

به نام خدا

تمرین سری ۶ ریاضی مهندسی

۱) هر یک از مساله های ناهمگن زیر را حل کنید :

1-1)

$$u_{tt} - 9u_{xx} = x, \quad 0 < x < \pi, t > 0$$

$$u(x, 0) = 2 \quad u_t(x, 0) = x + 1$$

$$u_x(0, t) = t \quad u_x(\pi, t) = 2t$$

1-2)

$$u_{xx} + u_{yy} = x + 2y, \quad 0 < x < \pi, 0 < y < \pi$$

$$u(x, 0) = x \quad u(x, \pi) = 2 \quad 0 \leq x \leq \pi$$

$$u(0, y) = y \quad u(\pi, y) = \cos(y) \quad 0 \leq y \leq \pi$$

1-3)

$$u_{xx} = u_t - x,$$

$$u(0, t) = 0 \quad u(1, t) = \frac{5}{6}$$

$$u(x, 0) = -\frac{1}{6}x^3$$

1-4)

$$9u_{xx} = u_t + xt, \quad 0 < x < 1 \quad t > 0$$

$$u(0, t) = t \quad u(1, t) = 1$$

$$u(x, 0) = 2x$$

1-5)

$$u_{xx} = u_{tt} - e^{-t} \sin(3x), \quad 0 < x < \pi \quad t > 0$$

$$u(0, t) = 0 \quad u(\pi, t) = 1$$

$$u(x, 0) = \frac{x}{\pi} \quad u_t(x, 0) = 0$$

۲) مسایل پواسن زیر را حل کنید :

2-1)

$$u_{xx} + u_{yy} = x + 1, \quad 0 < x < a, 0 < y < b$$

$$u(0, y) = u_y(x, b) = u_y(x, 0) = 0$$

$$u(a, y) = \alpha(y)$$

2-2)

$$u_{xx} + u_{yy} = xy, \quad 0 < x < \pi, 0 < y < 1$$

$$u(0, y) = u(\pi, y) = 0$$

$$u(x, 0) = 0 \quad u(x, 1) = 0$$

2-3)

$$\nabla^2 u = 1 \quad 0 < x < \pi, 0 < y < \pi$$

$$u(0, y) = 0 \quad u_x(\pi, y) = 0 \quad 0 \leq x \leq \pi$$

$$u_y(x, \pi) + hu(x, \pi) = x^2 \quad u_y(x, 0) = 0$$