Dark Matter Halo Bias

Shant Baghram
26 June 2013

I. HALO BIAS AND PEAK BACKGROUND SPLITTING

In the standard model of Cosmological structure formation based on the paradigm of Cold Dark matter, the observable luminous structures in Cosmology (i.e. galaxies and cluster of galaxies) are hosted by the Dark matter halos. The abundance of each halo is related to the background density of matter perturbations. Accordingly we define a halo bias parameter as the below:

$$b = \frac{\delta_h}{\delta_m}, \tag{1}$$

where δ_h is the halo density contrast, and δ_m is the background density perturbation of matter. One of the most prominent ways to calculate the bias parameter is the Peak-Background Splitting (PBS) method. This method is based on the splitting of density perturbation to short and long wavelength modes:

$$\delta(x) = \delta_s(x) + \delta_l(x), \tag{2}$$

where δ_s is the short wavelength mode corresponding to the local density contrast of a structure, and δ_l is the long wavelength mode of perturbation with the assumption of $\delta_s \gg \delta_l$. Now in order to find the bias parameter, we have to use the knowledge of non-linear structure formation. The starting point of non-linear perturbation theory is the spherical collapse, which introduce the criteria of the formation of the structures. In the spherical Collapse analysis we find a critical density $\delta_c \simeq 1.68$, where if a region of radius R, with corresponding mass of $M = 4\pi/3\bar{\rho}_m R^3$ has a density contrast higher than the threshold (i.e. $\delta(x) l \delta_c$), then this region will undergo a spherical collapse. Now in the context of PBS framework, we assert that the criteria of collapse is changed due to long mode density perturbation:

$$\delta(x) > \delta_c \rightarrow \delta_s > \delta_c - \delta_l \tag{3}$$

The equation above suggest that the long wavelength mode, change the critical density of collapse. In the word, in the case of positive(negative) long mode the number of structure will enhanced(decrease), while the number of void will have a(n) decrease(increase). Consequently, now we can define an effect height parameter as:

$$\nu_{eff} = \nu_c - \delta_l/\sigma(M, z) \tag{4}$$

where $\sigma(M, z)$ is the variance of matter perturbation, which depends on redshift and the mass scale and $\nu_c \equiv \delta_c/\sigma(M, z)$. (In general we define the height parameter as $\nu(M, z) = \delta(z)/\sigma(M, z)$, where in future we omit the arguments of ν)

Now we want to find the halo abundance density contrast, with the presence of long wavelength. The halo density contrast is defined as:

$$\delta_h(\vec{x}) = \frac{n(M, \nu_{eff}) - \bar{n}(M, \nu)}{\bar{n}(M, \nu)} \tag{5}$$

where $\bar{n}$ is the background number density. Now with the assumption that $\delta_l \ll \delta_s$ we can Taylor expand the the number density as:

$$n(M, \nu_{eff})(\mathbf{x}) \simeq \bar{n}(M, \nu) - \frac{\partial \bar{n}(M, \nu)}{\partial \nu} \frac{\delta_l(\mathbf{x})}{\sigma(M, z)} \tag{6}$$

Substituting the Eq.(6) into Eq.(5), and the fact that $\delta_l \equiv \delta_m$, we will find the bias parameter as:

$$b(M, z) = -\frac{1}{\sigma(M, z)} \frac{\partial \ln \bar{n}(M, \nu)}{\partial \nu} \tag{7}$$

In this point we can use the Press-Schechter number density of structures as below:

$$\bar{n}(M, \nu) = -2\frac{\bar{\rho}}{M^2} f(\nu) \frac{d \ln \sigma(M, z)}{d \ln M} \tag{8}$$

where $r\bar{h}o$ is the mean density of the universe and the fuctionality of height parameter is defined as:

$$f(\nu) = \frac{\nu}{\sqrt{2\pi}} e^{-\nu^2/2} \tag{9}$$

Now inserting the Press-Schechter mass function in Eq.(7), (as the only height dependence is in $f(\nu)$), we will have:

$$b(M, z) = -\frac{1}{\sigma(M, z)} \frac{\partial \ln f(\nu)}{\partial \nu} =^{P.S.} \frac{\nu^2 - 1}{\delta_c} \tag{10}$$

It is worth to mention that the bias parameter depends on the mass of structures we are probing and also the redshift through the height parameter $\nu = \delta_c/\sigma(M, z)$, where the mass and redshift dependence arise from the mass variance term.