امتحان پایان‌ترم فیزیک ۴

سؤال ۱

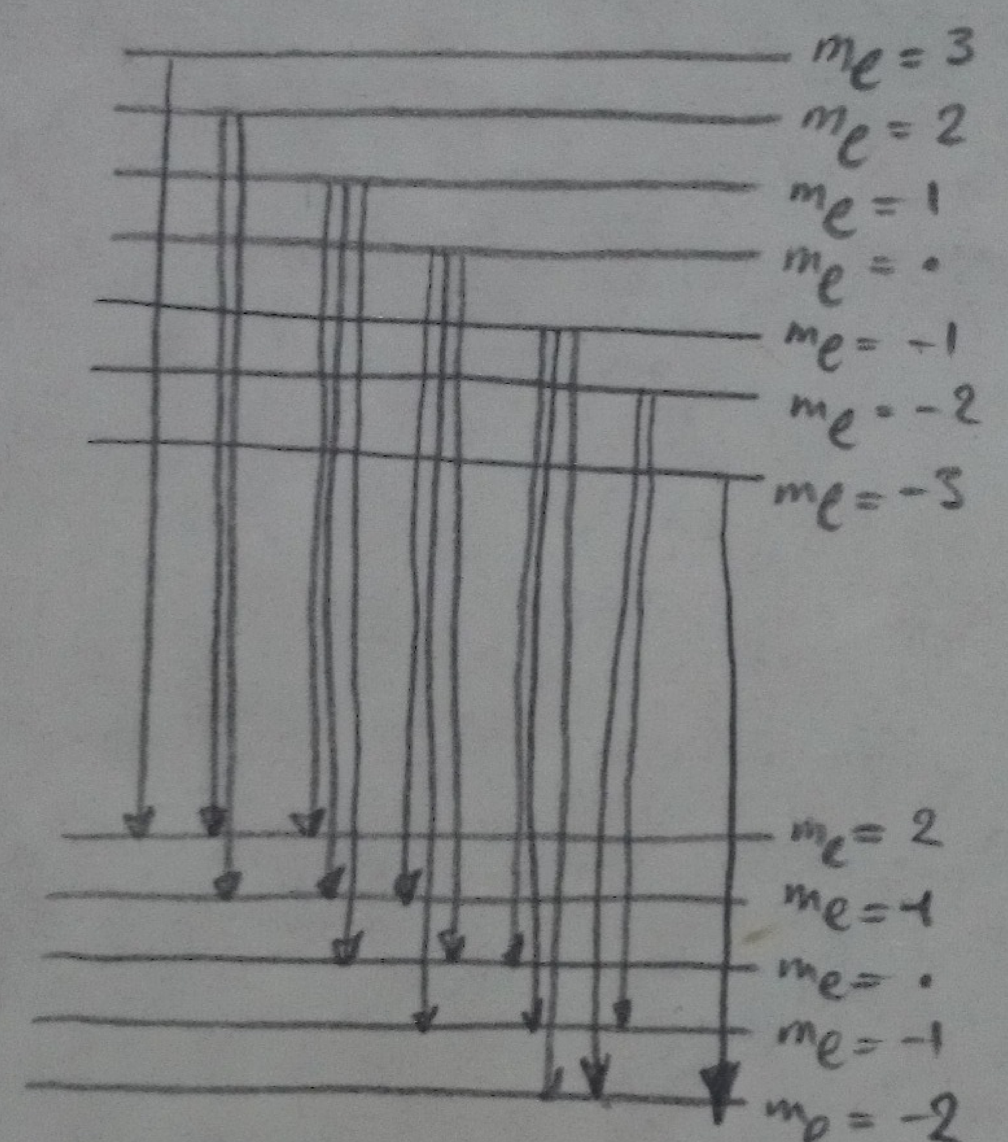
$$\mu = \frac{-e}{2m} L$$ /۲۵

الف) $L = n\hbar$ /۲۵ $\rightarrow \mu = \frac{-e}{2m} 2\hbar = 1.85 \times 10^{-23} \, J/T$ /۲۵

ب) $L = \sqrt{l(l+1)}\, \hbar$ /۲۵ ; $n=2 \longrightarrow l = 0, 1 \longrightarrow \begin{cases} \mu_{l=0} = 0 \\ \mu_{l=1} = \frac{-e}{2m}\sqrt{2}\,\hbar = 1.31 \times 10^{-23} \, J/T \end{cases}$ /۲۵ /۲۵

سؤال ۲

$l=3$



$l=2$

$$\Delta E = -\mu \cdot B$$ /۲۵

$$= \frac{-e}{2m} L_z B_z = \frac{-e}{2m} B_z m_z \hbar$$ /۲۵

/۷۵ $m_z = 0, \pm 1$ مقادیر مجاز m_z در گسیل فوتون

$$E = E_0 + \Delta E = \begin{cases} E_0 & m_z = 0 \\ E_0 - \frac{e\hbar}{2m} B & m_z = 1 \\ E_0 + \frac{e\hbar}{2m} B & m_z = -1 \end{cases}$$

$$E = h\nu = h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \begin{cases} 4000 \, \text{Å} & m_z = 0 \\ 4000/0149 \, \text{Å} & m_z = 1 \\ 3999/9851 \, \text{Å} & m_z = -1 \end{cases}$$ /۷۵

سؤال ۳

$$F = evB \xrightarrow{\text{حرکت دایره‌ای}} \frac{mv^2}{r} = evB \longrightarrow mv = erB$$ /۲۵ /۲۵

$$L = n\hbar \longrightarrow mvr = er^2B = n\hbar \longrightarrow r_n = \sqrt{\frac{n\hbar}{eB}} \longrightarrow v_n = \frac{n\hbar}{mr_n} = \frac{\sqrt{n\hbar eB}}{m}$$ /۲۵ /۲۵

$$E = \frac{1}{2}mv^2 = \frac{1}{2} m \frac{n\hbar eB}{m^2} = \frac{n\hbar eB}{2m}$$ /۵

سوال ۴

$$U(x)=U_0\left(e^{-2(x-x_0)/b}-2e^{-(x-x_0)/b}\right)$$

بسط تیلور $f(x)=f(x_0)+\frac{df}{dx}\Big|_{x_0}(x-x_0)+\frac{1}{2}\frac{d^2f}{dx^2}\Big|_{x_0}(x-x_0)^2+\cdots$

$$U(x)=U_0\left(1-\frac{2(x-x_0)}{b}+\frac{1}{2}\,4\frac{(x-x_0)^2}{b}+\cdots-2\left(1-\frac{x-x_0}{b}+\frac{1}{2}\left(\frac{x-x_0}{b}\right)^2+\cdots\right)\right)$$

$$U(x)=U_0\left(-1+\frac{(x-x_0)^2}{b^2}\right)$$ /۱۵

ب) $U(x)=\frac{1}{2}m\omega^2x^2 \longrightarrow E_n=\hbar\omega(n+1/2)$ /۱۵

نوسانگر هماهنگ

$$\frac{1}{2}m\omega^2=\frac{U_0}{b^2}\longrightarrow \omega=\sqrt{\frac{2U_0}{mb^2}}$$ /۰۵

سوال ۵

/۲۵ /۲۵

$$\frac{1}{2}mv_x^2=2k_BT \longrightarrow v_x=2\sqrt{\frac{k_BT}{m}}=5{,}755\times10^3\ m/s$$

$$t=\frac{x}{v_x}=\frac{6\times10^{-2}\,m}{5{,}755\times10^3\,m/s}=1.042\times10^{-5}\ s$$ /۲۵

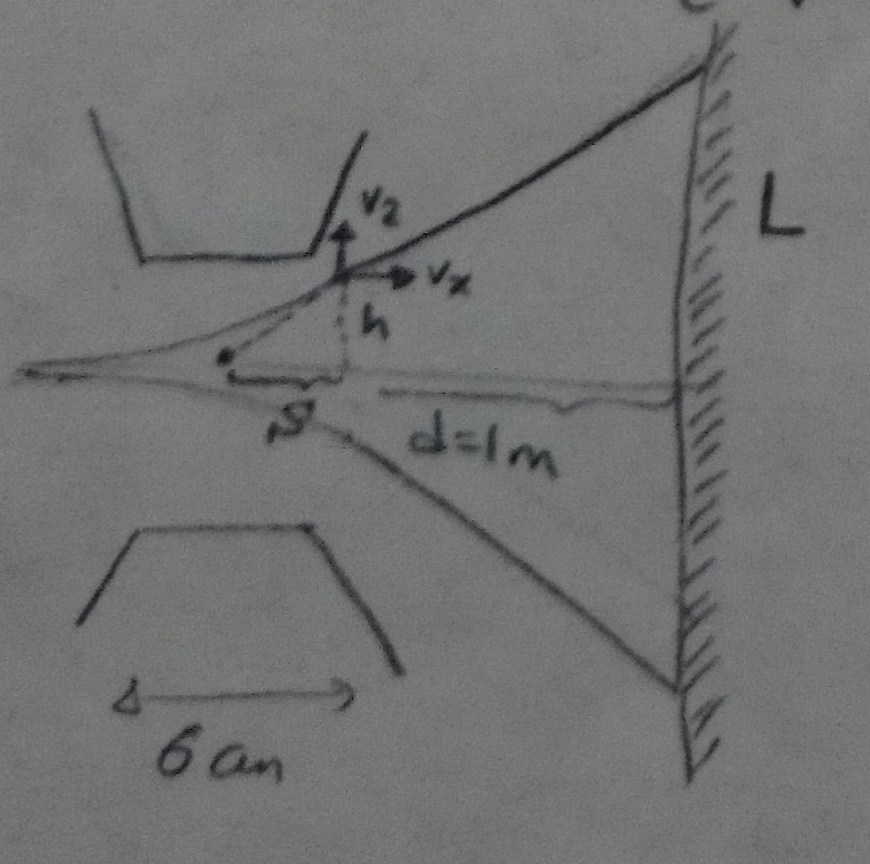
/۲۵ /۲۵ /۲۵

$$\underbrace{U=-\mu\cdot B\longrightarrow F_z=-\frac{dU}{dz}=\mu_s\frac{dB}{dz}}=\frac{e}{m_e}m_s\hbar\frac{dB}{dz}=\pm\frac{e\hbar}{2m_e}\frac{dB}{dz}=m_{H_2}a_z$$

/۲۵ /۲۵

$m_{H_2}\cong m_p$ /۲۵

$$\longrightarrow a_z=\pm\frac{e\hbar}{2m_em_p}\frac{dB}{dz}=\pm1{,}06\times10^6\ m/s^2$$ /۲۵



$$h=\frac{1}{2}a_zt^2+\cancel{v_zt}=\frac{1}{2}\times1{,}06\times10^6\times(1{,}042\times10^{-5})^2\Rightarrow$$

$$h=5{,}75\times10^{-5}\ m$$ /۲۵

$$v_z=a_zt=11.045\ m/s$$ /۲۵

/۲۵ $\frac{v_x}{v_z}=\frac{S}{h}\Rightarrow S=\frac{5{,}755\times10^3}{11{,}045}\times5{,}75\times10^{-5}=0.03\ m$

/۲۵

$$\frac{S}{S+d}=\frac{h}{L}\to L=0.002\longrightarrow\boxed{2L=4\ mm}$$ /۲۵

سوال ۷

الف)

$$\psi(x,t) = \alpha\, \psi_1(x)\, e^{-iE_1 t/\hbar} + \beta\, \psi_2(x)\, e^{-iE_2 t/\hbar}$$ ۱/۲۵

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a} \quad ; \quad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$ ۱/۲۵

$$|\psi(x,t)|^2 = (\alpha \psi_1 e^{-iE_1t/\hbar} + \beta \psi_2 e^{-iE_2t/\hbar})(\alpha \psi_1 e^{iE_1t/\hbar} + \beta \psi_2 e^{iE_2t/\hbar})$$

$$= |\alpha|^2 \psi_1^2 + |\beta|^2 \psi_2^2 + 2\alpha\beta\, \psi_1\psi_2 \cos(E_1-E_2)t/\hbar$$ ۱/۲۵

$$\int q|\psi(x,t)|^2 x\,dx = q|\alpha|^2 \frac{2}{a}\int_0^a \sin^2\frac{\pi x}{a}\, x\,dx + q|\beta|^2\frac{2}{a}\int_0^a \sin^2\frac{2\pi x}{a}\, x\,dx +$$

$$q\alpha\beta\frac{2}{a}\cos\left(\frac{(E_1-E_2)t}{\hbar}\right)\int_0^a \sin\frac{\pi x}{a}\sin\frac{2\pi x}{a}\, x\,dx = q\frac{2}{a}\left\{|\alpha|^2\int_0^a \frac{1-\cos\frac{2\pi x}{a}}{2}x\,dx + |\beta|^2\int_0^a\frac{1-\cos\frac{4\pi x}{a}}{2}x\,dx\right.$$

$$\left. + 2\alpha\beta\cos\left(\frac{(E_1-E_2)t}{\hbar}\right)\int_0^a\left(\frac{\cos\frac{\pi x}{a}+\cos\frac{3\pi x}{a}}{2}\right)x\,dx\right\}$$ ۱/۵

داریم $$\int_0^a x\cos\frac{m\pi x}{a}\,dx = \int_0^a \frac{a}{m\pi}\, x\, d\sin\frac{m\pi x}{a} = \frac{a}{m\pi}\, x\sin\frac{m\pi x}{a}\Big|_0^a - \frac{a}{m\pi}\int_0^a \sin\frac{m\pi x}{a}\,dx$$

$$= +\frac{a}{m\pi}\frac{a}{m\pi}\cos\frac{m\pi x}{a}\Big|_0^a = \left(\frac{a}{m\pi}\right)^2\left((-1)^m-1\right) \begin{cases} \text{زوج } m & = 0 \\ \text{فرد } m & = 2\left(\frac{a}{m\pi}\right)^2 \end{cases}$$

$$\Rightarrow \int q|\psi(x,t)|^2 dx = \frac{2q}{a}\left\{\frac{(|\alpha|^2+|\beta|^2)a^2}{4} + \alpha\beta\cos\left(\frac{(E_1-E_2)t}{\hbar}\right)\left[2\left(\frac{a}{\pi}\right)^2 + 2\left(\frac{a}{3\pi}\right)^2\right]\right\}$$

ب) $$\omega = \frac{E_2-E_1}{\hbar}$$ ۱/۲۵

ج)

$$\int q|\phi(x,t)|^2 x\,dx = q|\alpha|^2\frac{2}{a}\int_0^a \sin^2\frac{\pi x}{a}\, x\,dx + q|\beta|^2\frac{2}{a}\int_0^a \sin^2\frac{3\pi x}{a}\, x\,dx +$$

$$q\alpha\beta\frac{2}{a}\cos\left(\frac{(E_1-E_3)t}{\hbar}\right)\int_0^a \sin\frac{\pi x}{a}\sin\frac{3\pi x}{a}\, x\,dx = q\frac{2}{a}\left\{\frac{|\alpha|^2}{2}a + \frac{|\beta|^2}{2}a + 2\alpha\beta\cos\frac{(E_1-E_3)t}{\hbar}\int_0^a\left(\cos\frac{2\pi x}{a} + \cos\frac{4\pi x}{a}\right)x\right.$$

$$= \frac{2q}{a}\left\{\frac{|\alpha|^2}{4}a^2 + \frac{|\beta|^2 a^2}{4}\right\} = \text{Const} \longrightarrow$$ مستقل از زمان است، تابش نداریم. ۱/۵