به نام خدا

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1. Let us assume that the neutrino mass is small enough, so that the neutrinos are relativistic at decoupling. Show that the energy density after decoupling is

$$\rho_\nu = \frac{T_\nu^4}{\pi^2}\int_0^\infty dx x^2 \sqrt{\frac{x^2 + m_\nu^2/T_\nu^2}{e^x + 1}}$$

where

$$T_\nu \equiv T_{dec} a_{dec}/a$$

2. In supergravity theories, the superpartner of the graviton, the gravitino, is a spin-3/2 dark matter candidate. Assuming a negligible energy density of gravitinos after inflation, these particles would be produced most efficiently just after reheating. The rate at which Standard Model particles in the primordial soup would convert into gravitinos is $\Gamma_{3/2} \approx T^3/M_{pl}^2$. In this problem, you will estimate the relic density of gravitinos.

a) Argue that the normalized number of gravitinos, $N_{3/2} = n_{3/2}/s$, is given by:

$$N_{3/2} \approx \int_0^{T_R} \frac{dT}{T}\frac{\Gamma_{3/2}}{H}\frac{n}{s}$$

where s is the total entropy density and n is the particle density. Using that $n/s \approx O(1)$ and $H \approx T^2/M_{Pl}$, derive a simple expression for $N_{3/2}$ in terms of the reheating temperature T_R and M_{Pl} only.

b) Show that the energy density of gravitinos today would be:

$$\frac{\Omega_{3/2}h^2 \approx 0.1(\frac{T_R}{10^9 GeV})(m_3/2}{1GeV)}$$

For a reheating temperature of $T_R \approx 10^9 GeV$, gravitinos with masses around $m_{3/2} \approx 1GeV$ would hence constitute all of the dark matter. Note, however, that this scenario is severely constrained by its effects on BBN through the decay of heavier super symmetric particles.

3. Discuss the effect of the following suppositions on the production of helium and deuterium during primordial nucleosynthesis:
   - g_*: There were more relativistic species during BBN than we expected.

- G_F : The weak interaction strength was smaller than we thought.
- G: Newton's constant G is larger during BBN
- Q: The neutron-proton mass difference was larger than supposed.
- τ_n: The neutron lifetime was shorter than assumed.
- μ_ν: There were many more neutrinos than anti-neutrinos.