractive indices
- The thickness of each layer is a quarter wavelength: $\lambda/(4n_i) = \lambda_i/4$
- Reflected waves from the interfaces interfere constructively to generate a highly reflective coating over a optical wavelength range centered at λ_o
- The reflection coefficient for a particular boundary is similar to that calculated previously

$$R_{ij} = \left(\frac{n_i - n_j}{n_i + n_j}\right)^2, \; j = i \pm 1$$

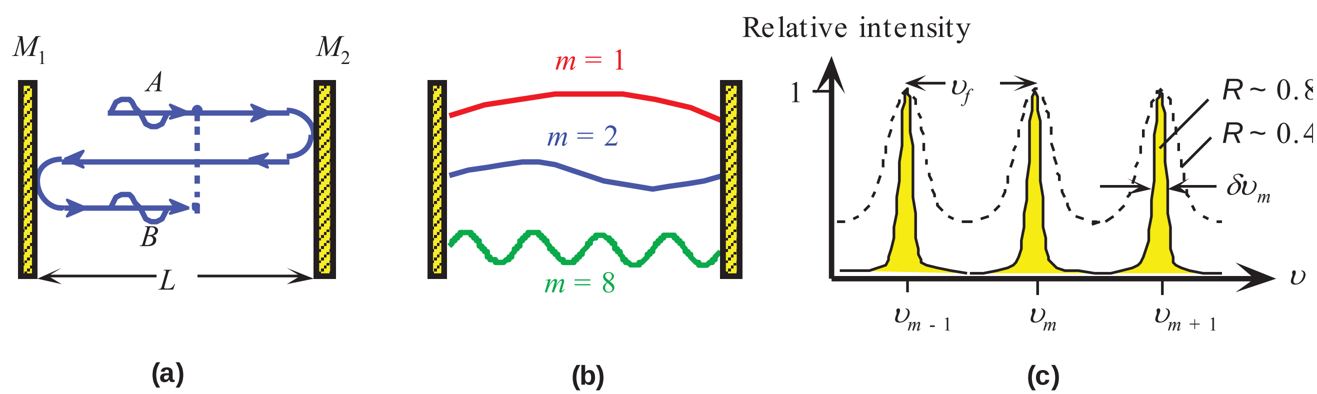
- Thus the reflection coefficients using values of i and j alternate throughout the mirror
- After several alternating reflectances, the transmission becomes exceedingly small and light is reflected back from the surface at values near unity (1).



Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers and its reflectance.

Multiple Interference and Optical Resonators

- Optical resonator stores energy or filters light only at certain frequencies
- Built by aligning two flat mirrors parallel to one another with free space in between them
- Reflections between mirror surfaces M1 and M2 lead to constructive and destructive interference the cavity
- This leads to stationary (or standing) EM waves in the cavity



Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, *modes*, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. *R* is mirror reflectance and lower *R* means higher loss from the cavity.

Multiple Interference and Optical Resonators

- Since the electric field at the mirrors must be zero, we can only fit integer multiples of half wavelengths into the cavity of length L

$$m\frac{\lambda}{2} = L \qquad m = 1,2,3\ldots$$

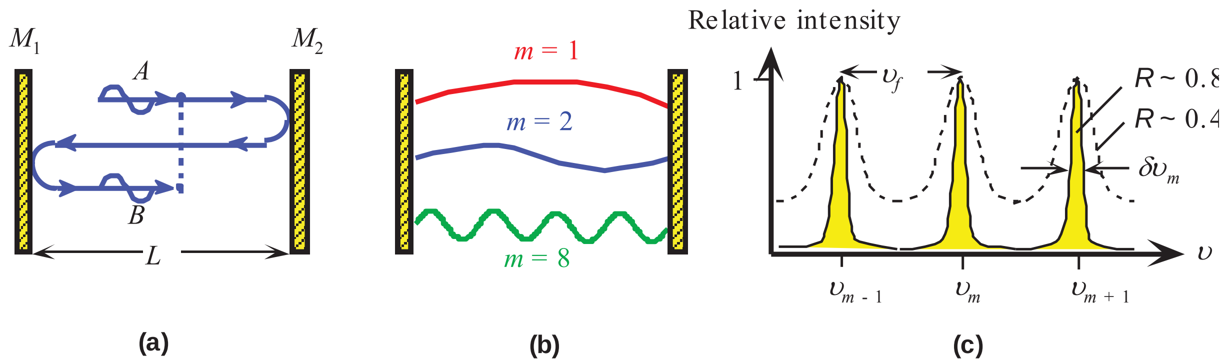
- Each cavity mode is defined by the m value, or the number of the half wavelengths that constructively interfering within the cavity.
- Resonant frequencies are the beat oscillation frequencies resonant in the cavity

$$v_m = m\frac{c}{2L} = mv_f$$

$$v_f = \frac{c}{2L}$$

- Free spectral range

$$\Delta v_m = v_{m+1} - v_m = v_f$$



Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, *modes*, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. *R* is mirror reflectance and lower *R* means higher loss from the cavity.

Fabry-Perot Optical Resonator

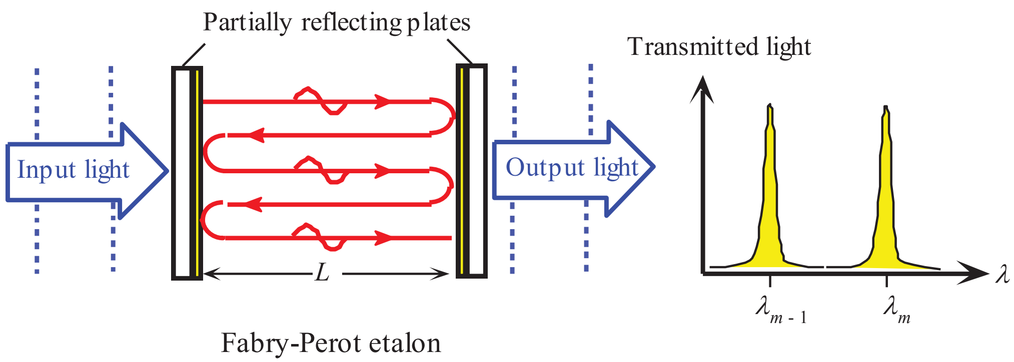
- Simple optical cavity that stores radiation energy only at certain frequencies
- Assume a wave A travels within the cavity and is reflected back and forth as wave B.
- The field and intensity of the cavity are:

$$E_{cavity} = A + Ar^2 e^{-2jkL} + Ar^4 j^{-4kL} + Ar^6 e^{-6kL} + \ldots \qquad E_{cavity} = \frac{A}{1 - r^2 e^{-jkL}}$$

$$I = \left|E_{cavity}\right|^2 = \frac{I_o}{(I-R)^2 + 4R\sin^2(kL)} \qquad I_{\max} = \frac{I_o}{(I-R)^2}$$

- Spectral width of the cavity: $\delta v_m = \frac{v_f}{F} \qquad F = \frac{\pi\sqrt{R}}{1-R}$

Where F is called the Finesse of the cavity which is the ratio of mode separation to spectral width. Thus as losses decrease, finesse increases. Also larger finesses lead to sharper mode peaks



$$I_{trans} = I_{incident} \frac{(1-R)^2}{(I-R)^2 + 4R\sin^2(kL)}$$

Transmitted light through a Fabry-Perot optical cavity.

Resonator Modes with Spectral Width

- Consider a Fabry-Perot optical cavity of air length = 100 microns with mirrors that have a 0.9 reflectance.
- Calculate the cavity mode nearest to 900 nm.

$$m = \frac{2L}{\lambda} = \frac{2\left(100\times10^{-6}m\right)}{\left(900\times10^{-9}m\right)} = 222.22$$

$$\lambda_m = \frac{2L}{m} = \frac{2\left(100\times10^{-6}m\right)}{222} = 900.90nm$$

- Calculate the separation of the modes and the spectral width of each mode.

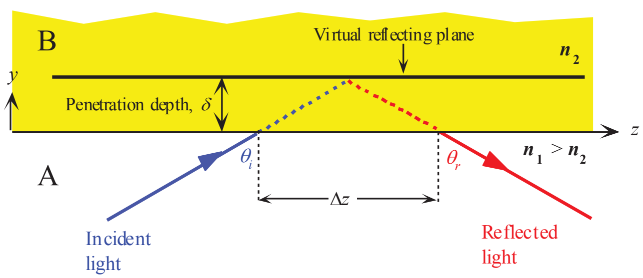
$$\Delta v_m = v_f = \frac{c}{2L} = \frac{3\times10^{8}m/s}{2\left(100\times10^{-6}m\right)} = 1.5\times10^{12}Hz$$

$$F = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{0.9}}{1-0.9} = 29.8$$

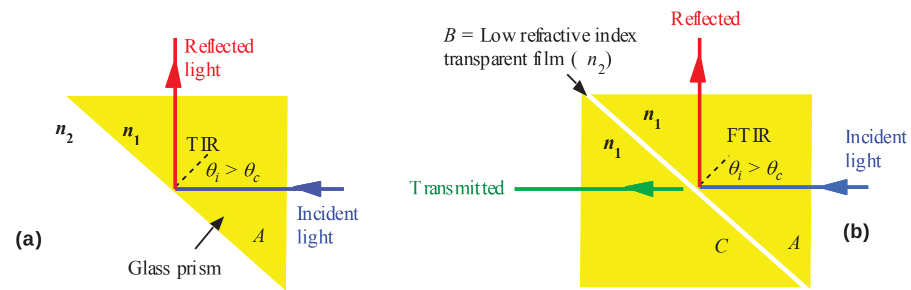
$$\delta v_m = \frac{v_f}{F} = 5.03\times10^{10}Hz$$

$$\delta\lambda_m = \left|\frac{c}{v_m^{\;2}}\right|\delta v_m = \left|\frac{\lambda_m}{c}\right|\delta v_m = 0.136nm$$ ½ bandwidth of resonator output

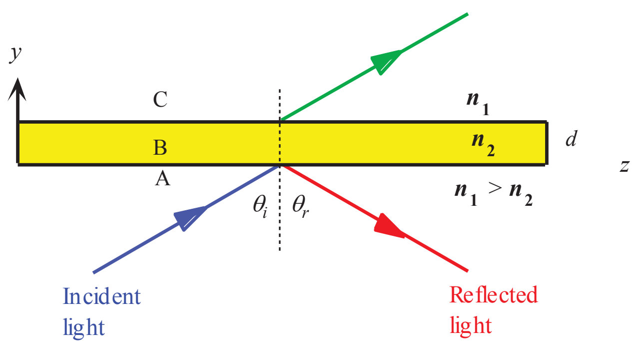
Optical Tuning



The reflected light beam in total internal reflection appears to have been laterally shifted by an amount Δz at the interface.



(a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.
(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.



When medium B is thin (thickness d is small), the field penetrates to the BC interface and gives rise to an attenuated wave in medium C. The effect is the tunnelling of the incident beam in A through B to C.

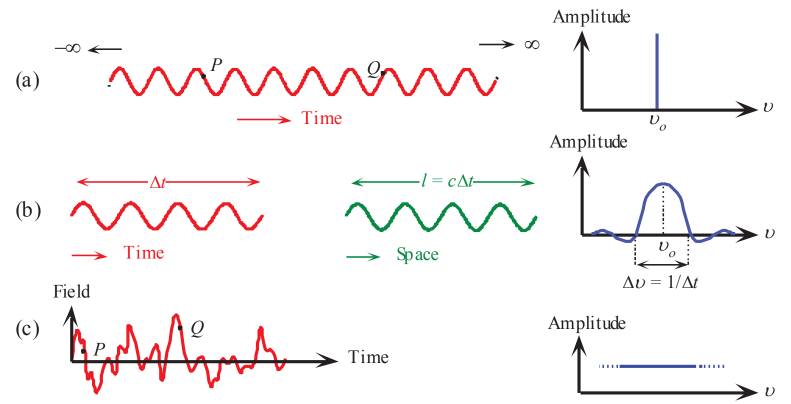
- Use concept of penetration depth associated with evanescent waves
- Frustrated Total Internal Reflection (FTIR) occurs when the thickness of the n2 medium is thinner than the penetration depth allowing the wave to partially transmit through medium
- Yields partial transmission into a TIR interface
- Transmitted beam contains some intensity, thus R is reduced below 1

Temporal and Spatial Coherence

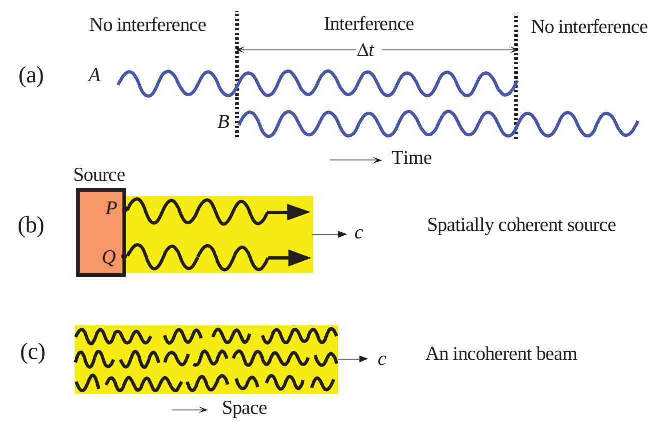
- Partial coherence is defined by the ability to predict the phase of any portion of the wave from any other portion
- Temporal coherence measures the extent at which two points on the wave are separated in time
- Coherence time: $\Delta t = \dfrac{l}{c}$
- Spatial coherence measures the extent at which two points are separated on the wave in space
- Coherence length: $l = c\Delta t$
- Spectral width: $\Delta v = \dfrac{1}{\Delta t}$
- Example: 589nm laser with spectral width of $5\text{x}10^{11}$Hz

$$\Delta t = 2\times 10^{-12} s$$

$$l = 0.6mm$$



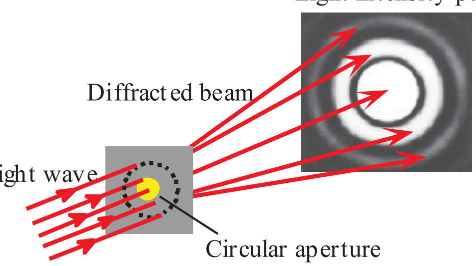
(a) A sine wave is perfectly coherent and contains a well-defined frequency υ_o. (b) A finite wave train lasts for a duration Δt and has a length l. Its frequency spectrum extends over $\Delta\upsilon = 1/\Delta t$. It has a coherence time Δt and a coherence length l. (c) White light exhibits practically no coherence.



(a) Two waves can only interfere over the time interval Δt. (b) Spatial coherence involves comparing the coherence of waves emitted from different locations on the source. (c) An incoherent beam.

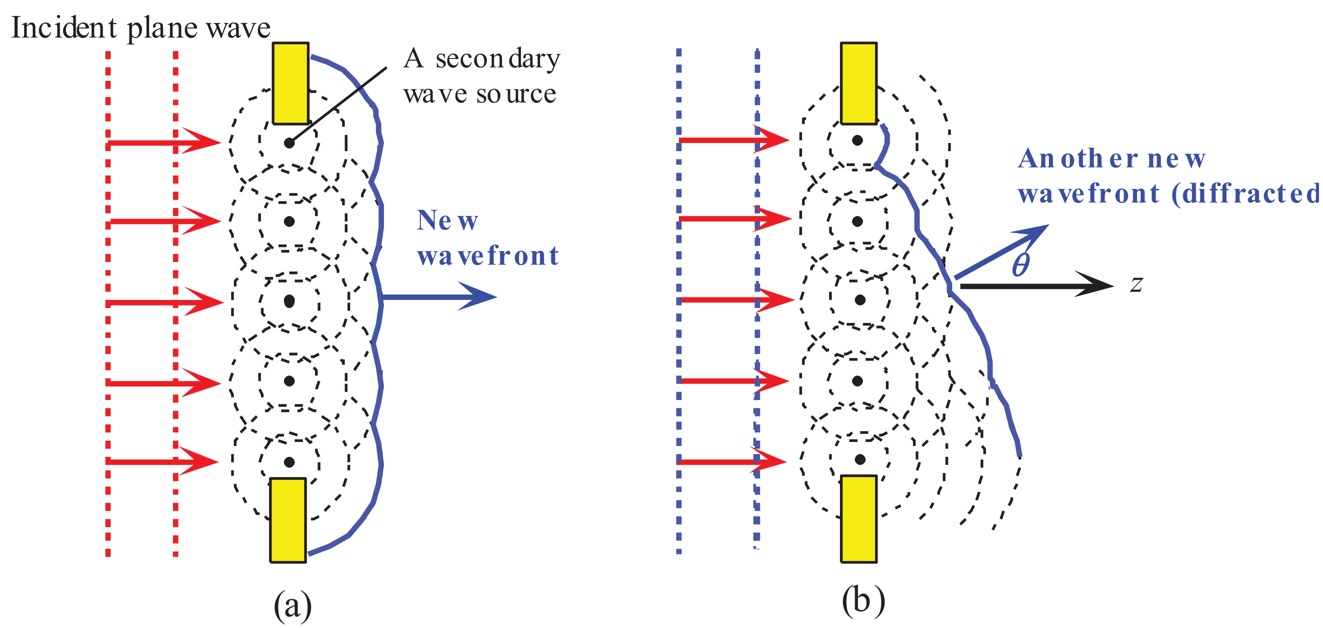
Introduction to Diffraction

- Airy rings are a diffraction pattern clearly visible when light passes through a circular aperture
- The diffracted beam does NOT correspond to the shadow of the aperture
- Instead the light imaged passed the aperture is the result of both light passing through the aperture and light scattered off the edges. The scattered light generates an interference pattern in the image
- Diffracted light from a distance generates the image in a planer wavefront: Fraunhofer Diffraction
- Diffracted light from a near by aperture images the surface with significant wavefront curvature: Fresnel Diffraction



A light beam incident on a small circular aperture becomes diffracted and its light intensity pattern after passing through the aperture is a diffraction pattern with circular bright rings (called Airy rings). If the screen is far away from the aperture, this would be a Fraunhofer diffraction pattern.

Introduction to Diffraction



(a) Huygens-Fresnel principles states that each point in the aperture becomes a source of secondary waves (spherical waves). The spherical wavefronts are separated by λ. The new wavefront is the envelope of the all these spherical wavefronts. (b) Another possible wavefront occurs at an angle θ to the z-direction which is a diffracted wave.

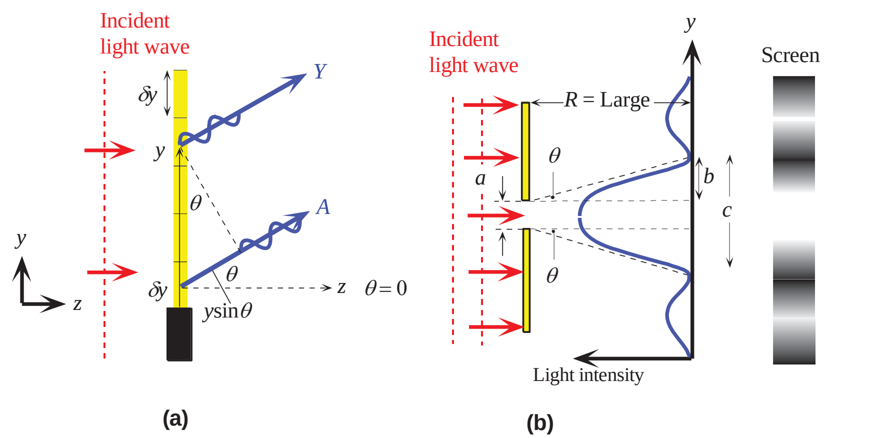
Introduction to Diffraction

- Light emitted from a point source

$$E \approx (\delta y) e^{-jk\sin\theta}$$

$$E(\theta) = C\int_{y=0}^{y=a} (\delta y) e^{-jk\sin\theta}$$

$$E(\theta) = \frac{Ce^{-jk\sin\theta} a \sin\left(\frac{1}{2} ka\sin\theta\right)}{\frac{1}{2} ka\sin\theta}$$



(a) The aperture is divided into N number of point sources each occupying δy with amplitude $\propto \delta y$. (b) The intensity distribution in the received light at the screen far away from the aperture: the diffraction pattern

- The single slit diffraction equation yields an intensity

$$I(\theta) = \left(\frac{Ca\sin\left(\frac{1}{2} ka\sin\theta\right)}{\frac{1}{2} ka\sin\theta}\right)^2 = I(0)\sin c\left(\frac{1}{2} ka\sin\theta\right)$$

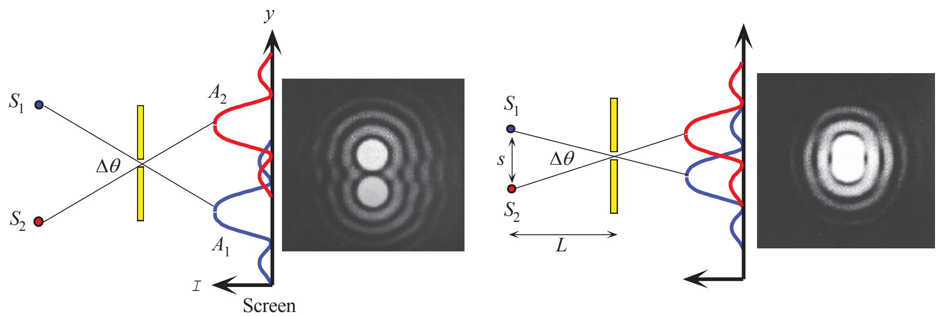
- With zero intensity points at

$$\sin\theta = \frac{m\lambda}{a}$$

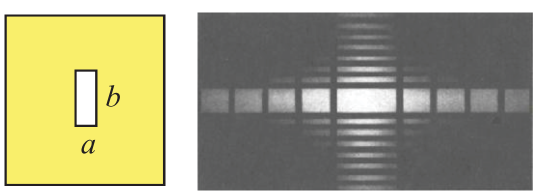
$$m = \pm 1, \pm 2, \ldots$$

$$\sin\theta = 1.22\frac{\lambda}{D}$$ where D is the diameter of the aperture

Image Resolution



Resolution of imaging systems is limited by diffraction effects. As points S_1 and S_2 get closer, eventually the Airy disks overlap so much that the resolution is lost.



The rectangular aperture of dimensions $a \times b$ on the left gives the diffraction pattern on the right.

- Consider 2 nearby coherent sources are imaged through an aperture of diameter D
- The two sources have an angular separation of $\Delta\theta$.
- As the points get closer together
  - angular separation becomes narrower
  - diffraction patterns overlap more
- According to the Rayleigh criterion, the two spots are just observable when the principle maximum of one diffraction pattern coincides with the minimum of another
- This minimum is obtained by the angular radius of the Airy disk

$$\sin\theta = 1.22\frac{\lambda}{D}$$

where D is the diameter of the aperture

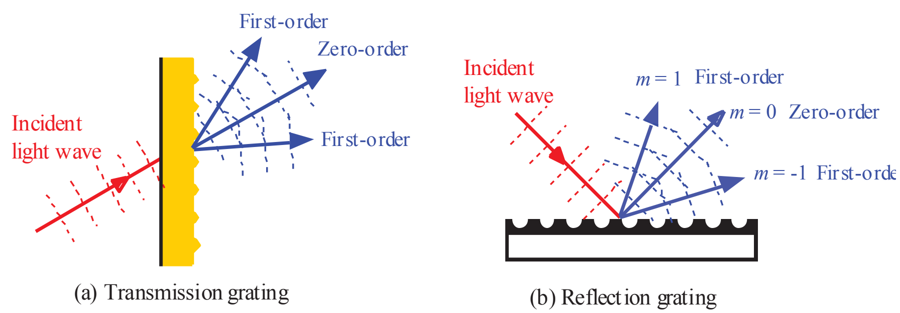
Diffraction Gratings

- Bragg Diffraction Condition

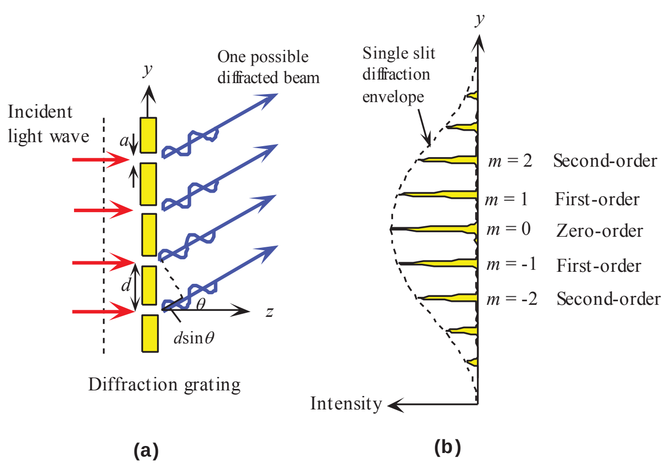
$$d\sin\theta = m\lambda$$
$$m = 0, \pm1, \pm2, \pm3, \ldots$$

- For light incident at an angle

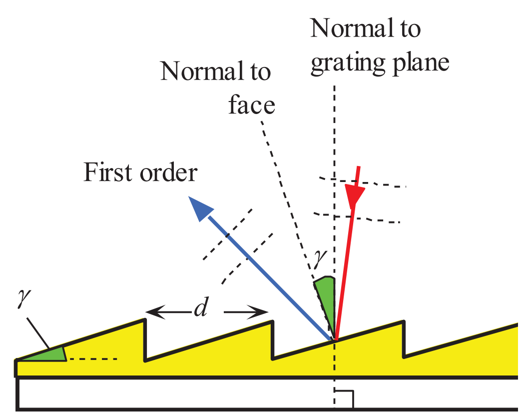
$$d(\sin\theta_m + \sin\theta_i) = m\lambda$$
$$m = 0, \pm1, \pm2, \pm3, \ldots$$



(a) Ruled periodic parallel scratches on a glass serve as a transmission grating. (b) A reflection grating. An incident light beam results in various "diffracted" beams. The zero-order diffracted beam is the normal reflected beam with an angle of reflection equal to the angle of incidence.



(a) A diffraction grating with *N* slits in an opaque scree. (b) The diffracted light pattern. There are distinct beams in certain directions (schematic)



Blazed (echelette) grating.