Curve Types

1. Simple curves with spirals
2. Broken Back – two curves same direction (avoid)
3. Compound curves: multiple curves connected directly together (use with caution) go from large radii to smaller radii and have $R(\text{large}) < 1.5 \times R(\text{small})$
4. Reverse curves – two curves, opposite direction (require separation typically for superelevation attainment)

Types of Circular Curves

- Simple Curve
- Compound Curves
- Broken-Back Curves

Broken-Back Curves should be avoided if possible. It is better to replace the Curves with a larger radius circular curve.

A tangent should be placed between reverse Curves.
Typical Configurations of Curves

- Spirals are typically placed between tangents and circular curves to provide a transition from a normal crown section to a superelevated one.
- Spirals are typically used at intersections to increase the room for large trucks to make turning movements.

Horizontal Alignment

- Objective:
  - Geometry of directional transition to ensure:
    - Safety
    - Comfort
- Primary challenge
  - Transition between two directions
  - Horizontal curves
- Fundamentals
  - Circular curves
  - Superelevation

Horizontal Alignment

1. Tangents
2. Curves
3. Transitions

- Curves require superelevation
  - Retard sliding,
  - Allow more uniform speed,
  - Allow use of smaller radii curves (less land)

Horizontal Curve Fundamentals

\[ T = R \tan \frac{\Delta}{2} \]

\[ L = \frac{\pi}{180} RD = \frac{100 \Delta}{D} \]

\[ D = \frac{100 \left( \frac{180}{\pi} \right)}{R} = \frac{18,000}{\pi R} \]

D = degree of curvature

(\Delta \approx L \approx D / 100)
**Horizontal Curve Fundamentals**

\[ E = R \left( \frac{1}{\cos \frac{\Delta}{2}} - 1 \right) \]
\[ M = R \left( 1 - \cos \frac{\Delta}{2} \right) \]

**Example**

- A horizontal curve is designed with a 1500 ft. radius. The tangent length is 400 ft. and the PT station is 20+00. What are the PI and PT stations?

**Superelevation**

\[ W \sin \alpha + f_s \left( W \cos \alpha + \frac{WV^2}{gR_v} \sin \alpha \right) = \frac{WV^2}{gR_v} \cos \alpha \]
\[ \tan \alpha + f_s = \frac{V^2}{gR_v} \left( 1 - f_s \tan \alpha \right) \]
\[ e + f_s = \frac{V^2}{gR_v} \left( 1 - f_s e \right) \]
\[ R_v = \frac{V^2}{g(f_s + e)} \]
Radius Calculation

- $R_{\text{min}}$ related to max. $f$ and max. $e$ allowed
- $R_{\text{min}}$ use max $e$ and max $f$ and design speed
- $f$ is a function of speed, roadway surface, weather condition, tire condition, and based on comfort
  - AASHTO: $0.5 \ @ 20$ mph with new tires and wet pavement to $0.35 \ @ 60$ mph
  - $f$ decreases as speed increases (less tire/pavement contact)

Selection of $e$ and $f_s$

- Practical limits on superelevation ($e$)
  - Climate
  - Constructability
  - Adjacent land use
- Side friction factor ($f_s$) variations
  - Vehicle speed
  - Pavement texture
  - Tire condition

Maximum $e$

- Controlled by 4 factors:
  - Climate conditions (amount of ice and snow)
  - Terrain (flat, rolling, mountainous)
  - Frequency of slow moving vehicles who might be influenced by high superelevation rates
  - Highest in common use = 10%, 12% with no ice and snow on low volume gravel-surfaced roads
  - 8% is logical maximum to minimized slipping by stopped vehicles

Side Friction Factor

From AASHTO's A Policy on Geometric Design of Highways and Streets 2004
Minimum Radius Tables

WSDOT Design Side Friction Factors

For Open Highways and Ramps

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<thead>
<tr>
<th>Design Speed (mph)</th>
<th>Side Friction Factor (f)</th>
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WSDOT Design Side Friction Factors

For Low-Speed Urban Managed Access Highways

Design Superelevation Rates - AASHTO

(from AASHTO's A Policy on Geometric Design of Highways and Streets 2004)
Radius Calculation (Example)

- Assume a maximum $e$ of 8% and design speed of 60 mph, what is the minimum radius?
  - $f_{\text{max}} = 0.12$ (from Green Book)
  - $R_{\text{min}} = \frac{V^2}{15(e + f)} = \frac{60^2}{15(0.08 + 0.12)}$
  - $R_{\text{min}} = 1200$ feet

Stopping Sight Distance

$$SSD = \frac{\pi}{180} R_s \Delta_s$$

$$\Delta_s = \frac{180(SSID)}{\pi R_v}$$

$$M_s = R_v \left[ 1 - \cos \left( \frac{90 SSD}{\pi R_v} \right) \right]$$

$$SSD = \frac{\pi R_v}{90} \left[ \cos^{-1} \left( \frac{R_v - M_s}{R_v} \right) \right]$$
Sight Distance Example

- A horizontal curve with $R = 800$ ft is part of a 2-lane highway with a posted speed limit of 35 mph. What is the minimum distance that a large billboard can be placed from the centerline of the inside lane of the curve without reducing required SSD? Assume $p/r = 2.5$ sec and $a = 11.2$ ft/sec$^2$

$$SSD = 1.47Vt + \frac{V^2}{30\left(\frac{a}{32.2} \pm G\right)}$$

$$= 1.47(35 \text{ mph})(2.5 \text{ sec}) + \frac{(35 \text{ mph})^2}{30\left(\frac{11.2}{32.2} \pm 0\right)} = 246 \text{ feet}$$

Sight Distance Example

- Now estimate the minimum distance that the billboard can be placed:

$$m = R\left[1 - \cos\left(\frac{90 \cdot SSD}{\pi R_v}\right)\right]$$

$$= R\left(1 - \cos\left(\frac{28.65(246)}{800}\right)\right) = 9.43 \text{ feet}$$

- in radians not degrees

Horizontal Curve Example

- Deflection angle of a $4^\circ$ curve is $55^\circ 25'$. PC at station 238 + 44.75. Find length of curve, $T$, and station of PT.
  - $D = 4^\circ$
  - $\Delta = 55^\circ 25' = 55.417^\circ$
  - $D = \frac{100 \times 180}{\pi R} = \frac{5729.58}{R} \rightarrow R = 1432.4 \text{ ft}$

$$L = \frac{2\pi R \Delta}{360} = \frac{2\pi(1432.4 \text{ ft})(55.417)}{360} = 1385.4 \text{ ft}$$

$$T = R \tan \frac{\Delta}{2} = 1432.4 \tan \frac{55.417}{2} = 752.3 \text{ ft}$$

Horizontal Curve Example

- Stationing goes around horizontal curve.
- What is station of PT?
  - PC = 238 + 44.75
  - L = 1385.42 ft = 13 + 85.42
  - Station at PT = (238 + 44.75) + (13 + 85.42) = 252 + 30.17
Superelevation Transition

Superelevation Runoff/Runout

Superelevation Runoff - WSDOT

Superelevation Transition

- Full Super
- Full Superelevation
- High Side
- Low Side
- Full Superelevation Runoff
- Normal Runoff
- Crown
- Tangent
- Length of Runoff
- Outside edge of traveled way
- Centerline profile
- Inside edge of traveled way
- Profile control centerline

TRAVELED WAY REVOLVED ABOUT CENTERLINE
Purpose of Transition Curves

- Provides path for vehicle to move from straight to a circular curve
- Improved appearance of curve to driver
- Allows introduction of superelevation and pavement widening

Characteristics of Transition Curve

- Should have constant rate of change of radius of curvature
- Transition should be equal to zero at start of straight and equal to radius of curvature at circular curve
- Allows passengers to adjust to change in rate of curvature

Types of Transition Curve

- Clothoid
  - Most commonly used
  - Will be examined in more detail
- Lemniscate
  - Used for large deflection angles on high speed roads
- Cubic Parabola
  - Unsuitable for large deflection angles

Geometry of Clothoid

- $K = L_p R$
  - $L_p =$ Length of plan transition
  - $R =$ Radius of circular curve
  - $K =$ constant

- Coordinates can be represented by
  - $x = l - l^5/40(RL_p)^2 - ...$
  - $y = l^3/6RL_p - l^7/336(RL_p)^3 + ...$

- $x$ and $y$ are measured along the tangent and at right angles from the tangent respectively.
Shift of Curve for Transition

- To accommodate the transition curve the circular curve is normally *shifted inwards* towards the centre of the curve.

Shift of Curve for Transition

- The shift can be calculated by:
  - **Shift** = \( S = \frac{L_p^2}{24R} \)
  - \( L_p \) is the length of transition
  - If \( S < 0.25 \text{m} \) then the transition is usually ignored or not required

Terminology

- When a transition curve is used:
  - **Tangent Length** = \((R + S) \tan \left( \frac{\Delta}{2} \right)\)
  - The distance from the IP to the TS = \((R + S) \tan \left( \frac{\Delta}{2} \right) + \frac{L_p}{2}\)
  - The circular arc length from SC – CS is reduced by \( L_p \):
    - \( \text{Arc} = R\Delta - L_p \)
Transition Length

- **Radial Acceleration Method:**
  - \( L_p = \frac{V^3}{46.73Ra} \)
  - \( L_p = \) length of plan transition
  - \( V = \) design speed (km/hr)
  - \( R = \) radius of circular curve
  - \( a = \) radial acceleration
  - \( a \) varies with design speed and design authority.
  - Typical values:
    - \( a = 0.6 \text{ m/sec}^2 \) for \( V = 40 \text{ km/hr} \) to \( 70 \text{ km/hr} \)
    - \( a = 0.45 \text{ m/sec}^2 \) for \( V = 80 \text{ km/hr} \) to \( 120 \text{ km/hr} \)
    - \( a = 0.3 \text{ m/sec}^2 \) for \( V > 120 \text{ km/hr} \)

Rate of Rotation of Pavement Method:
- Most calculations for plan transition are done in conjunction with the superelevation development length (\( L_e \)).
- Usually relies on design speed and rate of rotation of pavement:
  - \( L_p = L_e - 0.4V \)
  - \( L_e = (e_1 - e_2) \frac{V}{0.09} \)
    - Rotations of 2.5%/sec – this is most common, where \( e_1 = 0 \)
  - \( L_e = (e_1 - e_2) \frac{V}{0.126} \)
    - Rotation rates of 3.5%/sec

### Tables

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