Take Home
Special Topics in Cosmology Fall 2015

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Deadline: 11 Bahman 94 / 31 January 2016

December 20, 2015

1 ISW effect, bias of dark matter and galaxies and Weak Lensing to address the bias Problem. (Mollazadeh)

In order to determine the bias parameter in ISW-galaxy correlation, we can use the lensing data. In this arena, the cross-correlation between the lensing convergence and the projected density field can be calculated using the Limber approximation.
a)In small angular scales where the Limber approximation works show that the cross correlation is:

$$C_\ell^{\kappa g} = \int dz \frac{H(z)}{\chi^2(z)} K^g(z) K^\kappa(z) P(k = \frac{\ell + 1/2}{\chi(z)}, z) \tag{1}$$

where χ is the comoving distance, P is the matter power spectrum and K^κ is defined as:

$$K_z^\kappa = \frac{3\Omega_m H_0^2}{2H(z)} (1+z)\chi(z) \frac{\chi_* - \chi(z)}{\chi_*} \tag{2}$$

where χ_* is the comoving distance to the last scattering.
b) Find the relation of the galaxy Kernel K^g introduce in Eq.(1) with the galaxy distribution dN/dz and the bias parameter $b = \delta_g/\delta_m$.
c) Quantitatively show that cross correlation defined in Eq.(1) and also ISW-galaxy cross correlation will help to break the degeneracy of dark matter bias.
d) What about the CMB lensing? Can we use it as a probe to break the degeneracy?

2 Degeneracy of Neutrino Physics, Primordial Non Gaussianity and Modified Gravity on LSS observations. (Khaloei)

The cosmic shear power spectrum gives valuable information about the distribution of matter along the line of sight.
a)Show that the shear power spectrum is equal to convergence power spectrum.
b)Show that the convergence power spectrum in some limits become as below:

$$P_\kappa(\ell) = \frac{9}{4}\Omega_m^2 (\frac{H_0}{c})^4 \int_0^{\chi_{lim}} \frac{d\chi}{a^2(\chi)} P_\delta(\frac{\ell}{f_K(\chi)}; \chi) \times [\int_\chi^{\chi_{lim}} d\chi' n(\chi') \frac{f_K(\chi' - \chi)}{f_k(\chi')}]^2 \tag{3}$$

where Ω_m is the present value of matter density parameter, P_δ is the mater power spectrum, χ is the comoving distance, χ_{lim} is the limiting comoving distance of the sources. $n(\chi')$ is the redshift distribution of the sources and ℓ is modulus of 2-dimensional wavenumber vector perpendicular to the line of sight.

c)The convergence power spectrum can be derived from the two-point shear correlation functions. In particular, the $\xi_\pm$ correlation functions relate to the power spectrum as :

$$\xi_\pm(\theta) = \xi_{tt}(\theta) \pm \xi_{\times\times}(\theta) = \frac{1}{2\pi}\int_0^\infty d\ell \ell P_\kappa(\ell) J_{0,4}(\theta) \tag{4}$$

where $J_{0,4}$ are the Bessel functions of first kind. Discuss how this will help to relate the convergence power to observable quantities.
d) Discuss about the effect of Neutrinos and Modified gravity on the lensing observables, quantitatively and qualitatively.

3 Corrections on BAO: Non-linear effects - RSD - WL (Khoraminezhad)

The dark matter tracers (like galaxies) which are used to probe the BAO scale is displaced by weak lensing effect. a) Show that the observed correlation function of dark matter tracers due to lensing is modified as below:

$$\xi_{obs}(r) = \xi_s(r) + \delta\xi_{sm}(r) \tag{5}$$

where ξ_s is the correlation function in position of the source, $\delta\xi_{sm}$ is the correction to the lensing where:

$$\delta\xi_{sm}(r) = \Big[\langle\zeta^i(\vec{x})\zeta^j(\vec{x})\rangle - \langle\zeta^i(\vec{x})\zeta^j(\vec{y})\rangle\Big] \times \left[\frac{\delta_{ij}}{r}\frac{d\xi(r)}{dr} + \frac{r^i r^j}{r}\frac{d}{dr}(\frac{\xi'(r)}{r})\right] \tag{6}$$

where ζ is the displacement vector defined as $\vec{x}_o = \vec{x}_s + \vec{\zeta}(\vec{x}_s)$ (x_0 observed position and x_s is the real position).
b)Now by using the Limber approximation show that the corrected correlation function term is obtained as:

$$\delta\xi_{sm} = \xi''(\frac{I_1}{2} - I_2 + I_3) + \frac{\xi'}{r}(\frac{I_1}{2} - I_3) \tag{7}$$

where

$$I_i \equiv \frac{9\Omega_m^2 H_0^4}{4}\int\frac{dk}{2\pi k}\int_0^{\chi_s} d\chi W^2(\chi_s,\chi) P_\delta(k,\chi) K_i(k,\chi) \tag{8}$$

where χ is the comoving distance, χ_s is the comoving distance of source. P_δ is the matter power spectrum and K_i is defined as $K_1 = 1$, $K_2 = J_0(k\chi)$ and $K_1 = J_1(k\chi)/k\chi$. Also note that

$$W(\chi_s,\chi) = 2\chi_s(1 - \frac{\chi}{\chi_s}) \tag{9}$$

4 Too big to fail Problem and the kinematic of stars in dwarf galaxies. (Bagheri)

In order to solve the too big to fail problem and other galactic scale challenges of Dark Matter assume that our universe is ΛMDM. MDM means multi dark matter. The dark matter component of the Universe is made up of cold dark matter and warm dark matter with the fraction of:

$$f_W = \frac{\Omega_{WDM}}{\Omega_{WDM} + \Omega_{CDM}} \tag{10}$$

where Ω_{CDM} is the density parameter of cold dark matter, and Ω_{WDM} is the density parameter of Warm dark matter.
a) Find the the free streaming length of Warm dark with respect its fundamental properties.
b) How the matter power spectrum of matter $P(k,z) = Ak^{n_s}T^2(k)D^2(z)$ changed in MDM model.
c) How MDM can address too big to fail problem.
d)Use the Jeans equation derived from Boltzman equation in order to relate the position and line of sight velocity of stars to gravitational potential of a dwarf galaxy!

5 Inflation, Alternatives (Conformal Point of view) and observational effects (Khanbeigi)

Re-derive the Equations from the paper in the link $http://physics.princeton.edu/\ steinh/vaasrev.pdf$

6 A particle physics motivated inflationary model and observational prediction (Zahraie)

In order to find the Mukhanov-Sasaki equation,
a)Show that the second order gravity action with the perturbed metric can be rewritten as below:

$$S^{(2)} \sim d\tau d^3x \frac{1}{2}[(f')^2 - (\nabla f)^2 + \frac{a''}{a} f^2] \tag{11}$$

where the perturbed inflationary field is written as:

$$\phi(\tau, x) = \bar{\phi}(\tau) + \frac{f(\tau, x)}{a(\tau)} \tag{12}$$

where τ is the conformal time.
b) Relate the Mukhanov-Sasaki equation observables to spectral index of scalar perturbations.

7 Effect of the peculiar velocity of the host galaxy of SNe on distance measurement (Gholami)

The luminosity distance in perturbed universe is modified due to perturbations via Lensing, Sachs Wolfe, Integrated Sachs Wolfe and peculiar velocities. Shows that this change is obtained as below:

$$\begin{aligned}
\tilde{d}_L(z_s, n) &= (1+z_s)(\eta_o - \eta_s)\{1 - \frac{1}{(\eta_o - \eta_s)\mathcal{H}_s}\vec{v}_o.\hat{n} - (1 - \frac{1}{(\eta_o - \eta_s)\mathcal{H}_s})\vec{v}_s.\hat{n} \\
&- (1 - \frac{1}{(\eta_o - \eta_s)\mathcal{H}_s})\Psi_s - \frac{1}{(\eta_o - \eta_s)\mathcal{H}_s}\Psi_o + \frac{2}{(\eta_o - \eta_s)}\int_{\eta_s}^{\eta_o} d\eta\Psi \\
&+ \frac{2}{(\eta_o - \eta_s)\mathcal{H}_s}\int_{\eta_s}^{\eta_o} d\eta\dot{\Psi} - 2\int_{\eta_s}^{\eta_o} d\eta \frac{\eta - \eta_s}{\eta_o - \eta_s}\dot{\Psi} + \int_{\eta_s}^{\eta_o} \frac{(\eta - \eta_s)(\eta_o - \eta)}{(\eta_o - \eta_s)}\ddot{\Psi} \\
&- \int_{\eta_s}^{\eta_o} d\eta \frac{(\eta - \eta_s)(\eta_o - \eta)}{(\eta_o - \eta_s)}\nabla^2\Psi\}
\end{aligned} \tag{13}$$

where $\tilde{d}$ is the observed luminosity distance, z_s is the redshift of the observation, $\hat{n}$ is the direction of the observation. η_0 is the comoving time of the observer, η_s is the comoving time of the source. $\mathcal{H}_s$ is the conformal Hubble time. v_o is the peculiar velocity of the observer.

8 Assembly bias, halo merger tree and merger rates (Ayromlou)

In the context of the halo merger picture: a)Shows that the rate at which a halo with Mass M transits to a halo with mass between M and $M + \Delta M$ is given by the equation below:

$$P(\Delta M|M,t) d\ln\Delta M d\ln t = \frac{1}{\sqrt{2\pi}}[\frac{S_1}{(S_1 - S_2)}]^{3/2}\exp[-\frac{\delta_c^2(S_1 - S_2)}{2S_1S_2}]\times|\frac{d\ln\delta_c}{d\ln t}|\frac{\delta_c}{\sqrt{S_2}}|\frac{d\ln S_2}{d\ln\Delta M}|d\ln t d\ln\Delta M \tag{14}$$

where $S_1 = \sigma^2(M)$ and $S_2 = \sigma^2(M + \Delta M)$. In any finite time interval Δt
b) Find the halo bias with peak-background splitting method and peak-peak method with the assumption that the number density of structures is obtained via universality function.
c) Discuss about the Assembly bias, non-linear bias and stochastic bias quantitatively.

9 Effective Field Theory of LSS and Halo model (Chartab)

In the context of the halo model for non linear structure formation: a)Show that the non-linear power spectrum in small scale regime is obtained as:

$$P(k) = P^{1h}(k) + P^{2h}(k) \tag{15}$$

where P^{1h} is one halo term and P^{2h} is two halo term defined as below:

$$P^{1h}(k) = \int dmn(m)(\frac{m}{\bar{\rho}})^2|u(k|m)|^2 \tag{16}$$

$$P^{2h}(k) = \int dm_1n(m_1)(\frac{m_1}{\bar{\rho}})u(k|m_1)\int dm_2n(m_2)(\frac{m_2}{\bar{\rho}})u(k|m_2)P_{hh}(k|m_1,m_2) \tag{17}$$

where $u(k|m)$ is the Fourier transform of the NFW profile. P_{hh} is the halo power spectrum.
b)Discuss how the fourier transform of the NFW profile depends on concentration parameter and how it changes with redshift.
c)Discuss how we can relate the halo matter power spectrum to linear matter power spectrum.