Role of Safety Inventory

- Forecasts are rarely completely accurate
  - If average demand is 1000 units per week, then half the time actual demand will be greater than 1000, and half the time actual demand will be less than 1000;
  - What happens when actual demand is greater than 1000?
- If you kept only enough inventory in stock to satisfy average demand, half the time you would run out!
- Safety inventory: Inventory carried for the purpose of satisfying demand that exceeds the amount forecasted in a given period

Two Questions to Answer

1. What is the appropriate level of safety inventory to carry?
2. What actions can be taken to improve product availability while reducing safety inventory?

- Average inventory is therefore cycle inventory plus safety inventory
- There is a fundamental tradeoff:
  - Raising the level of safety inventory provides higher levels of product availability and customer service.
  - Raising the level of safety inventory also raises the level of average inventory and therefore increases holding costs.
  - Very important in high-tech or other industries where obsolescence is a significant risk
Determining Safety Inventory Level

- Measuring demand uncertainty
- Measuring product availability
- Replenishment policies
- Evaluating cycle service level and fill rate
- Evaluating safety level given desired cycle service level or fill rate
- Impact of required product availability and uncertainty on safety inventory

Appropriate Level of Demand Uncertainty

- Appropriate level of safety inventory is determined by:
  - supply or demand uncertainty
  - desired level of product availability
- Higher levels of uncertainty require higher levels of safety inventory given a particular desired level of product availability
- Higher levels of desired product availability require higher levels of safety inventory given a particular level of uncertainty

Measuring Demand Uncertainty

- Demand has a systematic component and a random component
  - Estimate of the random component is the measure of demand uncertainty
  - Estimated by the standard deviation of demand
- Notation
  - $D = \text{Average demand per period}$
  - $\sigma_D = \text{standard deviation of demand per period}$
  - $L = \text{lead time}$
    - Time between when an order is placed and when it is received
  - Uncertainty of demand during lead time is what is important

Measuring Demand Uncertainty

- $P = \text{demand during lead time}$
  $$P = D_L = \sum_{i=1}^{L} D_i$$
- $\Omega = \text{std dev of demand during k periods}$
  $$\Omega = \sqrt{\sum_{i=1}^{k} \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} \sigma_i \sigma_j}$$
- Coefficient of variation
  $$cv = \frac{\sigma}{\mu}$$
Measuring Product Availability

- Product availability:
  - Firm’s ability to fill an order out of available inventory.
- Stockout:
  - Order arrives when product is not available.
- Availability measures:
  - Product fill rate ($f_f$):
    - Fraction of demand that is filled from available inventory
  - Order fill rate:
    - Fraction of orders that are filled from available inventory
  - Cycle service level:
    - Fraction of replenishment cycles that end with all the demand met

Replenishment Policies

- Replenishment policy:
  - When to reorder and how much to reorder decisions.
1. Continuous review:
   - An order of size $Q$ is placed when the inventory level reaches the reorder point $ROP$.
2. Periodic review:
   - Inventory is checked at regular intervals and an order is placed to raise the inventory to a specified threshold.

Continuous Review Policy

$L$: Lead time for replenishment
$D$: Average demand per unit time
$\sigma_D$: Demand stdv per period
$D_L$: Mean demand during lead time
$\sigma_L$: Demand stdv during lead time
$\text{CSL}$: Cycle service level
$ss$: Safety inventory
$ROP$: Reorder point

\[
D_L = DL \\
\sigma_L = \sqrt{L} \sigma_D \\
CSL = F(ROP, D_L, \sigma_L) \\
ss = F^{-1}(CSL) \times \sigma_L \\
ROP = D_L + ss
\]

\[
\text{Average Inventory} = \frac{Q}{2} + ss
\]

Example 1

Estimating Safety Inventory (Continuous Review Policy)

- Assume that weekly demand for Palms at B&M Computer World is normally distributed, with a mean of 2500 and a stdv of 500. The manufacturer takes 2 weeks to fill an order placed by the B&M manager. The store manager currently orders 10,000 Palms when the inventory on hand drops to 6,000.

  - Evaluate the safety inventory carried by B&M and
  - the average inventory carried by B&M.
  - Also evaluate the average time spent by a Palm at B&M.
Example 1

Average demand per week, \( D = 2,500 \)
Standard deviation of weekly demand, \( \sigma_D = 500 \)
Average lead time for replenishment, \( L = 2 \) weeks
Reorder point, \( ROP = 6,000 \)
Average lot size, \( Q = 10,000 \)

Safety inventory, \( s_s = ROP - DL = 6,000 - 5,000 = 1,000 \)

Cycle inventory = \( Q/2 = 10,000/2 = 5,000 \)

Average inventory = cycle inventory + safety inventory = 5,000 + 1,000 = 6,000

Average flow time = average inventory/throughput = 6,000/2,500 = 2.4 weeks

Example 2

Estimating Cycle Service Level (Continuous Review Policy)

- Weekly demand for Palms at B&M is normally distributed, with a mean of 2,500 and a stdv of 500. The replenishment lead time is 2 weeks. Assume that the demand is independent from one week to the next.

  - Evaluate the CSL resulting from a policy of ordering 10,000 Palms when there are 6,000 Palms in inventory.

Fill Rate

- Proportion of customer demand satisfied from stock
- Stockout occurs when the demand during lead time exceeds the reorder point

  - \( ESC \): expected shortage per cycle,
  - \( S_s \): Safety inventory,
  - \( Q \): Order quantity,
  - \( F \): Cumulative density function of demand,
  - \( f \): density function of demand.

  \[
  fr = 1 - \frac{ESC}{Q}
  \]

  \[
  ESC = \int_{x=ROP}^{\infty} (x - ROP)f(x)dx = -\frac{1}{\sigma_s} \left[ F_s \left( \frac{3\sigma_s}{\sigma L} \right) + \sigma_Lf_s \left( \frac{3\sigma_s}{\sigma L} \right) \right]
  \]

  \( \ast \) Proof in Appendix 11C
Example 3

Evaluating Fill Rate

- Evaluate the fill rate resulting from the policy of ordering 10,000 Palms when there are 6,000 Palms in inventory, for Example 2.

Factors Affecting Fill Rate

- Increase safety inventory:
  - Fill rate increases
  - Cycle service level increases

- Increase lot size:
  - Fill rate increases
  - Cycle service level does not change

- Next:
  - Appropriate level of safety inventory given a CSL or fill rate?

Example 3

Lot size, $Q = 10,000$
Average demand during lead time, $D_l = 5,000$
Standard deviation of demand during lead time, $\sigma_l = 707$

Safety inventory, $s = Q - D_l = 10,000 - 5,000 = 5,000$

$$
ESQ = -1.000[1 - NORMDIST(10,000/707, 0, 1)]
+ 707 \text{NORMDIST}(10,000/707, 0, 1) \approx 25
$$

$$
fr = (Q - ESC)/Q = (10,000 - 25)/10,000 = 0.9975
$$

Safety Inventory Given CSL or FR

- In many practical settings,
  - Firms have a desired level of product availability and design replenishment polices that achieve this.

- The desired level of product availability
  - May be determined by trading off between inventory and stockout costs.
  - This trade-off is discussed in Chapter 12.

- Evaluating Safety Inventory Given
  - Desired Cycle Service Level
  - Desired Fill Rate
Given Cycle Service Level

- Identify safety inventory $ss$ such that:

\[ P(\text{demand during lead time} < D_L + ss) = \text{CSL} \]

\[ F(D_L + ss, D_L, \sigma_L) = \text{CSL} \]

\[ ss = F^{-1}(\text{CSL}, D_L, \sigma_L) - D_L \]

\[ ss = F_S^{-1}(\text{CSL}) \times \sigma_L \]

Example 4

Evaluating Safety Inventory Given CSL

- Weekly demand for Lego at a Wal-Mart store is normally distributed, with a mean of 2500 boxes and a stdv of 500. The replenishment lead time is two weeks. Assuming a continuous-review replenishment policy, evaluate the safety inventory that the store should carry to achieve a CSL of 90 percent.

Example 4

Given Fill Rate

- Identify safety inventory $ss$ such that:

\[ \text{ESC} = (1 - f_r)Q \]

\[ - nss \left[ 1 - F_D \left( \frac{ss}{\sigma_L} \right) \right] + nL \int \frac{ss}{\sigma_L} \]

- No closed-from solution.
- Use GOALSEEK in MS Excel.
Example 5

Evaluating Safety Inventory Given FR

- Weekly demand for Legos at a Wal-Mart store is normally distributed, with a mean of 2500 boxes and a standard deviation of 500. The replenishment lead time is two weeks. The store manager currently orders replenishment lots of 10,000 boxes from Lego. Assuming a continuous-review replenishment policy, evaluate the safety inventory the store should carry to achieve a fill rate of 97.5 percent.

\[
\text{ESC} = (1 - f)Q = (1 - 0.975)10,000 = 250
\]

\[
\text{ESC} = 250 = ss\left[1 - F\left(\frac{SS}{\sigma_L}\right)\right] + \sigma_L \frac{SS}{\sigma_L}
\]

\[
= ss\left[1 - F\left(\frac{SS}{707}\right)\right] + 707 \sigma_L \frac{SS}{707}
\]

\[
250 = ss[1 - \text{NORMDIST}(ss/707)] + 707\text{NORMDIST}(ss/707)
\]

Try Different Values of ss

<table>
<thead>
<tr>
<th>Fill Rate</th>
<th>Safety Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.5%</td>
<td>67</td>
</tr>
<tr>
<td>98.0%</td>
<td>183</td>
</tr>
<tr>
<td>98.5%</td>
<td>321</td>
</tr>
<tr>
<td>99.0%</td>
<td>499</td>
</tr>
<tr>
<td>99.5%</td>
<td>767</td>
</tr>
</tbody>
</table>

Impact of Availability and Uncertainty

- Required safety inventory increases, as
  - desired product availability increases,
  - demand uncertainty increases.

- Managerial levers to reduce safety inventory without reducing product availability
  - reduce supplier lead time, \( L \) (better relationships with suppliers)
  - reduce uncertainty in demand, \( \sigma_L \) (better forecasts, better information collection and use)
Managerial Levers on Safety Inventory

- What actions can be taken to improve product availability while reducing safety inventory?
  - Impact of supply uncertainty on safety inventory
  - Impact of aggregation on safety inventory
  - Impact of replenishment policies on safety inventory

Impact of Supply Uncertainty

- \( D \): Average demand per period
- \( \sigma_D \): Stdv of demand per period
- \( L \): Average lead time
- \( s_L \): Stdv of lead time

\[
D_L = D L
\]
\[
\sigma_L = \sqrt{L \sigma_D^2 + D^2 s_L^2}
\]

Example 6

Impact of lead time uncertainty on safety inventory

- Daily demand for PCs at Dell is normally distributed, with a mean of 2,500 and a standard deviation of 500. A key component in PC assembly is the hard drive. The hard drive supplier takes an average of \( L = 7 \) days to replenish inventory at Dell. Dell is targeting a CSL of 90 percent for its hard drive inventory.
  - Evaluate the safety inventory of hard drives that Dell must carry if the standard deviation of the lead time is seven days.
  - Dell is working with the supplier to reduce the standard deviation to zero. Evaluate the reduction in safety inventory that Dell can expect as a result of this initiative.
Impact of Aggregation

2 ways to serve demand in the \( k \) regions:
- To have local inventories in each region,
  - Total safety inventory = \( \sum_{i=1}^{k} F^{-1}_{c}(CSL) \times \sqrt{L} \times \sigma_i \)
- To aggregate all inventories into one centralized facility.
  - If aggregate demand is IID normal:
    - Total safety inventory = ?

How aggregation affects forecast accuracy and safety inventories?

Impact of Aggregation

\[
D^C = \sum_{i=1}^{n} D_i \\
\sigma^C_D = \sqrt{\sum_{i=1}^{n} \sigma^2_i} \\
\sigma^C_L = \sqrt{L} \sigma^C_D \\
s_s = F^{-1}_{s}(CSL) \times \sigma^C_L
\]

Example 7

Impact of correlation on value of aggregation

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>Disaggregate Safety Inventory</th>
<th>Aggregate Safety Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>36.24</td>
<td>18.12</td>
</tr>
<tr>
<td>0.2</td>
<td>36.24</td>
<td>22.92</td>
</tr>
<tr>
<td>0.4</td>
<td>36.24</td>
<td>26.88</td>
</tr>
<tr>
<td>0.6</td>
<td>36.24</td>
<td>30.32</td>
</tr>
<tr>
<td>0.8</td>
<td>36.24</td>
<td>33.41</td>
</tr>
<tr>
<td>1.0</td>
<td>36.24</td>
<td>36.24</td>
</tr>
</tbody>
</table>
Information Centralization

- Virtual aggregation
- Information system
  - allows access to inventory records in all warehouses
  - In case of a stockout, another warehouse can fill the order
- Better responsiveness, higher product availability, but reduced safety inventory

Specialization

- Stock all items in each location or stock different items at different locations?
  - Different products may have different demands in different locations (e.g., snow shovels)
- Specializing the distribution network with:
  - Fast-moving items stocked at decentralized locations
  - Slow-moving items stocked at a centralized locations.
- Benefits of aggregation can be affected by:
  - coefficient of variation of demand
  - value of item

Example 8

Impact of coefficient of variation on value of aggregation

- Assume that Grainger, has 1600 stores distributed throughout the United States. Consider two products: large electric motors and industrial cleaners. Demand experienced by each store is independent, and supply lead time for both motors and cleaner is four weeks. Grainger has a holding cost of 25 percent.
- Evaluate the reduction in safety inventories that will result if they are removed from retail stores and carried only in a centralized DC, for each product. Assume a desired CSL of 0.95.

Value of Aggregation at Grainger

<table>
<thead>
<tr>
<th></th>
<th>Motors</th>
<th>Cleaner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory in stock in each store:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean weekly demand per store</td>
<td>20</td>
<td>3,200</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>2.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Safety inventory per store</td>
<td>12.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Value of safety inventory</td>
<td>111,200</td>
<td>526,400</td>
</tr>
<tr>
<td>Total safety expenditure</td>
<td>$405,600,000</td>
<td>$15,752,000</td>
</tr>
</tbody>
</table>
Product Substitution

- Use of one product to satisfy the demand for another product:
  - Manufacturer-driven substitution
  - Customer-driven substitution

Component Commonality

- Using common components in a variety of different products
- With increasing product variety, component commonality is a key to reducing supply chain inventories without hurting product availability.

Example 9

Value of component commonality

- Assume that Dell is to manufacture 27 different PCs with three distinct components: processor, memory, and hard drive.
  1. Under the disaggregate option: Dell designs specific components for each PC, resulting in $3 \times 27 = 81$ distinct components.
  2. Under the common-component option, Dell designs PCs such that three distinct processors, three distinct memory units, and three distinct hard drives can be combined to create 27 different PCs.
- Monthly demand for each PCs is normal IID, with a mean of 5000 and a stdv of 3000. The replenishment lead time for each component is one month. Dell is targeting a CSL of 95 percent for component inventory.
  - Evaluate the safety inventory requirements with and without the use of component commonality.

Value of Component Commonality

<table>
<thead>
<tr>
<th>Number of Finished Products per Component</th>
<th>Safety Inventory</th>
<th>Marginal Reduction in Safety Inventory</th>
<th>Total Reduction in Safety Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>399,699</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>282,630</td>
<td>117,069</td>
<td>117,069</td>
</tr>
<tr>
<td>3</td>
<td>230,766</td>
<td>51,864</td>
<td>168,933</td>
</tr>
<tr>
<td>4</td>
<td>199,849</td>
<td>30,917</td>
<td>199,850</td>
</tr>
<tr>
<td>5</td>
<td>178,751</td>
<td>21,098</td>
<td>220,948</td>
</tr>
<tr>
<td>6</td>
<td>163,176</td>
<td>15,575</td>
<td>236,723</td>
</tr>
<tr>
<td>7</td>
<td>151,072</td>
<td>12,104</td>
<td>248,627</td>
</tr>
<tr>
<td>8</td>
<td>141,315</td>
<td>9,787</td>
<td>259,384</td>
</tr>
<tr>
<td>9</td>
<td>133,233</td>
<td>8,082</td>
<td>266,466</td>
</tr>
</tbody>
</table>
Postponement

- The ability of a supply chain to delay product differentiation or customization until closer to the time the product is sold
- Goal is to have common components in the supply chain for most of the push phase and move product differentiation as close to the pull phase as possible

Impact of Replenishment Policies

- Continuous review policies
  - P (demand during lead time < $D_l + s$) = CSL
- Periodic review policies
  - P (demand during $L + T < OUL$) = CSL

  $OUL$: Order up to level

  Mean demand during $T + L$ periods, $D_{T+L} = (T + L)D$
  Standard deviation of demand during $T + L$ periods, $\sigma_{T+L} = \sqrt{T + L}\sigma_D$

  $OUL = D_{T+L} + s$

  $s = F_0^{-1}(CSL) \times \sigma_{T+L}$

  Average lot size, $Q = D_T = DT$

Homework 7

Chapter 11

- Exercises:
  - 15
  - 16