

# Logics for Access Control and Security

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## 1 Introduction

- The Problem
- Why Logics?
- Applications of Logics in Security

## 2 A Calculus for Access Control in DS

- Basic Concepts
- The Basic Logic
- Roles and Delegation
- Extensions
- Access Control Decision Algorithm

## 3 A Propositional Policy Algebra for Access Control

- The Problem
- Basic Concepts
- Syntax
- Semantics
- The Algebra of Operators
- Determinism, Consistency, and Completeness

## 4 Summary and Conclusions

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# The Problem

- Three ingredients are essential for security in computing systems:
  - 1 A trusted computing system
  - 2 Authentication
  - 3 Authorization
- The ingredients are fairly understood in centralized systems. What about in Distributed Systems?
- Difficulties with DS:
  - Scale
  - Communication
  - Booting
  - Loading
  - Authentication
  - Authorization



# Why Logics?

- ① Clean Foundation: hence formal guarantees
- ② Flexibility
- ③ Expressiveness: possible to describe protocols & policies at reasonable level of abstraction.
- ④ Independency from implementation: important in heterogeneous distributed systems
- ⑤ Declarativeness: Users not required any programming ability
- ⑥ Having Ability of Verification
- ⑦ ...

# Applications of Logics in Security

- ❶ Logic-based Policy Specification
- ❷ Policy Composition Frameworks
  - Constraint-based Approach
  - Deontic Approach
  - Algebraic Approach
- ❸ Policy Evaluation and Verification
- ❹ Trust Management
- ❺ Model Checking for Protocol Verification
- ❻ Intrusion Detection

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# Basic Concepts

## Principal

Participants in a distributed systems are called *agents*, and the symbols the represents agents in logical expressions are called *principals*.

- Users and Machines
- Channels
- Conjunction of principals ( $A \wedge B$ )
- Groups
- Principals in roles ( $A \text{ as } R$ )
- Principals on behalf of principals ( $B \text{ for } A$ ) or ( $B|A$ )

# Basic Concepts

$A \wedge B$ :  $A$  and  $B$  as cosigners. A request from  $A \wedge B$  is a request that both  $A$  and  $B$  make.

$A \vee B$ : represents a group of which  $A$  and  $B$  are the sole members.

$A \text{ as } R$ : a principal  $A$  in role  $R$ .

$B|A$ : the principal obtained when  $B$  speaks on behalf of  $A$ , not necessarily with a proof that  $A$  has delegated authority to  $B$ . we pronounce it  *$B$  quoting  $A$* .

$B|A$  says  $s$  if  $B$  says that  $A$  says  $s$ .

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## Basic Concepts

*B for A*: the principal obtained when *B* speaks on behalf of *A* with appropriate delegation certificates.

*B for A says s* when *A* has delegated authority to *B* and *B* says that *A* says *s*.

$A \Rightarrow B$  (*A implies B*) or (*A speaks for B*): *A* is a member of group *B*. *A* is at least as powerful as *B*.

$$A \Rightarrow B \text{ iff } A = A \wedge B.$$

*A says s*: *says* is a modal operator. *A says s* is a formula that means the principal *A* believes that formula *s* is true.

$\supset$ : logical implication.

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$\supset$ : logical implication.

# Basic Concepts

*A controls s*:  $(A \text{ says } s) \supset s$ .

**ACL** is a list of assertions like  $(A \text{ controls } s)$ . If  $s$  is clear, ACL is a list of principals trusted on  $s$

## Corollary

$$\frac{B \text{ controls } s \wedge A \Rightarrow B}{\therefore A \text{ controls } s}$$

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# The Basic Logic

## A Calculus of Principals

### Some Axioms

- ①  $\wedge$  and  $|$  are primitive operators of calculus of principals.
- ②  $\wedge$  is associative, commutative, and idempotent
  - Principals form a **semilattice** under  $\wedge$ .
- ③  $|$  is associative.
  - Principals form a **semigroup** under  $|$ .
- ④  $|$  distributes over  $\wedge$ .
  - Multiplicativity implies monotonicity.

### Corollary

*Principals structure **multiplicative semilattice semigroup**, which is isomorphism with **binary relations with union and composition**.*

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# The Basic Logic

## A Logic of Principals and Their Statements

### Syntax

The **formulas** are defined inductively as follows:

- 1 a countable supply of primitive propositions  $p_0, p_1, \dots$  are formulas;
- 2 if  $s$  and  $s'$  are formulas then so are  $\neg s$  and  $s \wedge s'$ ;
- 3 if  $A$  and  $B$  are principal expressions then  $A \Rightarrow B$  is a formula;
- 4 if  $A$  is a principal expression and  $s$  is a formula then  $A \text{ says } s$  is a formula.

# The Basic Logic

## A Logic of Principals and Their Statements

### Axioms

- ① The basic axioms for normal modal logic:
  - if  $s$  is an instance of a propositional-logic tautology then  $\vdash s$ ;
  - if  $\vdash s$  and  $\vdash (s \supset s')$  then  $\vdash s'$ ;
  - $\vdash A \text{ says } (s \supset s') \supset (A \text{ says } s \supset A \text{ says } s')$ ;
  - if  $\vdash s$  then  $\vdash A \text{ says } s$ , for every  $A$ .
- ② The axioms of the calculus of principals:
  - if  $s$  is a valid formula of the calculus of principals then  $\vdash s$ .
- ③ The axioms connect the calculus to the modal logic:
  - $\vdash (A \wedge B) \text{ says } s \equiv (A \text{ says } s) \wedge (B \text{ says } s)$ ;
  - $\vdash (B|A) \text{ says } s \equiv B \text{ says } A \text{ says } s$ ;
  - $\vdash (A \Rightarrow B) \supset ((A \text{ says } s) \supset (B \text{ says } s))$ .

which is equivalent to  $(A = B) \supset ((A \text{ says } s) \equiv (B \text{ says } s))$ .

# The Basic Logic

## Semantics

### Kripke Semantics

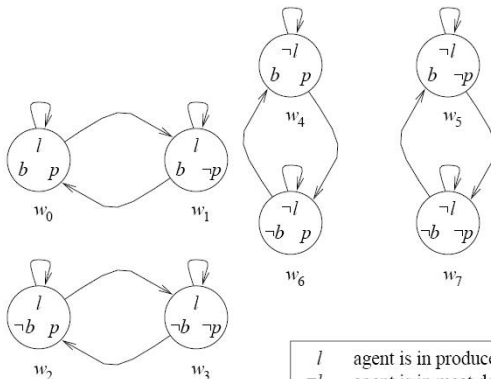
A structure  $\mathcal{M}$  is a tuple  $\langle \mathcal{W}, \omega_0, \mathcal{I}, \mathcal{J} \rangle$ , where:

- $\mathcal{W}$  is a set (a set of possible worlds)
- $\omega_0$  is distinguished element of  $\mathcal{W}$
- $\mathcal{I}$  is an interpretation function  
 $\mathcal{I} : \text{PropositionSymbols} \rightarrow \mathcal{P}(\mathcal{W})$ .  
( $\mathcal{I}(s)$  is a set of worlds where the proposition symbol is true)
- $\mathcal{J}$  is an interpretation function  
 $\mathcal{J} : \text{Principals} \rightarrow \mathcal{P}(\mathcal{W} \times \mathcal{W})$

# The Basic Logic

## Semantics

### Example



$l$	agent is in produce department
$\neg l$	agent is in meat department
$b$	the bananas are yellow
$\neg b$	the bananas are green
$p$	the pork is fresh
$\neg p$	the pork is spoiled

# The Basic Logic

## Semantics

The meaning function  $\mathcal{R}$  extends  $\mathcal{J}$  as follows:

- $\mathcal{R}(A_i) = \mathcal{J}(A_i)$
- $\mathcal{R}(A \wedge B) = \mathcal{R}(A) \cup \mathcal{R}(B)$
- $\mathcal{R}(A|B) = \mathcal{R}(A) \circ \mathcal{R}(B)$

# The Basic Logic

## Semantics

The meaning function  $\mathcal{E}$  extends  $\mathcal{I}$  as follows:

- $\mathcal{E}(p_i) = \mathcal{I}(p_i)$
  - $\mathcal{E}(\neg s) = \mathcal{W} - \mathcal{E}(s)$
  - $\mathcal{E}(s \wedge s') = \mathcal{E}(s) \cap \mathcal{E}(s')$
  - $\mathcal{E}(A \text{ says } s) = \{\omega \mid \mathcal{R}(A)(\omega) \subseteq \mathcal{E}(s)\}$
  - $\mathcal{E}(A \Rightarrow B) = \mathcal{W}$  if  $\mathcal{R}(B) \subseteq \mathcal{R}(A)$  and  $\emptyset$  otherwise
- $$\mathcal{R}(C)(\omega) = \{\omega' \mid \omega \mathcal{R}(C) \omega'\}$$

# The Basic Logic

## Semantics

### Soundness

The axioms are **sound**, in the sense that if  $\vdash s$  then  $\models s$ .

### Completeness

Although useful for our application, the axioms are **not complete**.

For example, the formula

$C \text{ says } (A \Rightarrow B) \equiv ((A \Rightarrow B) \vee (C \text{ says false}))$

is valid but not provable.

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# The Basic Logic

## On Idempotence

- The idempotence of  $|$  and  $for$  is intuitively needed.
  - $A|A = A$ :  $A$  says  $A$  says  $s$  and  $A$  says  $s$  are equal.
  - $G|A$  in an ACL postulates  $G|G|A$  and  $G|G \Rightarrow G$ .
  - Idempotence impose more complexity. e.g., it yields  $(A \wedge B) \Rightarrow (B|A)$ . On a request of  $A \wedge B$  we need to check both  $(A|B)$  and  $(B|A)$ .
- We unable to find a sensible condition on **binary relations** that would force idempotence and would be preserved by **union** and **composition**.

### Corollary

*The authors preferred to do without **idempotence** and rely on assumptions of the form  $G|G \Rightarrow G$ .*

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# Roles

## What Roles Are For?

- "Least Privilege" principle.
- For diminishing power of a user ( $A$  as  $R$ ).
- For limiting untrusted software.

## Roles, Groups, and Resources

- Roles may be related to Groups. e.g.,  $G_{role}$  related to group  $G$ .  $A$  as  $G_{role}$  means  $A$  act in the role of member of  $G$ .
- We do allow roles related to groups but this relation is not formal.
- Roles may correspond to a set of resources.

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# Roles

## The Encoding

### Definition (Role)

In the binary relation model, roles are subsets of the identity relations:  $1 \Rightarrow R$ .

A principal  $A$  in role  $R$  is defined as  $A \text{ as } R$  which is equal to  $A|R$ .

A special principal  $1$ , the *identity*, believes everything that is true and nothing that is not.

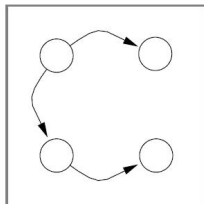
$$\mathcal{R}(1)(\omega) = \omega, \quad \forall \omega \in \mathcal{W}$$

# Roles

## The Encoding

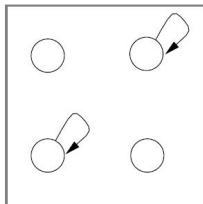
Roles reduce privileges.

$$\mathcal{R}(A) \circ \mathcal{R}(R_1) \subseteq \mathcal{R}(A)$$



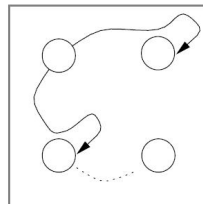
An arbitrary principal relation  $\mathcal{R}(A)$  ...

o



... composed with a role relation  $\mathcal{R}(R)$  ...

=



... gives a new relation that is always a subset of  $\mathcal{R}(A)$ .

# Roles

## The Encoding

### Role Properties

- **Monotonicity**, **multiplicativity**, and **associativity** of  $|$  offer formal advantages.
  - $|$  is **monotonic**: if  $R \Rightarrow R'$  and  $A \Rightarrow A'$ , then  $(A \text{ as } R) \Rightarrow (A' \text{ as } R')$ . [**role hierarchy** is possible]
  - $|$  is **multiplicative**:  $A \wedge B$  in a role  $R$  is identical to the conjunction of  $A$  and  $B$  both in role  $R$ .  
 $(A \wedge B) \text{ as } R = (A \text{ as } R) \wedge (B \text{ as } R)$ .  
Similarly,  $A \text{ as } (R \wedge R') = (A \text{ as } R) \wedge (A \text{ as } R')$ .
  - $|$  is **associative**: is useful for interaction between roles and delegation.  $B|(A \text{ as } R) = (B|A) \text{ as } R$  and also  $B \text{ for } (A \text{ as } R) = (B \text{ for } A) \text{ as } R$ .

# Roles

## The Encoding

### Role Properties

- All roles are:
  - idempotent ( $R|R = R$ )
  - commute with one another ( $R|R' = R'|R$ ).

These yield the following:

- $A \text{ as } R \text{ as } R = A \text{ as } R$
- $A \text{ as } R \text{ as } R' = A \text{ as } R' \text{ as } R$
- We assume:
  - $A \Rightarrow (A \text{ as } R)$  for all  $A$ .



# Delegation

## Definition (Delegation)

The ability of a principal  $A$  to give to another principal  $B$  the authority to act on  $A$ 's behalf.

- Different mechanisms embody the concept of delegation in different settings.
- Delegation relies on authentication. It is often convenient to view delegation as independent.
- We consider 3 instances of delegations:
  - Delegation without certificates;
  - Delegation with certificates;
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# Delegation

## The Forms of Delegation

### Assumptions for Delegation

- Synchronized clocks are available.
- All principals can perform digital signature.
- The formula  $K \text{ says } X$  represents the certificate  $X$  encrypted under  $K^{-1}$ .
- A certificate authority  $S$  provides certificates for the principals' public keys.

# Delegation

## The Forms of Delegation

### 1. Delegation Without Certificate

$B$  sends the signed request along with  $A$ 's name:  $K_B \text{ says } A \text{ says } r$



When  $C$  receives  $r$ , must believe that  $K_S$  is  $S$ 's public key:  $K_S \Rightarrow S$



$C$  obtains a certificate encrypted under the inverse of  $K_S$ :  
 $K_S \text{ says } (K_B \Rightarrow B)$



These yield:  $S \text{ says } (K_B \Rightarrow B)$

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$C$  trusts  $S$  for such statement:  $S$  controls  $(K_B \Rightarrow B)$



We obtain that  $C$  has  $B$ 's key:  $K_B \Rightarrow B$



$C$  sees a message as:  $K_B$  says  $A$  says  $r$ .



$C$  obtains:  $B$  says  $A$  says  $r = (B|A)$  says  $r$ .



$C$  checks ACL for  $r$ . If  $B|A$  exist in the ACL,  $r$  access is granted.





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# Delegation

## The Forms of Delegation

It is preferable for  $A$  to issue a delegation certificate that proves the delegation to  $B$ .

### 2. Delegation With Certificate

After mutual authentication,  $A$  issues a certificate to  $B$  under  $A$ 's key:  $K_A$  says ( $B$  serves  $A$ )



Checking of public-key certificates from  $S$  yields:  
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If  $C$  trusts  $A$  on this statement,  $C$  gets:  
 $((B|A) \text{ says } r) \wedge (B \text{ serves } A)$



In our theories, this implies:  $(B \text{ for } A) \text{ says } r$



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Omitting the authentication between  $B$  and  $C$ , leaving the responsibility of authenticating  $B$  solely to  $A$ .

### 3. Delegation Without Delegate Authentication

After mutual authentication,  $A$  issues a certificate to  $B$  under  $A$ 's key. The cert. includes a key  $K_d$  and  $B$ 's name. [ $K_d^{-1}$  is secret key]



$B$  can present  $A$ 's certificate to  $C$  along with request under the delegation key  $K_d$ .



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# Delegation

## The Encoding

### Delegation Properties

- *for* is monotonic
- *for* is multiplicative. This follows:  

$$(B \wedge B') \text{ for } (A \wedge A') = (B \text{ for } A) \wedge (B \text{ for } A') \wedge (B' \text{ for } A) \wedge (B' \text{ for } A')$$
- $B \text{ for } A$  is always defined, even if  $A$  has not delegated to  $B$ .

We have:

- $(B|A) \wedge (C \text{ for } A) \Rightarrow ((B \wedge C) \text{ for } A)$
- $(B|A) \wedge (C \text{ for } A) \Rightarrow (B \text{ for } A)$

# Delegation

## The Encoding

### Delegation Properties

- *for* possesses a weak associativity property:

$$(C \text{ for } (B \text{ for } A)) \Rightarrow (C \text{ for } B) \text{ for } A$$

- $(A \wedge (B|A)) \Rightarrow (B \text{ for } A)$ , because  $|$  is multiplicative.
- If  $A = A|A$  then  $A = A \text{ for } A$ , the idempotence of  $|$  implies the idempotence of *for*:  $A \text{ for } A = A$ .

# Extensions

The Basic logic can be extended in many ways, such as:

- ➊ Adding Intersection
- ➋ Adding Subtraction
- ➌ Adding Variables

# Extensions

## Intersection

An intersection operation  $\cap$  permits the construction of groups from groups.

- It can only applied to atomic symbols.  $(A_j | A_i) \cap A_k$  is not valid
- Conjunction is strictly weaker than intersection:  
 $(A \cap B) \Rightarrow (A \wedge B), (A \wedge B) \not\Rightarrow (A \cap B)$
- **Application:** restricting access to only a particular member of a group.

The ACL entry  $A \wedge G$  grants access to

- $A$ , if  $A$  is a member of  $G$ ,
- $A \wedge B$ , if  $A$  and  $B$  are members of  $G$ .

In contrast,  $A \cap G$  just grant access to

- $A$ , if  $A$  is a member of  $G$ .



# Extensions

## Subtraction

Group subtraction ( $-$ ) provides the ability to specify that certain named individuals or subgroups within a supergroup should be **denied access**.

- $G'' - G$  means all members of  $G''$ , except for those members of  $G''$  only via  $G$ .
- In distributed systems **nonmembership** may not be available.
- It is not very useful in distributed systems, just for groups which are managed by centralized subsystems.

# Extensions

## Variables

Adding variables can increase the **expressiveness**, but more **complexity**.

- Variables can be included in ACLs.
- **Example:** Existing  $(y|x)$  *controls*  $s$  in an ACL, results in granting access to  $(B|A)$ , by matching
  - $x$  with  $A$ ,
  - $y$  with  $B$ .

# Access Control Decision Algorithm

- The calculus of principals is undecidable.
- There are decidable variants of this calculus.

# Access Control Decision Algorithm

## A General Access Control Problem

### The Parts of an Instance of A.C. Decision Problem

- A principal  $P$  that is making the request.  
[An expression in the calculus of principals]
- A statement  $s$  represents what is being requested or asserted.  
[The service provider does not need to derive it logically from other statements.]
- Assumptions state implications among principals.  
[Assumptions about group membership (e.g.,  $P_i \Rightarrow G_i$ ) and idempotence (e.g.,  $G|G \Rightarrow G$ )]
- Roles  $R_0, R_1, \dots$   
[Certain atomic symbols which may be obvious from their name]
- An ACL for  $s$ .  
[A list of expressions  $E_0, E_1, \dots$  of principals that are trusted on  $s$ .]

# Access Control Decision Algorithm

## A General Access Control Problem

### The Basic Problem of Access Control

Deciding whether by having

- $\bigwedge_i (P_i \Rightarrow G_i)$ , derived from the assumptions
- $\bigwedge_i (E_i \text{ controls } s)$ , derived from the ACL

can we imply *P controls s*.

# Outline

- 1 Introduction
  - The Problem
  - Why Logics?
  - Applications of Logics in Security
- 2 A Calculus for Access Control in DS
  - Basic Concepts
  - The Basic Logic
  - Roles and Delegation
  - Extensions
  - Access Control Decision Algorithm
- 3 **A Propositional Policy Algebra for Access Control**
  - The Problem
  - Basic Concepts
  - Syntax
  - Semantics
  - The Algebra of Operators
  - Determinism, Consistency, and Completeness
- 4 Summary and Conclusions
- 5 References

# The Problem

## Problem

- Information are governed by **Policies**.
- When information is shared, it is necessary **compare**, **contrast**, and **compare** the underlying security policies.
- Problems arise in doing so is diversity and incompatibility of
  - requirements,
  - security models,
  - security policies,
  - enforcement mechanisms.

## Proposed Solution

Presenting a **policy composition framework** at the propositional level for access control.

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# Basic Concepts

## Definition (Permission and Permission Set)

- A **permission** is an ordered pair  $(object, \pm action)$ .

**Example:**  $(file1, +read)$ , or  $(file1, -write)$

- A **permission set** is a set of such permissions.

**Example:**  $\{(file1, +read), (file1, -write)\}$

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# Basic Concepts

## Definition (Nondeterministic Transformers)

- Permission Set Transformation:  $(s, PermSet) \mapsto (s, PermSet')$
- Nondeterministic Transformers of Permission Sets:  
 $(s, SetOfPermSet) \mapsto (s, SetOfPermSet')$

**Example:**

$(A, \emptyset) \mapsto (A, \{\{(f1, +r), (f1, -w)\}, \{(f1, -r), (f1, +w)\}\})$

It allows  $A$  two choices:

$\{(f1, +r), (f1, -w)\}$  or

$\{(f1, -r), (f1, +w)\}$

# Basic Concepts

## Definition (Policy)

A **policy** is interpreted as nondeterministic transformers on permission set assignments to subjects.  
Operations on policies are interpreted as relational or set theoretical operations on such nondeterministic transformers.

# Basic Concepts

## Internal vs. External Operator

There are two types of **policy composition** operators. Suppose  $P_1 = (C_1, \emptyset) \mapsto (C_1, \{(check, +read), (check, +write)\})$  and  $P_2 = (C_1, \emptyset) \mapsto (C_1, \{(check, +read), (check, +approval)\})$ .

- 1 **Internal Operators:** E.g., internal union,  
 $P_1 \cup P_2 = (C_1, \emptyset) \mapsto$   
 $(C_1, \{(check, +read), (check, +write), (check, +approval)\})$  .
- 2 **External Operators:** E.g., external union,  
 $P_1 \sqcup P_2 = (C_1, \emptyset) \mapsto$   
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# Basic Concepts

## Individual vs. Set Proposition

For **conditional authorization** we have two types of propositions:

- 1 **Individual Proposition:** applies to an object.  
E.g., *Can read any check with a face value greater than \$ 10,000.*
- 2 **Set Proposition:** applies to a set of objects.  
E.g., *if the total value of all checks to be read by a clerk is more than \$10,000.*

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# Syntax

## Definition

- $P_{atomic}$  is terminal symbol taken from a set of atomic policies *POL*
- $\phi_{atomic}$  is a terminal symbol taken from a set of atomic propositions *PROP*
- $\Phi_{atomic}$  is a terminal symbol taken from a set of atomic set (second order) propositions *SETP*
- Definition of **policies**, **propositions** and **set propositions**:

$$P := P_{atomic} | P \sqcup P | P \sqcap P | P \boxplus P | \neg P | (\phi :: P) | (P \parallel \phi) | P \cup P | \\ P \cap P | P - P | \neg P | (\phi : P) | (p \vdash \phi) | \odot P | P; P | P^* | \\ min(P) | max(P) | oCom(P) | cCom(P)$$

$$\phi := \phi_{atomic} | \phi \wedge \phi | \phi \vee \phi | \neg \phi$$

$$\Phi := \Phi_{atomic} | \Phi \wedge \Phi | \Phi \vee \Phi | \neg \Phi$$

# Syntax

## Introduction to the Notations

- $\sqcup$   $\rightsquigarrow$  external union
- $\sqcap$   $\rightsquigarrow$  external intersection
- $\boxminus$   $\rightsquigarrow$  external difference
- $\neg$   $\rightsquigarrow$  external negation
- $\phi ::$   $\rightsquigarrow$  external scoping
- $\parallel \phi$   $\rightsquigarrow$  external provisioning
- $;$   $\rightsquigarrow$  sequential composition
- $*$   $\rightsquigarrow$  closure (extension of sequential composition)
- $\cup$   $\rightsquigarrow$  internal union

# Syntax

## Introduction to the Notations

- $\cap$   $\rightsquigarrow$  internal intersection
- $-$   $\rightsquigarrow$  internal difference
- $\neg$   $\rightsquigarrow$  internal negation
- $\phi :$   $\rightsquigarrow$  internal scoping
- $! \phi$   $\rightsquigarrow$  internal provisioning
- $\odot$   $\rightsquigarrow$  invalidate permissions
- $\text{min}, \text{max}$   $\rightsquigarrow$  for conflict resolution, denial/permission take precedence
- $\text{oCom}, \text{cCom}$   $\rightsquigarrow$  open and close policy

# Semantics

## Basic Building Blocks of the Semantics

### Definition (Subjects, Objects, and Permissions)

- (1) **Subjects:** Let  $\mathcal{S} = \{s_i, i \in \mathbb{N}\}$  be a set of subjects.
- (2) **Objects:** Let  $\mathcal{O} = \{o_i, i \in \mathbb{N}\}$  be a set of objects.
- (3) **Signed Actions:** Let  $\mathcal{A} = \{a_i, i \in \mathbb{N}\}$  be a set of action terms.  
Then  $\mathcal{A}^\pm = \mathcal{A}^+ \cup \mathcal{A}^-$ , where  $\mathcal{A}^+ = \{+a : a \in \mathcal{A}\}$  and  $\mathcal{A}^- = \{-a : a \in \mathcal{A}\}$  is the set of signed action terms.
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# Semantics

## Basic Building Blocks of the Semantics

### Definition (Subjects, Objects, and Permissions)

- (5) *Authorizations*:  $(s, PermSet)$  is an authorization if one of the following conditions holds:
- $s$  is either a subject or a role and  $PermSet \subseteq \mathcal{O} \times \mathcal{A}^{\pm}$
  - $s$  is a subject and  $PermSet$  is a role. The notation  $\mathcal{AU}(\mathcal{S}, \mathcal{R}, \mathcal{O}, \mathcal{A})$  denotes the set of all authorizations.
- (6) *Permission-Prohibition Triples*:  $(s, o, \pm a)$  where  $s \in \mathcal{S}, o \in \mathcal{O}, a \in \mathcal{A}^{\pm}$ . The notation  $\mathcal{T}(\mathcal{S}, \mathcal{R}, \mathcal{O}, \mathcal{A})$  denotes the set of all permission-prohibition triples.



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# Semantics

## Basic Building Blocks of the Semantics

### Definition (State)

A state is a pair of mappings  $(M_{prop}, M_{setProp})$ , where  $M_{prop} : PROP \mapsto \mathcal{P}(T)$  and  $M_{setProp} : SETP \mapsto \mathcal{P}(\mathcal{P}(T))$ .

### Definition (Interpreting Atomic Policies)

An interpretation of atomic policies  $M_{AtPolicy}$  is a mapping from  $STATES \times POL \times (S \cup R) \times \mathcal{P}(\mathcal{O} \times \mathcal{A}^{\pm}) \mapsto (S \cup R) \times \mathcal{P}(\mathcal{P}(\mathcal{O} \times \mathcal{A}^{\pm}))$  satisfying the condition that  $s' = s$  for any  $(s', PermSet') \in M_{AtPolicy}(St)(p)(s, PermSet)$ .

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# Semantics

## Basic Building Blocks of the Semantics

### Definition (Negating Permissions Sets)

If  $PS \subseteq \mathcal{O} \times \mathcal{A}_{\pm}$  is a permission set, then

$$-PS = \{(o, -a) : (o, +a) \in PS\} \cup \{(o, +a) : (o, -a) \in PS\}.$$

If  $r \in \mathcal{R}$  is a role, then  $(o, -a) \in -r$  iff  $(o, +a) \in r$  and

$(o, +a) \in -r$  iff  $(o, -a) \in r$ .

# Semantics

## Interpreting Policy Operators

### Definition (Interpreting Policy Operators)

An interpretation  $M_{AtPolicy}$  of atomic policies is extended to an interpretation  $M_{policy}$  nonatomic policies using the following definition:

- (1)  $M_{policy}(St)(p) = M_{AtPolicy}(St)(p)$  for all atomic policies  $p$  and states  $St$ .
- (2)  $M_{policy}(St)(p \sqcup q)(s, PermSet) =$   
 $M_{policy}(St)(p)(s, PermSet) \cup M_{policy}(St)(q)(s, PermSet).$
- (3)  $M_{policy}(St)(p \sqcap q)(s, PermSet) =$   
 $M_{policy}(St)(p)(s, PermSet) \cap M_{policy}(St)(q)(s, PermSet).$
- (4)  $M_{policy}(St)(p \boxminus q)(s, PermSet) =$   
 $M_{policy}(St)(p)(s, PermSet) \setminus M_{policy}(St)(q)(s, PermSet).$

# Semantics

## Interpreting Policy Operators

- (5)  $M_{policy}(St)(\uparrow p)(s, PermSet) = \{(s, PS) : PS \in \mathcal{PS}\} \setminus M_{policy}(St)(p)(s, PermSet).$
- (6)  $M_{policy}(St)(\phi :: p)(s, PermSet) = M_{policy}(St)(p)(s, PermSet),$  if  $(s, PermSet) \in M_{setProp}(St)(\phi),$  and  $\emptyset$  otherwise.
- (7)  $M_{policy}(St)(p)(p \parallel \phi)(s, PermSet) = M_{prop}(St)(p)(s, PermSet) \cap M_{setProp}(St)(\phi).$
- (8)  $M_{policy}(St)(p; q)(s, PermSet) = \{(s, PermSet_1) \in M_{policy}(St')(q)(s, PermSet_2) : \text{for some } (s, PermSet_2) \in M_{policy}(St)(p)(s, PermSet)\}.$
- (9) To define  $M_{policy}(St)(p^*)(s, PermSet),$  inductively define  $M_{policy}(St)(p^n)$  using the following rules:
  - (a)  $M_{policy}(St)(p^1) = M_{policy}(St)(p).$
  - (b)  $M_{policy}(St)(p^{n+1}) = M_{policy}(St)((p; p^n) \cup p^n).$
  - (c)  $M_{policy}(St)(p^*) = \bigcup_{n \in \omega} M_{policy}(St)(p^n).$
- (10)  $M_{policy}(St)(p \cup q)(s, PermSet) = \{(s, PermSet_p \cup PermSet_q) : (s, PermSet_p) \in M_{policy}(St)(p)(s, PermSet_p) \text{ and } (s, PermSet_q) \in M_{policy}(St)(q)(s, PermSet_q)\}$

# Semantics

## Interpreting Policy Operators

- (11)  $M_{policy}(St)(p \cap q)(s, PermSet) = \{(s, PermSet_p \cap PermSet_q) : (s, PermSet_p) \in M_{policy}(St)(p)(s, PermSet_p) \text{ and } (s, PermSet_q) \in M_{policy}(St)(q)(s, PermSet_q)\}$
- (12)  $M_{policy}(St)(p - q)(s, PermSet) = \{(s, PermSet_p \setminus PermSet_q) : (s, PermSet_p) \in M_{policy}(St)(p)(s, PermSet_p) \text{ and } (s, PermSet_q) \in M_{policy}(St)(q)(s, PermSet_q)\}$
- (13)  $M_{policy}(St)(\neg p)(s, PermSet) = \{(s, -PermSet') : (s, PermSet) \in M_{policy}(St)(p)(s, PermSet')\}$ .
- (14)  $M_{policy}(St)(\phi : p)(s, PermSet\{(o, a) \notin M_{prop}(St)(\phi)\}) = M$  if  $M_{policy}(St)(p)(s, PermSet) = M$ .
- (15)  $M_{policy}(St)(p \setminus \phi)(s, PermSet) = M$  provided that  $M_{policy}(St)(p)(s, PermSet) = \{(s, PermSet'\{(o, a) : (s, o, a) \notin M_{prop}(St')(\phi)\})\}$ , where  $M_{policy}(St)(p) = St'$ .
- (16)  $M_{policy}(St)(\max(p))(s, PermSet) = \{(s, PermSet_1) : PermSet_1 = PermSet_2 \setminus \{(o, -a) : (o, +a), (o, -a) \in PermSet_2\} \text{ for some } (s, PermSet_2) \in M_{policy}(St)(p)(s, PermSet)\}$ .

# Semantics

## Interpreting Policy Operators

- (17)  $M_{policy}(St)(\min(p))(s, PermSet) = \{(s, PermSet_1) : PermSet_1 = PermSet_2 \setminus \{(o, +a) : (o, +a), (o, -a) \in PermSet_2\} \text{ for some } (s, PermSet_2) \in M_{policy}(St)(p)(s, PermSet)\}.$
- (18)  $M_{policy}(St)(\odot(p))(s, PermSet) = (s, \emptyset).$
- (19)  $M_{policy}(St)(cCom(p))(St)(s, PermSet) = \{(s, PermSet_1) : PermSet_1 = PermSet_2 \cup \{(o, -a) : (o, -a), (o, +a) \notin PermSet_2\} \text{ for some } (s, PermSet_2) \in M_{policy}(St)(p)(s, PermSet)\}.$
- (20)  $M_{policy}(St)(oComp)(s, PermSet) = \{(s, PermSet_1) : PermSet_1 = PermSet_2 \cup \{(o, +a) : (o, -a), (o, +a) \notin PermSet_2\} \text{ for some } (s, PermSet_2) \in M_{policy}(St)(p)(s, PermSet)\}.$



# The Algebra of Operators

## The Algebra of External Operators

### Theorem (1)

*External operator namely,  $(\mathcal{POL}, \sqcup, \sqcap, \neg, 1_{pol}, 0_{pol})$  form a Boolean algebra under the following interpretation of  $1_{pol}$  and  $0_{pol}$*

$$M_{policy}(St)(1_{pol})(s, PermSet) = (s, \mathcal{P}(PS))$$

$$M_{policy}(St)(0_{pol})(s, PermSet) = (s, \emptyset)$$

*for each  $s \in \mathcal{S} \cup \mathcal{R}$  and  $St \in STATES$ .*

In summary, theorem 1 says that we can manipulate policy operators just as operators in propositional logic and therefore use the same disjunctive or conjunctive normal forms etc.

# The Algebra of Operators

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# The Algebra of Operators

## The Algebra of External Operators

### Theorem (2)

Let  $p, p_1, p_2$ , and  $p_3$  be policy terms and  $I = (M_{policy}, M_{prop}, M_{setProp})$  be an interpretation. Then the following properties are valid in  $I$ :

1. *Idempotency of conjunctions and disjunctions:*

- $p \sqcup p = p$
- $p \sqcap p = p$

2. *Distributivity of composition over unions and intersections:*

- $(p_1 \sqcup p_2); p_3 = (p_1; p_2) \sqcup (p_1; p_3)$
- $p_1; (p_2 \sqcup p_3) = (p_1; p_2) \sqcup (p_1; p_3)$
- $(p_1 \sqcap p_2); p_3 \subseteq (p_1; p_3) \sqcap (p_2; p_3)$
- $p_1; (p_2 \sqcap p_3) \subseteq (p_1; p_2) \sqcap (p_1; p_3)$

# The Algebra of Operators

## The Algebra of External Operators

### Theorem (2)

#### 3. *properties of the scoping operator*

- $\Phi :: (\Psi :: p) == (\Phi \wedge \Psi) :: p$
- $\Phi :: (p_1 \sqcup p_2) = (\Phi :: p_1) \sqcup (\Phi :: p_2)$
- $\Phi :: (p_1 \sqcap p_2) = (\Phi :: p_1) \sqcap (\Phi :: p_2)$
- $\Phi :: (p_1 ; p_2) = (\Phi :: p_1) ; p_2$
- $\Phi :: (p_1 \boxminus p_2) = (\Phi :: p_1) \boxminus (\Phi :: p_2)$

#### 4. *Properties of the provisioning operator*

- $(p \parallel \Phi) \parallel \Psi = p \parallel (\Phi \wedge \Psi)$
- $(p_1 \sqcup p_2) \parallel \Phi = (p_1 \parallel \Phi) \sqcup (p_2 \parallel \Phi)$
- $(p_1 \sqcap p_2) \parallel \Phi = (p_1 \parallel \Phi) \sqcap (p_2 \parallel \Phi)$
- $(p_1 ; p_2) \parallel \Phi = p_1 ; (p_2 \parallel \Phi)$
- $(p_1 \boxminus p_2) \parallel \Phi = (p_1 \parallel \Phi) \boxminus (p_2 \parallel \Phi)$

# The Algebra of Operators

## The Algebra of External Operators

- In summary, **theorem 2** shows how policy operators interact with each other.
- It also shows that scoping and provisioning operators distribute over unions, intersections, and sequencing operators. So, they can be used to express composed policies.

# The Algebra of Operators

## The Algebra of Internal Operators

- Internal operators do not satisfy  $p \cup p = p$  and  $p \cap p = p$ , So they do not form a Boolean algebra.
- There are some properties for internal operators which are hold for all policies.

# Determinism, Consistency, and Completeness

## Determinism

### Determinism

- 1 Determinism with respect to Authorizations
- 2 Determinism with respect to an Interpretation
- 3 Deterministic Policy
- 4 Ultra Deterministic Policy

**Determinism** is closed under some (and not all) internal and external operators.

# Determinism, Consistency, and Completeness

## Consistency

### Consistency

- 1 Contradictory Permission Sets
- 2 Policies Consistence for Authorizations
- 3 Policies Consistent with respect to an Interpretation
- 4 Consistent Policy

Consistency is closed under some (and not all) internal and external operators.



# Determinism, Consistency, and Completeness

## Completeness

### Completeness

- 1 Complete Permission Sets
- 2 Policies Complete for Authorizations
- 3 Policies Complete with respect to an Interpretation
- 4 Complete Policy

**Completeness** is closed under some (and not all) internal and external operators.

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- The Problem
- Why Logics?
- Applications of Logics in Security

## 2 A Calculus for Access Control in DS

- Basic Concepts
- The Basic Logic
- Roles and Delegation
- Extensions
- Access Control Decision Algorithm

## 3 A Propositional Policy Algebra for Access Control

- The Problem
- Basic Concepts
- Syntax
- Semantics
- The Algebra of Operators
- Determinism, Consistency, and Completeness

## 4 Summary and Conclusions

## 5 References

- *Advantages of using logics in security*  
[Clean Foundation, Flexibility, Declarativeness, Ability of Verification, Independency from Implementation.]
- *A sample of logic-based policy specification*: A calculus for access control
- *A sample of algebraic approach in policy composition*: A propositional algebra for access control

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## References

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# Thanks for your attention ...

## Questions?