سئوال ۱

$L = pr \xrightarrow[r=\text{const}]{\text{سیر دایره ای}} \Delta L = \Delta p\, r$ (۱۵)

$x = r\theta \xrightarrow{\quad''\quad} \Delta x = r\Delta\theta$ (۱۵)

$\Delta x \Delta p = r\Delta\theta \dfrac{\Delta L}{r} \geq \hbar/2$ (۱۵)

نمره ۲

$\Delta\theta = 2\pi$ توقعیت زاویه ای مشخص $\longrightarrow$ $\Delta L \geq \dfrac{\hbar}{4\pi}$ (۱۵)

سئوال ۲ الف)

$\psi(x) = \int_{-\infty}^{\infty} A(k)\, e^{ikx}\, dk = \int_{-\infty}^{0} N e^{\alpha k} e^{ikx}\, dk + \int_{0}^{\infty} N e^{-\alpha k} e^{ikx}\, dk$ (۱۵)

نمره ۴

$= \dfrac{N}{\alpha + ix}\, e^{ixk}\, e^{\alpha k}\Big|_{-\infty}^{0} + \dfrac{N}{-\alpha + ix}\, e^{ikx}\, e^{-\alpha k}\Big|_{0}^{\infty} = \dfrac{N}{\alpha + ix} - \dfrac{N}{-\alpha + ix} = \dfrac{-2i\alpha N}{\alpha^2 + x^2}$ (۱۵)

توابع نمایی که حداکثر مقدار ۱ و حداقل مقدار ۰ می پذیرند

ب)

$E = \dfrac{p^2}{2m}$ (۲۰)

$E = \hbar\omega \;;\; p = \hbar k$ (۱۰)

$\longrightarrow \hbar\omega = \dfrac{\hbar^2 k^2}{2m} \longrightarrow \omega = \dfrac{\hbar k^2}{2m}$ (۱۵)

$v_g = \dfrac{d\omega}{dk} = \dfrac{\hbar k}{m}$ (۱۲۵) $\qquad v_p = \dfrac{\omega}{k} = \dfrac{\hbar k}{2m}$ (۱۲۵)

ج)

$A(k, t=0) = N e^{-\alpha k^2} \longrightarrow \psi(x, t=0) = \int_{-\infty}^{\infty} N e^{-\alpha k^2} e^{ikx}\, dk = N \int_{-\infty}^{\infty} e^{-\alpha\left(k^2 - \frac{ix}{\alpha}k\right)}\, dk$

$= N \int_{-\infty}^{\infty} e^{-\alpha\left(k - \frac{ix}{2\alpha}\right)^2} e^{-x^2/4\alpha}\, dk = N\sqrt{\dfrac{\pi}{\alpha}}\, e^{-x^2/4\alpha}$

$|\psi(x, t=0)|^2 = \dfrac{N^2\pi}{\alpha} e^{-x^2/2\alpha} \longrightarrow |\psi(x, t=0)|^2 = \dfrac{1}{e} |\psi(x, t=0)|^2_{max} \longrightarrow x = \pm\sqrt{2\alpha}$

$\rightarrow \Delta x = 2\sqrt{2\alpha}$ عرض پهنای پک

۰٫۵

$$\psi(x,t) = \int_{-\infty}^{\infty} N\, dk\; e^{-ak^2}\, e^{i\omega t}\, e^{ikx} = \int_{-\infty}^{\infty} N\, dk\; e^{-\overbrace{\left(\alpha - \frac{i\hbar t}{2m}\right)}^{\gamma} k^2}\, e^{ikx}$$

$$= N\int_{-\infty}^{\infty} dk\; e^{-\gamma\left(k^2 - \frac{ix}{\gamma}k\right)} = N\sqrt{\frac{\pi}{\gamma}}\, e^{-x^2/4\gamma} = N\sqrt{\frac{\pi}{\alpha - i\hbar t/2m}}\; e^{\frac{-x^2}{4(\alpha - i\hbar t/2m)}}$$

۰٫۲۵

$$|\psi(x,t)|^2 = \psi(x,t)\,\psi^*(x,t) = \frac{N^2\pi}{\alpha^2 + \hbar^2 t^2/4m^2}\exp\left(\frac{-\alpha x^2}{2(\alpha^2 + \hbar^2 t^2/4m^2)}\right) = \frac{N^2\pi}{\alpha^2 + \frac{\hbar^2 t^2}{4m^2}}\, e^{-1}$$

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$$\to x^2 = \frac{2}{\alpha}\left(\alpha^2 + \frac{\hbar^2 t^2}{4m^2}\right) \longrightarrow x = \pm\sqrt{\frac{2}{\alpha}\left(\alpha^2 + \frac{\hbar^2 t^2}{4m^2}\right)} \longrightarrow \Delta x = 2\sqrt{\frac{2}{\alpha}\left(\alpha^2 + \frac{\hbar^2 t^2}{4m^2}\right)}$$

۰٫۵

۰٫۵

سؤال ۳

$$2d\sin\theta = n\lambda \to 2\times 10^{-9}\sin\theta = 0.8\times 10^{-9}\, n \to n = \frac{2}{0.8}\sin\theta \leqslant ۲٫۵ \to$$

$n = 1,۲$ مرتبه اول و دوم ۰٫۵

۱ نمره

سوال ۴

$$\left(\frac{P^2}{2m}+V(x)\right)\psi(x)=E\psi(x) \xrightarrow{-a/2<x<a/2} \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi(x) \quad (۰/۵)$$

$$\rightarrow \frac{d^2\psi(x)}{dx^2}=\frac{-2mE}{\hbar^2}\psi(x)=-k^2\psi(x) \longrightarrow \psi(x)=A\sin kx+B\cos kx \quad (۰/۵)$$

شرط مرزی $\psi(x=a/2)=0 \longrightarrow A\sin\frac{ka}{2}+B\cos\frac{ka}{2}=0$ (1)

$\psi(x=-a/2)=0 \longrightarrow -A\sin\frac{ka}{2}+B\cos\frac{ka}{2}=0$ (2) (۱/۵)

(1)+(2): $2B\cos\frac{ka}{2}=0 \xrightarrow{B\neq 0} \frac{ka}{2}=\frac{(2n_1-1)\pi}{2}$ (۱/۲۵) $\quad n_1=1,2,\dots \rightarrow A=0$

(1)−(2) $2A\sin\frac{ka}{2}=0 \xrightarrow{A\neq 0} \frac{ka}{2}=n_2\pi$ (۱/۲۵) $\quad n_2=1,2,\dots \rightarrow B=0$

(ب۱) $\rightarrow \psi(x)=\begin{cases} A\sin k_2x & (۱/۲۵) \quad k_2=\frac{2n_2\pi}{a} \\ B\cos k_1x & (۱/۲۵) \quad k_1=\frac{(2n_1-1)\pi}{a} \end{cases}$

(ب) $\rightarrow k_2^2=\frac{2mE_2}{\hbar^2} \rightarrow E_2=\frac{\hbar^2}{2m}\left(4n_2^2\frac{\pi^2}{a^2}\right)$

$\rightarrow k_1^2=\frac{2mE_1}{\hbar^2} \rightarrow E_1=\frac{\hbar^2}{2m}(2n_1-1)^2\frac{\pi^2}{a^2}$

$$\int_{-a/2}^{a/2}|\psi(x)|^2dx=1 \longrightarrow \begin{cases} A^2\int_{-a/2}^{a/2}\sin^2k_2x\,dx=1 \longrightarrow A=\sqrt{2/a} \\ B^2\int_{-a/2}^{a/2}\cos^2k_1x\,dx=1 \longrightarrow B=\sqrt{2/a} \end{cases}$$

ج) حالت پایه $\rightarrow n_1=1$ (۱/۵) $\rightarrow E_1=\frac{\hbar^2}{2m}\frac{\pi^2}{a^2}$

$$\psi(x)=\sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$$

$\Delta x^2=\langle x^2\rangle-\langle x\rangle^2$ (۱/۵)

$$\langle x\rangle=\int_{-a/2}^{a/2}\frac{2}{a}\cos^2\frac{\pi x}{a}\,x\,dx=0$$

$$\langle x^2\rangle=\int_{-a/2}^{a/2}\frac{2}{a}\cos^2\frac{\pi x}{a}\,x^2dx=\frac{2}{a}\int_{a/2}^{a/2}x^2\left(\frac{1+\cos\frac{2\pi x}{a}}{2}\right)dx=\frac{2(a/2)^3}{3a}$$

$$+\frac{1}{a}\int_{-a/2}^{a/2}\frac{a}{2\pi}x^2\,d\sin\frac{2\pi x}{a}=\frac{a^2}{12}-\int_{-a/2}^{a/2}\frac{1}{\pi}\sin\frac{2\pi x}{a}\,x\,dx=\frac{a^2}{12}-\frac{a^2}{2\pi^2}$$