The Effects of Imbalanced Phase Shifters Loss on Phased Array Gain

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***Abstract*—In the presence of imbalanced insertion loss of phase-shifters, satisfying phase conjugate condition does not necessarily lead to the maximum phased array gain. This paper introduces *noncoherent* beamforming and uses lab-measured characteristics of varactor-based phase shifters to prove that by perturbing coherency partially, the array gain can increase significantly, e.g., 1.6 dB for a 12 element array. Moreover, it is shown that reduction in the average insertion loss increases the array efficiency by almost the same amount. These results are extendible to the optical beamforming networks where different lengths of fiber are used to make true time delay lines.**

***Index Terms*—Array efficiency, insertion loss, noncoherent beamforming, phased array antennas, phase shifter.**

I. Introduction

INSERTION loss of conventional phase shifters impairs the beamforming performance of phased array antennas. Analog IC designers have mostly focused on achieving linear phase shift; hence, the effect of Insertion Loss (IL) on the beamforming-the main purpose of using phase shifters—has been neglected. An increase in the receiver Noise Figure due to the unequal values of IL has been addressed earlier [1]; however, most of such analyses assume that IL is constant over the control voltage range, and the strongest signal can be achieved through coherent beamforming.

Different types of analog phase shifters have been developed so far [2]–[7], and the key element of most of them is a voltage variable capacitor (varactor). As the reverse bias voltage of the varactor varies, the capacitance and the phase of the total impedance change. Although this mechanism provides a linear or quasi-linear phase shift versus bias voltage, it causes two major problems. First, the insertion loss is not constant for all desired phase shifts, and second, it does not vary linearly versus the required phase shift (see Fig. 12 in [2] and Fig. 14 in [3]). In [4] a Ku-band ferroelectric phase shifter on Silicon was introduced, where IL changed from 0.4 to 2.6 dB at 15 GHz. In [5] an optimally loaded phase shifter with continuous phase shift from 0° to 360° at 20 GHz was reported. Its IL varied from 1.7 dB to 4.2 dB for the bias voltages ranging from 0 to -10 v (see Fig. 7 in [5]). As the authors correctly mentioned the majority of the circuit loss was due to the varactor diodes. In [6] a Ku-band MMIC analog phase shifter (12–14 GHz) was presented with a phase shift of more than 180° and an IL of 3.6±1.1 dB. Finally, a tunable phase shifter implemented in 0.18-μm CMOS process for monolithic microwave integrated circuit (MMIC) applications was presented in [7], where the average IL of the phase shifter was 0.3 dB with a variation of ±0.8 dB. Despite the different implementation methods and operational frequencies, all of these phase shifters [2]–[7] suffer from non-uniform IL versus control (bias) voltage. The goal of this paper is to analyze the drawbacks of phase shifter's imbalanced IL on array efficiency, and propose a compensation method.

In the following, first the maximum array gain theorem is reviewed and a general solution to the coherent beamforming, based on adding a constant phase to all phase shifters, is presented. Next, noncoherent beamforming is introduced, and the lab measured data for varactor-based phase shifters are used to compare the performance of coherent and noncoherent beamforming. The effect of reducing the variations of the IL on the array efficiency is discussed in Section IV.

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II. Coherent Beamforming

A. Maximum Array Gain Theorem

Fig. 1 shows a linear array of N identical omni-directional elements separated by a distance d. A source radiates at angle θ relative to the normal to the array. The received signals pass through phase shifters, represented by weights $w_1, w_2, \ldots w_N$, and combine by the power-combiner, denoted by $\mathbf{\Sigma}$. Each weight is a complex number with magnitude A_i and phase ψ_i,

$$w_i = A_i e^{j\psi_i}. \tag{1}$$

We assume that the received power by each element is equal to unity, so the total received signal by the combiner ($\mathbf{\Sigma}$) is

$$S(\psi_1, \ldots, \psi_N) = \sum_{m=1}^{N} w_m e^{-j\phi_m} = \sum_{m=1}^{N} (A_m e^{j\psi_m}) e^{-j\phi_m} \tag{2}$$

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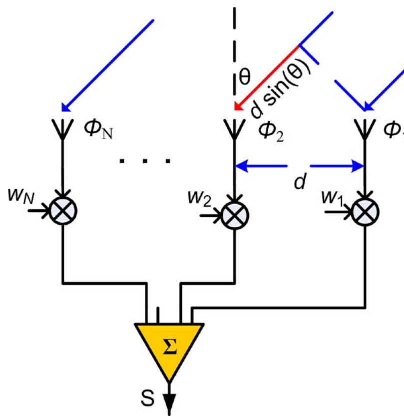

Fig. 1. Simplified block diagram of a linear array of N identical elements.

where ϕ_m is the phase of the received signal by element m. If the source radiates at wavelength λ, the *phase-lag* between elements 1 and m is related to λ, d and θ by

$$\Delta\phi_m = \phi_m - \phi_1 = 2\pi(m-1)\frac{d\sin(\theta)}{\lambda}. \tag{3}$$

The aim of beamforming is to compensate such phase-lags by adjusting phase shifters. The maximum array gain theorem states that the array gain reaches its maximum value when each element's weight (w_i) is proportional to the complex conjugate of the element's gain [8]. When elements are identical, each phase shifter must be adjusted to provide the phase conjugate of the corresponding received signal. So, regarding our sign convention in (2), for each phase shifter we must have

$$\psi_m = \phi_m \quad 1 \le m \le N. \tag{4}$$

As a result, the exponential terms in (2) vanish and all received signals become coherent at power combiner, hence, the total received signal, S_{Coh}, is the sum of the individual magnitudes

$$S_{Coh} = S(\phi_1, \ldots, \phi_N) = \sum_{m=1}^{N} A_m. \tag{5}$$

This theorem is valid if the amplitude of w_i, i.e. A_i, is independent of its phase (ψ_i), but as discussed in the introduction, this assumption is not valid for the current analog phase shifters [2]–[7]. We will show when A_i varies, noncoherent addition of the signals results in a stronger sum.

Fig. 2 shows the normalized insertion loss $(|S_{12}|)$ and the insertion phase $(\angle S_{12})$ of a Ku-band varactor-based phase shifter [9]. By normalization, we mean that all constant losses such as copper loss have been neglected. Insertion phase in Fig. 2(c) varies almost linearly versus the bias voltage, but insertion loss in Fig. 2(b) shows a nonlinear behavior. If v_i is the control voltage of phase shifter i, (1) can be expressed as

$$w_i = A(v_i)e^{j\psi(v_i)}. \tag{6}$$

Since phase shift, $\psi_i(v_i)$, is a monotonic function of the bias voltage, the phase shifter weights can be denoted in the form

$$w_i = A(\psi_i)e^{j\psi_i} \tag{7}$$

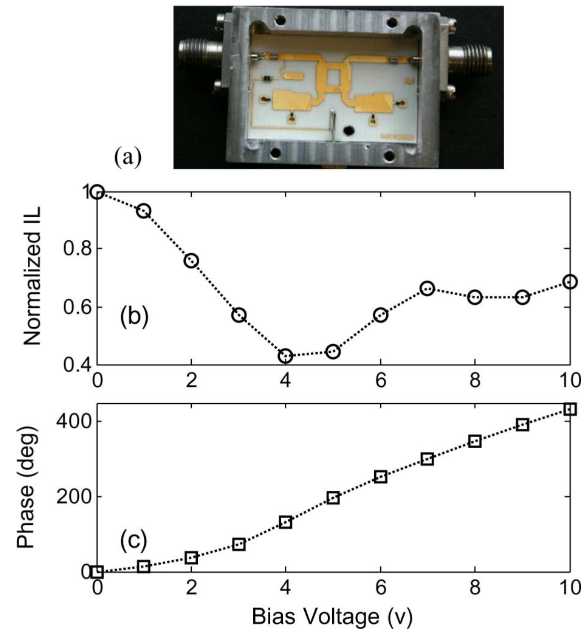

Fig. 2. A reflective-type phase-shifter with 90° hybrid coupler (a) fabricated phase shifter, (b) measured normalized IL, (c) phase shift versus bias voltage.

which implies that IL is a function of the required phase shift. Combining (3), (4) and (7), (5) can be expressed in terms of $A(\psi)$

$$S_{Coh} = \sum_{m=1}^{N} A(\phi_1 + \Delta\phi_m). \tag{8}$$

If IL were constant versus the bias voltage, $A(\psi) = A_0$, for all values of ϕ_1, S_{Coh} would have been a constant equal to NA_0.

B. Phase-Added Coherent Beamforming

If we add a constant phase $(\delta\psi)$ to the coherent phase shift of each phase shifter, clearly the phase differences between elements do not change. However, when IL varies as in Fig. 2(b), where $A(\phi_m + \delta\psi) \neq A(\phi_m)$, the amplitude of the total received signal will change. It is possible to find a proper $\delta\psi$ that provides a stronger sum, such that

$$S_{Coh} \le |S(\phi_1 + \delta\psi, \ldots, \phi_N + \delta\psi)|. \tag{9}$$

From (2) and (4), $S(\phi_1 + \delta\psi, \ldots, \phi_N + \delta\psi)$ can be calculated

$$S(\phi_1 + \delta\psi, \ldots, \phi_N + \delta\psi) = \sum_{m=1}^{N} A(\phi_1 + \delta\psi + \Delta\phi_m)e^{j\delta\psi}. \tag{10}$$

The magnitude of $S(\phi_1 + \delta\psi, \ldots, \phi_N + \delta\psi)$ is given by

$$|S(\phi_1 + \delta\psi, \ldots, \phi_N + \delta\psi)| = \sum_{m=1}^{N} A(\phi_1 + \delta\psi + \Delta\phi_m). \tag{11}$$

To illustrate the advantage of adding a constant phase, consider an array of two elements, $N = 2$, with half-wavelength spacing. The source direction (θ in Fig. 1), varies from 0–180°. Fig. 3(a) compares the coherent array gain, S_{Coh}, and the added-phase gain, i.e. $S(\phi_1 + \delta\psi, \phi_2 + \delta\psi)$, for the best $\delta\psi$ which gives the

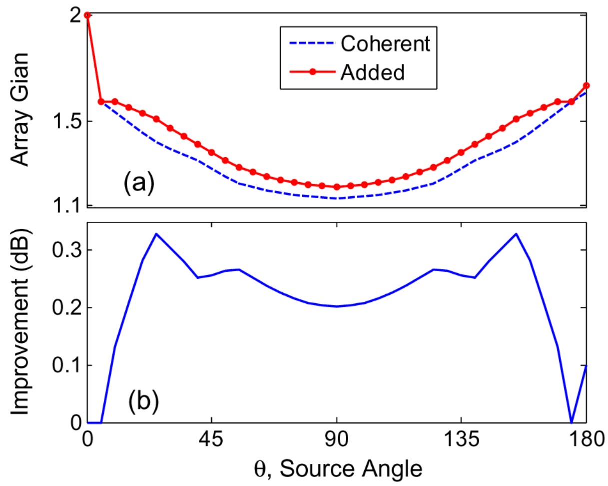

Fig. 3. Comparison of coherent and phase-added coherent beamforming. (a) Array gain, (b) improvement in the array gain due to adding a constant phase.

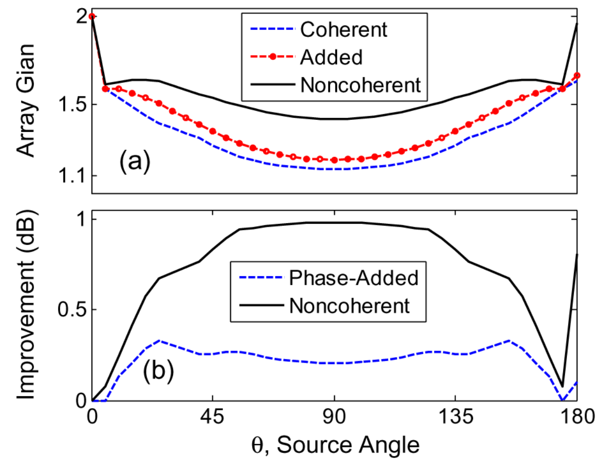

Fig. 4. Comparison of 3 different beamforming methods for a 2-element array. (a) Maximum array gain and, (b) improvement in the array gain due to noncoherent beamforming, compared to coherent beamforming.

highest array gain. The best $\delta\psi$ depends on the source direction. Fig. 3(b) shows the improvement in the array gain achieved by adding a constant phase, which varies from 0 to 0.33 dB, and its maximum occurs at $\theta = 24^\circ$. So when IL varies with voltage, the coherent beamforming ($\delta\psi = 0^\circ$) does not necessarily bring about the maximum gain. If the array efficiency, η, is defined as

$$\eta = \frac{\sum_{m=1}^{N} w_m e^{-j\phi_m}}{N} = \frac{S}{N} \tag{12}$$

the minimum of η occurs when $\theta = 90^\circ$, which is 61.4% for added-phase and 56.7% for coherent beamforming.

III. Noncoherent Beamforming

In this section, we investigate if coherency can be violated partially to obtain a larger total received signal. Suppose for each element k in Fig. 1, the applied phase shift is ζ_k degrees more than the required coherent phase shift, i.e. $\psi_k = \phi_k + \zeta_k$. Remembering (2), we define the *noncoherent* array gain as

$$S_{Non} = S(\phi_1+\zeta_1, \ldots, \phi_N+\zeta_N) = \sum_{m=1}^{N} \left[A(\phi_m + \zeta_m)e^{j\zeta_m}\right]. \tag{13}$$

The magnitude of S_{Non} is then shown in (14) at the bottom of the page, which is less than $\Sigma A(\phi_m + \zeta_m)$, the sum of amplitudes. To maximize $|S_{Non}|$, we have to calculate the gradient of $|S_{Non}|$ or $|S_{Non}|^2$ with respect to ζ_k, $(1 \le k \le N)$ and find its roots. There is no general analytical solution to find these roots for all values of k. We have proposed an iterative gradient approximation method [10] to find the best approximation of ζ_k, $(1 \le k \le N)$ which maximizes $|S_{Non}|$

$$\zeta_k(n+1) = \zeta_k(n) + \frac{1}{2}\mu\hat{G}_k(n) \tag{15}$$

where n is the current iteration, μ is the step size and $\hat{G}_k(n)$ is the k component of the approximate gradient derived from

$$\hat{G}_k(n) = \frac{|S_{Non}(\zeta_k(n)+\delta)| - |S_{Non}(\zeta_k(n)-\delta)|}{2\delta}. \tag{16}$$

In (16), δ is a phase perturbation applied by phase shifter k to estimate the partial derivative of $|S_{Non}|$ relative to ζ_k. Fig. 4 compares the performance of noncoherent and coherent beamforming for the 2-element array in Section II-B.

A significant improvement in the array gain is observed for $\theta > 5^\circ$. For example for $\theta = 90^\circ$ the array gain obtained by the noncoherent beamforming is close to 1 dB better than that of the coherent beamforming. Also the array efficiency increases to 71% compared to 56.7% for coherent beamforming.

IV. Improvement in the Gain of 2D Arrays

A. Case Study

To extend the results of previous sections to two dimensional (2D) arrays, we consider an array of 12 identical omni-directional elements arranged in a planar rectangular structure as depicted in Fig. 5. The inter-element spacing is $\lambda/2$ and $\lambda = 2.4$ cm (Ku-band). All antennas are followed by identical phase shifters like Fig. 2, and connected to a 12 to 1 power combiner. The source scans a region in the spherical coordinates from

$$|S_{Non}| = \sqrt{\left(\sum_{m=1}^{N}\left[A(\phi_m+\zeta_m)\cos(\zeta_m)\right]\right)^2 + \left(\sum_{m=1}^{N}\left[A(\phi_m+\zeta_m)\sin(\zeta_m)\right]\right)^2} \tag{14}$$

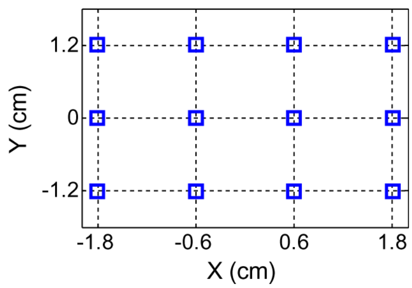

Fig. 5. Geometry of the planar array consisted of 12 identical elements.

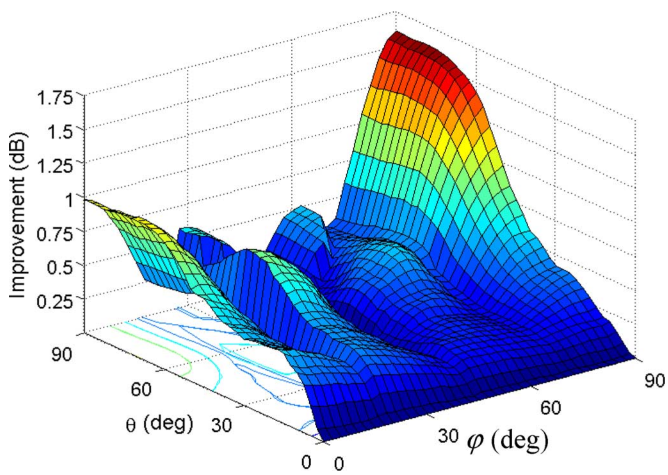

Fig. 6. Improvement in the gain of the 12-element phased array antenna due to noncoherent beamforming for different source directions.

$\theta = 0°$ to $90°$ and $\varphi = 0°$ to $90°$ in steps of $2.5°$. Both coherent and noncoherent beamforming methods are performed for each source direction. The noncoherent algorithm is executed for 50 iterations to assure the convergence of the beamforming. Fig. 6 illustrates the improvement in the array gain due to the noncoherent beamforming. While for the broadside direction ($\theta = 0°$) the improvement is zero as expected, it increases to 1.6 dB for the end-fire direction where θ and φ are both 90°. The average array gain improvement over the whole region is 0.44 dB. Moreover, the maximum increase in the array efficiency for this 12-element array is 24.7% and the mean value is 5.9%.

Fig. 7 shows the effects of the noncoherent algorithm on the phase and IL of each phase shifter when the source is at $(\theta, \varphi) = (46°, 20°)$. It is seen that the gain improvement is 0.8 dB. In this case all phases of the noncoherent beamforming are smaller than the coherent beamforming, hence we observe more than 40% increase in the amplitude of elements 1, 4, and 10, according to Fig. 2(c). However the amplitude of element 7 has dropped by 18%, because the proposed noncoherent beamforming compromises between coherency and loss.

B. Reducing the Phase Shifter Insertion Loss

Assume there is another phase shifter, named PS2, whose IL is the square root of the phase shifter in Fig. 2(b), named PS1, with the same phase-voltage characteristics. Hence the average IL reduces from −1.77 dB to −0.92 dB. Fig. 8 illustrates the array efficiencies of the 12 element phased array, obtained by noncoherent and coherent methods for both phase shifters. The

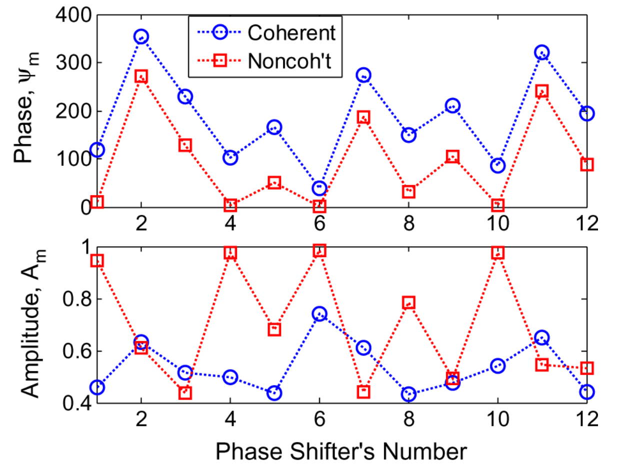

Fig. 7. Phase and amplitude of all phase shifters, when noncoherent and coherent methods are applied to the 12-element array. The source is at $(\theta, \varphi) = (46°, 20°)$, and the array gain improvement is 0.8 dB.

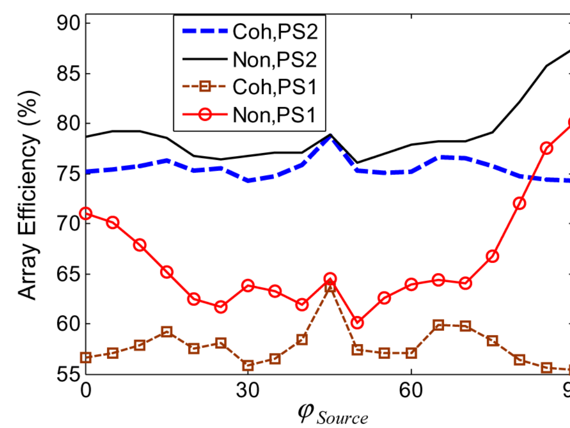

Fig. 8. Array efficiency of coherent and noncoherent beamforming for the low loss and regular phase shifters. The target is located at $\theta = 90°$.

TABLE I
BEHAVIOR OF THE ARRAY EFFICIENCY FOR COHERENT AND NONCOHERENT BEAMFORMING WITH REGULAR (PS1) AND LOW LOSS (PS2) PHASE SHIFTERS

Phase Shifter	Beamforming Method	η_{MEAN}	η_{MIN}	η_{MAX}
PS1	Coherent	57.6%	55.3%	63.8%
	Noncoherent	65.8%	59.0%	80.1%
PS2	Coherent	75.5%	74.3%	78.8%
	Noncoherent	79.0%	76.1%	87.5%

θ coordinate of the source is fixed at 90° but its φ coordinate varies from 0°–90°. Table I summarizes the results, and shows that if PS1 is replaced with PS2, the mean array efficiencies of the coherent and noncoherent methods increase by 17.9% and 13.2% respectively, which corresponds to 1.17 dB and 0.8 dB increase in the array gain. So, decreasing IL variations narrows the gap between the coherent and noncoherent beamforming as it is expected.

V. CONCLUSION

We reviewed the coherent beamforming relations for a receiver array (Fig. 1), and showed when the insertion loss of phase shifter varies with the applied control voltage as in Fig. 2(b), by adding a constant phase to the coherent phase-shifts the array gain improves. For example, for a 2-element array the array gain can increase up to 0.33 dB (Fig. 3). We introduced the *noncoherent* beamforming which states that in the presence of imbalanced IL, a moderate degree of noncoherency enhances the array gain (Fig. 7), and illustrated that for a planar 12-element array a maximum gain improvement of 1.6 dB can be obtained (Fig. 6). The improvement factor of the noncoherent beamforming depends on the geometry of the array and direction of arrival of the source signal. These outcomes are valid for all beamforming networks which use elements with a variable IL such as true time delay lines, where attenuation varies with fiber length. Reduction of IL variations increases the array efficiency and narrows the gap between the noncoherent and coherent array efficiencies (Fig. 8). The alternative to noncoherent beamforming is to redesign the beamforming network by adding high-gain (or variable gain) and low-noise amplifiers before or after each phase shifter, which increases the complexity and cost of the system. Instead, noncoherent beamforming does not add to the total cost and has a significant performance for low cost phased arrays.

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