به نام خدا

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1. a)We know Solutions for the expansion when the Universe contained either matter ($p = 0$) or radiation ($p = \frac{\rho c^2}{3}$). Suppose we have a more general equation of state, $p = (\gamma - 1)\rho c^2$, where γ is a constant in the range $0 < \gamma < 2$. Find solutions for $\rho(a)$, $a(t)$ and hence $\rho(t)$ for universes containing such matter. Assume $k = 0$ in the Friedmann equation. what is the solution if $p = -\rho c^2$?

   b)Using your answers, what value of γ would be needed so thath ρ has the same time dependence as the curvature term $\frac{k}{a^2}$? Find the solution $a(t)$ to the full Friedmann equation for a fluid with this γ, assuming negative k.

   c)Now consider the case $k < 0$, with a universe containing only matter ($p = 0$) so that $\rho = \frac{\rho}{a^3}$. what is the solution $a(t)$ in a situation where the final term of the Friedmann equation dominates over the density term? how does the density of matter vary with time? Is domination by the curvature term a stable situation that will continue forever?

2. The deceleration parameter is defined by equation:

   $$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2} = -\frac{a(t_0)\ddot{a}(t_0)}{\dot{a}^2(t_0)}$$

   Use the acceleration equation:

   $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

   and the definition of critical density to show that a radiation-dominated universe has $q_0 = \Omega_0$.

3. Show that for a pressureless universe with a cosmological constant has a deceleration parameter given by

   $$q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda(t_0)$$

4. The most likely cosmology describing our own Universe has a flat geometry with a matter density of $\Omega_0 \approx 0.3$ and a cosmological constant with $\Omega_\Lambda(t_0) \approx 0.7$. What will the values of Ω and Ω_Λ be when the Universe has expanded to be five times its present size? Use an approximation suggested by this result to find the late-time solution to the Friedmann equation for our Universe. What is the late-time value of the deceleration parameter q?

5. In a matter-dominated open Universe, the present age of the Universe is given by the intimidating formula:

$$H_0 t_0 = \frac{1}{1-\Omega_0} - \frac{\Omega_0}{2(1-\Omega_0)^{\frac{3}{2}}} \cosh^{-1}\left(\frac{2-\Omega_0}{\Omega_0}\right)$$

Demonstrate that in the limiting case of an empty Universe $\Omega_0 \longrightarrow 0$ we get $H_0 t_0 = 1$, and in the limiting case of a flat Universe $\Omega_0 \longrightarrow 1$ we recover the result $H_0 t_0 = \frac{2}{3}$

(Useful formulas:
$\cosh^{-1}(x) \approx \ln(2x)$ for large x,
$\cosh^{-1}[\frac{1+x}{1-x}] \approx 2\sqrt{x} + \frac{2x^{\frac{3}{2}}}{3}$ for small x.)