

# Machine learning theory

## Introduction

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2. Supervised learning
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## Introduction

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## Definition (Mohri et. al., 2018)

Computational **methods** that use **experience** to **improve performance** or to make accurate predictions.

## Definition (Mitchell, 1997)

A computer program is said to learn from experience  $E$  with respect to some class of tasks  $T$  and performance measure  $P$ , if its performance at tasks in  $T$ , as measured by  $P$ , improves with experience  $E$ .

## Example (Spam classification)

- ▶ **Task:** determine if emails are spam or non-spam.
- ▶ **Experience:** incoming emails with human classification.
- ▶ **Performance Measure:** percentage of correct decisions.



We need machine learning because

1. Tasks are too complex to program but they are performed by animals/humans such as driving, speech recognition, image understanding, and etc.
2. Tasks beyond human capabilities such as weather prediction, analysis of genomic data, web search engines, and etc.
3. Some tasks need adaptivity. When a program has been written down, it stays unchanged. In some tasks such as **optical character recognition** and **speech recognition**, we need the behavior to be adapted when new data arrives.



Machine learning algorithms based on the information provided to the learner can be classified into different groups.

1. **Supervised/predictive** vs **unsupervised/descriptive** vs **reinforcement** learning.
2. **Batch** vs **online** learning
3. **Passive** vs **Active** learning.
4. **Cooperative** vs **adversarial** teachers.



## 1. Supervised learning:

- ▶ **Classification:**
  - ▶ Document classification and spam filtering.
  - ▶ Image classification and handwritten recognition.
  - ▶ Face detection and recognition.
- ▶ **Regression:**
  - ▶ Predict stock market price.
  - ▶ Predict temperature of a location.
  - ▶ Predict the amount of PSA.

## 2. Unsupervised/descriptive learning:

- ▶ Discovering clusters.
- ▶ Discovering latent factors.
- ▶ Discovering graph structures (correlation of variables).
- ▶ Matrix completion (filling missing values).
- ▶ Collaborative filtering.
- ▶ Market-basket analysis (frequent item-set mining).

## 3. Reinforcement learning:

- ▶ Game playing.
- ▶ robot navigation.



- ▶ A key concept in machine learning is **uncertainty**.
- ▶ Data comes from a process that **is not completely known**.
- ▶ This lack of knowledge is indicated by modeling the process as a **random process**.
- ▶ The process actually may be **deterministic**, but we don't have access to **complete knowledge** about it, we model it as **random** and we use the **probability theory** to analyze it.

## Supervised learning

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- ▶ In supervised learning, the goal is to find a mapping from inputs  $X$  to outputs  $t$  given a labeled set of input-output pairs

$$S = \{(x_1, t_1), (x_2, t_2), \dots, (x_m, t_m)\}.$$

$S$  is called **training set**.

- ▶ In the simplest setting, each training input  $x$  is a  $D$ -dimensional vector of numbers.
- ▶ Each component of  $x$  is called **feature**, **attribute**, or **variable** and  $x$  is called **feature vector**.
- ▶ In general,  $x$  could be a complex structure of object, such as an image, a sentence, an email message, a time series, a molecular shape, a graph.
- ▶ When  $t_i \in \{-1, +1\}$  or  $t_i \in \{0, 1\}$ , the problem is **classification**.
- ▶ When  $t_i \in R$ , the problem is known as **regression**.

## Supervised learning

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### Classification



- ▶ Given a hypothesis space  $H$ , the learning algorithm should find a particular hypotheses  $h \in H$  to approximate  $C$  as closely as possible.
- ▶ We choose  $H$  and the aim is to find  $h \in H$  that is similar to  $C$ . This reduces the problem of learning the class to the easier problem of finding the parameters that define  $h$ .
- ▶ Hypothesis  $h$  makes a prediction for an instance  $x$  in the following way.

$$h(x) = \begin{cases} 1 & \text{if } h \text{ classifies } x \text{ as a positive example} \\ 0 & \text{if } h \text{ classifies } x \text{ as a negative example} \end{cases}$$



- ▶ In real life, we don't know  $c(x)$  and hence cannot evaluate how well  $h(x)$  matches  $c(x)$ .
- ▶ We use a small subset of all possible values  $x$  as the **training set** as a representation of that concept.
- ▶ **Empirical error (risk)/training error** is the proportion of training instances such that  $h(x) \neq c(x)$ .

$$\hat{\mathbf{R}}(h) = \frac{1}{m} \sum_{i=1}^m \mathbb{I}[h(x_i) \neq c(x_i)]$$

- ▶ When  $\hat{\mathbf{R}}(h) = 0$ ,  $h$  is called a **consistent hypothesis** with dataset  $S$ .
- ▶ For many examples, we can find infinitely many  $h$  such that  $\hat{\mathbf{R}}(h) = 0$ . But which of them is better than for prediction of future examples?
- ▶ This is the problem of **generalization**, that is, how well our hypothesis will correctly classify the future examples that are not part of the training set.



- ▶ The **generalization capability of a hypothesis** usually measured by the true error/risk.

$$\mathbf{R}(h) = \mathbb{P}_{x \sim \mathcal{D}} [h(x) \neq c(x)]$$

- ▶ We assume that  **$H$  includes  $C$** , that is there exists  $h \in H$  such that  $\hat{\mathbf{R}}(h) = 0$ .
- ▶ Given a hypothesis class  $H$ , it may be the cause that we cannot learn  $C$ ; that is there is no  $h \in H$  for which  $\hat{\mathbf{R}}(h) = 0$ .
- ▶ Thus in any application, we need to make sure that  $H$  is **flexible enough**, or has **enough capacity** to learn  $C$ .

## Supervised learning

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Regression



- ▶ In regression,  $c(x)$  is a continuous function. Hence the training set is in the form of

$$S = \{(x_1, t_1), (x_2, t_2), \dots, (x_m, t_m)\}, t_k \in \mathbb{R}.$$

- ▶ In regression, there is noise added to the output of the unknown function.

$$t_k = f(x_k) + \epsilon \quad \forall k = 1, 2, \dots, m$$

$f(x_k) \in \mathbb{R}$  is the unknown function and  $\epsilon$  is the random noise.

- ▶ The explanation for the noise is that there are extra hidden variables that we cannot observe.

$$t_k = f^*(x_k, z_k) + \epsilon \quad \forall k = 1, 2, \dots, N$$

$z_k$  denotes hidden variables

- ▶ Our goal is to approximate the output by function  $g(x)$ .
- ▶ The empirical error on the training set  $S$  is

$$\hat{\mathbf{R}}(h) = \frac{1}{m} \sum_{k=1}^m [t_k - g(x_k)]^2$$

- ▶ The aim is to find  $g(\cdot)$  that **minimizes the empirical error**.
- ▶ We assume that a hypothesis class for  $g(\cdot)$  has a small set of parameters.

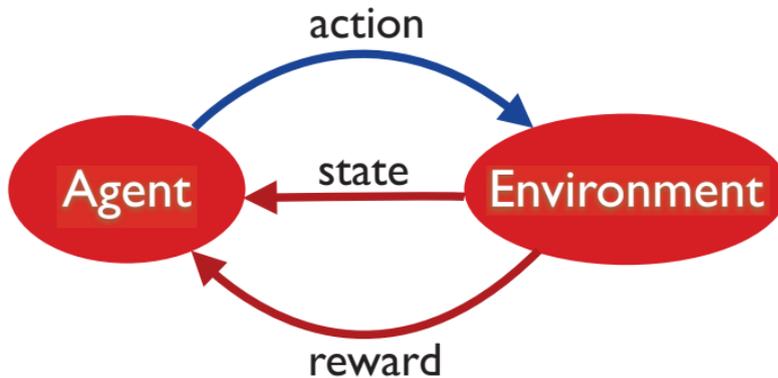
## Reinforcement learning

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- ▶ Reinforcement learning is what to do (**how to map situations to actions**) so as to maximize a scalar reward/reinforcement signal
- ▶ The learner is not told which actions to take as in **supervised learning**, but discover which actions yield the most reward by trying them.
- ▶ The **trial-and-error** and **delayed reward** are the two most important feature of **reinforcement learning**.
- ▶ Reinforcement learning is defined not by characterizing **learning algorithms**, but by characterizing **a learning problem**.
- ▶ Any algorithm that is well suited for solving the given problem, we consider to be a **reinforcement learning**.
- ▶ One of the challenges that arises in reinforcement learning and other kinds of learning is **tradeoff** between **exploration** and **exploitation**.

- ▶ A key feature of reinforcement learning is that it explicitly considers the **whole problem** of a **goal-directed agent** interacting with an **uncertain environment**.



## Unsupervised learning

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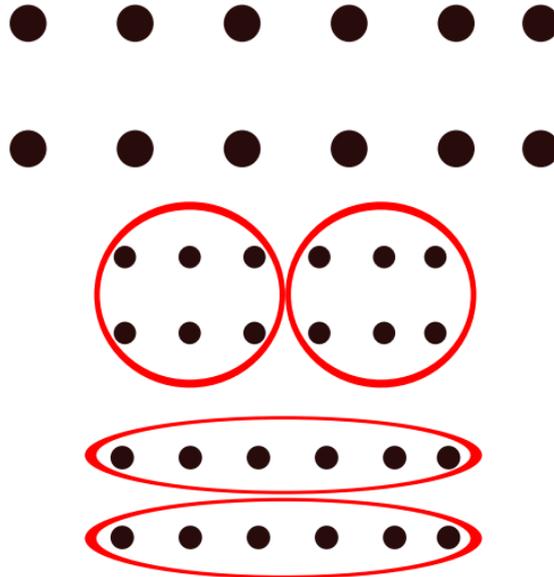
- ▶ Unsupervised learning is fundamentally problematic and subjective.
- ▶ Examples :
  1. **Clustering** : Find natural grouping in data.
  2. **Dimensionality reduction** : Find projections that carry important information.
  3. **Compression** : Represent data using fewer bits.
- ▶ Unsupervised learning is like supervised learning with missing outputs (or with missing inputs).



- ▶ Given data

$$X = \{x_1, x_2, \dots, x_m\}$$

learn to **understand** the data, by re-representing it in some intelligent way.



## Machine learning theory

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What is machine learning theory?

1. What are the intrinsic properties of a given learning problem that make it hard or easy to solve?
2. How much do you need to know ahead of time about what is being learned in order to be able to learn it effectively?
3. Why are **simpler hypotheses** better?
4. How do we **formalize** machine learning problems (for eg. online , statistical)?
5. How do we pick the right model to use and what are the tradeoffs between various models?
6. How many instances do we need to see to learn to given accuracy?
7. How do we design learning algorithms with provable guarantees on performance?



1. Suppose that you have a coin that has an unknown probability  $\theta$  of coming up heads.
2. We must determine this probability as accurately as possible using experimentation.
3. Experimentation is to repeatedly tossing the coin. Let us denote the two possible outcomes of a single toss by **1 (for head)** and **0 (for tail)**.
4. If you toss the coin  $m$  times, then you can record the outcomes as  $x_1, \dots, x_m$ , where each  $x_i \in \{0, 1\}$  and  $\mathbb{P}[x_i = 1] = \theta$  independently of all other  $x_i$ 's.
5. What would be a reasonable estimate of  $\theta$ ? By Law of Large Numbers, in a long sequence of independent coin tosses, the relative frequency of heads will eventually approach the true value of  $\theta$  with high probability. Hence,

$$\hat{\theta} = \frac{1}{m} \sum_i x_i$$

6. Using **Chernoff bound**, we have

$$\mathbb{P} \left[ |\hat{\theta} - \theta| > \epsilon \right] \leq 2e^{-2\epsilon^2 m}$$

7. Equivalently,

$$m \geq \frac{1}{2\epsilon^2} \log \left( \frac{2}{\delta} \right),$$

where  $1 - \delta$  specifies the confidence of estimation.



1. There are two basic questions
  - 1.1 How large of a sample do we need to achieve a given accuracy with a given confidence?
  - 1.2 How efficient can our learning algorithm be?
2. The first question is within **statistical learning theory**.
3. The second question is within **computational learning theory**.
4. However, there are some overlaps between these two fields.

## Outline of course

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## Outline of course

1. Introduction
2. Part 1 (Theoretical foundation)
  - 2.1 Consistency and PAC models
  - 2.2 Learning by uniform convergence
  - 2.3 Empirical and structural risk minimization
  - 2.4 Growth functions, VC-dimension, covering number, ...
  - 2.5 Learning by non-uniform convergence, SRM, and MDL
  - 2.6 Generalization bounds
  - 2.7 Regularization and stability of algorithms
  - 2.8 Analysis of kernel learning
  - 2.9 Computational complexity and running time of learning algorithms
  - 2.10 PAC-MDP model for reinforcement learning
  - 2.11 Theoretical foundation of clustering
  - 2.12 Convex learning problems
3. Part 2 (Analysis of algorithms)
  - 3.1 Linear classification
  - 3.2 Boosting
  - 3.3 SVM and Kernel based learning
  - 3.4 Regression
  - 3.5 Learning automata
  - 3.6 Reinforcement learning
  - 3.7 Ranking



- 3.8 Online learning
- 3.9 Active learning
- 3.10 Semi-supervised learning
- 3.11 Deep learning
- 4. Part 3 (Advanced topics)
  - 4.1 Radamacher Complexity
  - 4.2 PAC-Bayes theory
  - 4.3 Universal learning
  - 4.4 Other advanced topics
- 5. Part 4 (Applications)
  - 5.1 Learnability of some problems in Social networks
  - 5.2 Analysis of some graph-based problems
  - 5.3 Generalization bound for Domain adaptation
  - 5.4 Generalization bound for sampling-based algorithms



- ▶ **Evaluation:**

Mid-term exam	25%	1401/01/ 24
Final exam	35%	
Homeworks	25%	
Quiz	10%	
Paper & Project	10%	Explore a theoretical or empirical question and present it.
- ▶ Deadline for choosing paper: 1401/02/01  
**You should send the selected paper to Mr. Soudani with me as a cc.**
- ▶ Class Link: <https://vc.sharif.edu/ch/beigy>
- ▶ Course Website:  
<http://ce.sharif.edu/courses/00-01/2/ce718-1/>  
<http://sharif.edu/~beigy/14002-40718.html>
- ▶ **TAs :**
  - Heydar Soudani
  - Mohammad Hossein Mojab
  - Pooya Moeini



1. [IEEE Trans on Pattern Analysis and Machine Intelligence](#)
2. [Journal of Machine Learning Research](#)
3. [Pattern Recognition](#)
4. [Machine Learning](#)
5. [Neural Networks](#)
6. [Neural Computation](#)
7. [Neurocomputing](#)
8. [IEEE Trans. on Neural Networks and Learning Systems](#)
9. [Annuals of Statistics](#)
10. [Journal of the American Statistical Association](#)
11. [Pattern Recognition Letters](#)
12. [Artificial Intelligence](#)
13. [Data Mining and Knowledge Discovery](#)
14. [IEEE Transaction on Cybernetics \(SMC-B\)](#)
15. [IEEE Transaction on Knowledge and Data Engineering](#)
16. [Knowledge and Information Systems](#)

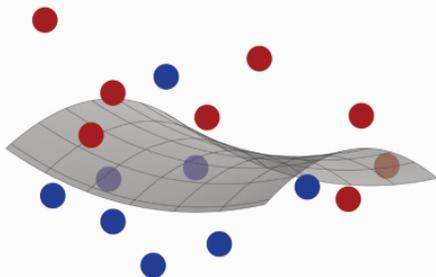


1. [Neural Information Processing Systems \(NIPS\)](#)
2. [International Conference on Machine Learning \(ICML\)](#)
3. [European Conference on Machine Learning \(ECML\)](#)
4. [Asian Conference on Machine Learning \(ACML\)](#)
5. [Conference on Learning Theory \(COLT\)](#)
6. [Algorithmic Learning Theory \(ALT\)](#)
7. [Conference on Uncertainty in Artificial Intelligence \(UAI\)](#)
8. [Practice of Knowledge Discovery in Databases \(PKDD\)](#)
9. [International Joint Conference on Artificial Intelligence \(IJCAI\)](#)
10. [IEEE International Conference on Data Mining series \(ICDM\)](#)

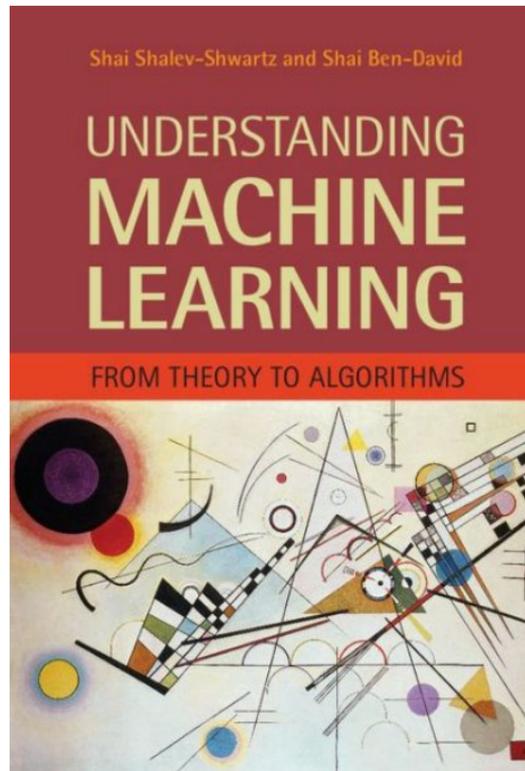
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