Project Planning and Control Methods

Lecture #9-P1
(Cost Estimation)

Project Cost Planning

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Outline

- Introduction
- Project cost estimation elements
- Deterministic project cost estimating
- Stochastic estimation
- Estimating in practice

Introduction

- Project cost planning includes the processes of *cost estimating* and *budgeting* so that we can have a cost baseline (or budget) set for controlling project costs during the project's execution phase!
- Two main processes are involved in project cost planning:
  - Estimating the cost: The process of developing an approximation of the money needed to complete project activities. The estimated money is going to be used for supplying resources (including people, equipment, and material) required for carrying out the project!
  - Developing the budget: The process of aggregating the estimated costs of individual activities or work packages to establish an authorized cost baseline.

Project cost estimation elements

- **Project cost estimation at different stages:**

  We may need the project estimation at different stages. However, estimations at early stages and before gaining a reasonable understanding from the project usually have reduced accuracies (can go up to 50% inaccuracy) and need a good source of related information from past projects.

  The purpose of estimations developed at different stages are different. Examples of these stages are:

  - Project definition stage (by owner): Estimations at this stage are developed for giving a sense of project scale to the project owner.
  - Design phase (by designer): In different stages of the design phase owners might ask designers to develop cost estimations of the designed facility to match it with their budgets and to ask for proper adjustments to the design or to prepare the tender!
  - Proposal development stage (by bidders): Estimations at this stage are developed for winning a profitable project
  - Detailed implementation plan development stage (by contractor): Estimations at this stage are developed for forming the project cost baseline

Project cost estimation elements

- **Different types of project costs**:
  - **Project direct costs:** Costs can be *directly* linked to specific project activities. Examples: Field workers' hourly salary, material spent in the project and cost of rental equipment used for doing specific project activities.
  - **Project indirect costs:** Costs can NOT be *directly* linked to specific project activities. Examples: Field workers' monthly benefits; office space rent; general supplies, the costs of furniture, general fixtures, and general equipment; and administrative costs.
    - **Project overhead costs:** Costs required for operating the *project* (also know as project operation cost) but are difficult to be subdivided and linked directly to specific activities. Examples**:** Project manager's, site superintendent's and site safety supervisor's salaries; general supplies spent on-site camps; computers used for the project.
    - **General and administrative costs**: Cost required for operating the *organization*. Examples of that can be: Salaries for contract department staff, top management and main office service personnel.

Project cost estimation elements

- **Note**: The *detail* estimation of project's *indirect* costs is usually very difficult and is usually skipped by project estimating group. For estimating *indirect* costs plus *company's expected benefit* in a project, usually a *mark up* percentage is estimated (according to the past project estimation experiences, level of competition, company's priorities, historical data, or markup norms in the business). This mark up is added to the estimated project's *direct* costs (which is estimated from project activities) to form the total project estimated costs.
- **Example**: We have estimated direct cost of 8-storey building project 500 MT, with the mark up rate of 15% total estimated cost of the project (including all indirect costs and expected profit) as: 1.15 * 500 MT = 575 MT
- *Mark up* is considered as a golden number in most companies and is kept secrete (specially when estimation is prepared for bidding on a project).
  - High mark up results in losing the bids!
  - Low mark up results in money loss!

Project cost estimation elements

- So estimating project costs consists of two parts:
  - Estimating project direct costs (or cost of project activities/ work packages/ phases) (more explanations in several next slides)
  - Finalize a mark up percentage for estimating indirect costs (overhead + general and administrative costs + expected profit) from direct costs. Finalizing mark up percentage is done according to many effective factors such as *level of competition, number of project's on hand*, *company's interest on winning the project*, *previous projects' estimated and actual costs*, *past experiences* and/ or *mark up norms in the business*.
    - For small or regular projects, the mark up value is set by the project estimator according to the instructions and guidelines set by company (president, contracting department , project planning department and/ or estimating department)
    - For big and critical projects the mark up value is usually decided by top management (president, top management committee, etc.)

Deterministic project cost estimating

- **Deterministic estimation**:

  When an activity duration, resources and related cost rates are deterministic, we can estimate an activity cost (AC) by multiplying number of resources worked (W) for/ spent (M) for an activity multiply by the resource cost rate (Rw for indispensable resource rates and Rm for material resources rates):

$$AC = \sum Wi * Rwi + \sum Mj * Rmj$$

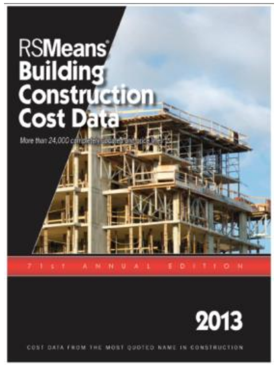
  - **Example:** What is activity cost for “Order form sheets required” for 100 m2 form-sheets at the rental rate of 9 TToman/m2. The activity is done by one purchaser spending 8 hours. The company pays 10 TToman per hour to the purchaser:

$$AC = 8 * 10 + 100 * 9 = 980 \text{ TToman}$$

Question: How can we estimate cost rates?

Deterministic project cost estimating

- **Estimating cost rates**:
  - **Based on determined rates**: Examples rates in the signed contracts with the workers, rates of purchased items and proforma invoices received!
  - **Historical rates:** From past projects and experiences
  - **Price inquiry:** Directing contacting suppliers or referring to the suppliers' official price list
  - **Cost index (فهرست بها):** Construction (and in general engineering and technical services) cost indexes for different engineering and technical services, resources and materials are usually updated by regulatory bodies (e.g., in Iran it is regulated by "strategic planning and supervision"سازمان برنامه و بودجه http://bpms.mporg.ir) or for-profit engineering and technical organizations (e.g., RSMeans annually is published by Reed Construction Data, Means® in north America www.RSMeans.com)



Deterministic project cost estimating

- **Estimating cost rates**:
  - **Cost index (cont`d):**
    - Cost indexes in the country have been divided into 15 different sections!
    - It is tried that all types of activities, resources and material required in each section be covered!

فهرست بهای واحد پایه رشته آبخیزداری و منابع طبیعی ۱۳۸۸	1
فهرست بهای واحد پایه رشته ابنیه رسته ساختمان و ساختمان صنعتی سال ۱۳۸۸	2
فهرست بهای واحد پایه رشته تاسیسا ت برقی رسته ساختمان و ساختمان صنعتی سال ۱۳۸۸	3
فهرست بهای واحد پایه رشته چاه ۱۳۸۸	4
فهرست بهای واحد پایه رشته خطوط انتقال آب ۱۳۸۸	5
فهرست بهای واحد پایه رشته شبکه جمع آوری و انتقال فاضلاب ۱۳۸۸	6
فهرست بهای واحد پایه رشته ساخت و ترمیم قنات ۱۳۸۸	7
فهرست بهای واحد پایه رشته تاسیسا ت مکانیکی ۱۳۸۸	8
فهرست بهای واحد پایه رشته راه ، باند فرودگاه و زیرسازی راه آهن رسته راه و ترابری سال ۱۳۸۸	9
فهرست بهای واحد پایه رشته راهداری ۱۳۸۸	10
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فهرست بهای واحد پایه رشته آبیاری تحت فشار ۱۳۸۸	13
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Deterministic project cost estimating

- **Estimating cost rates**:
  - **Cost index (cont`d):**
    - Make sure you have read the instructions when using the cost index book!

فهرست بهای واحد پایه رشته ابنیه

رسته ساختمان و ساختمان صنعتی

سال ۱۳۸۸

فهرست مطالب — شماره صفحه

Deterministic project cost estimating

- Estimating cost rates:
  - Cost index (cont`d):

فصل هشتم. بتن درجا

فهرست بهای واحد پایه رشته ابنیه سال ۱۳۸۸

شماره	شرح	واحد	بهای‌واحد(ریال)	مقدار	بهای‌کل(ریال)
۰۸۰۱۰۱	تهیه و اجرای بتن با شن و ماسه شسته طبیعی یا شکسته، با ۱۰۰ کیلو گرم سیمان در متر مکعب بتن.	مترمکعب	۲۷۹،۰۰۰		
۰۸۰۱۰۲	تهیه و اجرای بتن با شن و ماسه شسته طبیعی یا شکسته، با ۱۵۰ کیلو گرم سیمان در متر مکعب بتن.	مترمکعب	۳۰۹،۵۰۰		
۰۸۰۱۰۳	تهیه و اجرای بتن با شن و ماسه شسته طبیعی یا شکسته با مقاومت فشاری مشخصه ۱۲ مگاپاسکال.	مترمکعب	۳۴۹،۵۰۰		
۰۸۰۱۰۴	تهیه و اجرای بتن با شن و ماسه شسته طبیعی یا شکسته با مقاومت فشاری مشخصه ۱۶ مگاپاسکال.	مترمکعب	۳۸۳،۰۰۰		
۰۸۰۱۰۵	تهیه و اجرای بتن با شن و ماسه شسته طبیعی یا شکسته با مقاومت فشاری مشخصه ۲۰ مگاپاسکال.	مترمکعب	۴۱۳،۰۰۰		
۰۸۰۱۰۶	تهیه و اجرای بتن با شن و ماسه شسته طبیعی یا شکسته با مقاومت فشاری مشخصه ۲۵ مگاپاسکال.	مترمکعب	۴۴۱،۵۰۰		

Deterministic project cost estimating

- **Estimating cost rates:**
  - **Cost index (cont`d):**
    - Adjusting factors (ضرایب تعدیل): May be applied to different locations (location adjusting factors) or to cover inflation (time adjusting factors)!

ضرایب منطقه ای استان کرمان

شماره ردیف	نام منطقه	ضریب کارهای ساختمانی	ضریب کارهای تاسیسات مکانیکی وبرق
١٦	سرماشیر	١/١٢ (یک‌ودوازده صدم)	١/٢١ (یک‌وبیست‌ویک صدم)
١٧	کهنوج	١/١٧ (یک‌وهفده صدم)	١/٢١ ("" "")
١٨	گلباف	١/٩٦ (یک‌وششصدم)	١/٢١ ("" "")
١٩	ارزوئیه صوغان	١/١٢ (یک‌ودوازده صدم)	١/٢١ ("" "")
٢٠	راویز	١/٠٦ (یک و ششصدم)	١/٢١ ("" "")

- You have got more information on cost indexes and their usage in your "Estimating course" in undergraduate courses!

Deterministic project cost estimating

- **Estimating cost rates:**
  - **Cost Capacity Factor Estimates (mainly for plant construction):**
    - This type of project estimation is usually use at early stages, e.g., during project definition and prefeasibility studies. Cost capacity factors apply to changes in size of projects/ facilities of similar types.
    - Cost capacity factor estimates are formulized as: $C_2 = C_1 \left(\frac{Q_2}{Q_1}\right)^m$ where:

      C2 = estimated cost of new facility

      C1 = [estimated] cost of known facility

      Q2 = size of new facility

      Q1 = size of known facility

      m = cost capacity exponent factor which depends on type of industry.
    - With no additional information it is suggested value of ***m*** be set as ***0.6****. However construction of different industries there are some more specific m values provided by engineering companies which can be referred to.*

Deterministic project cost estimating

- **Estimating cost rates**:
  - **Cost Capacity Factor Estimates (cont'd):**
    - **Example**: Cost Capacity Factors (provided by F. C. Jelen, Cost & Optimization Engineering, McGraw-Hill Book Co., New York, 1970, p. 312) for cost capacity exponent of different types of plants!

Process	Unit	Cost-Capacity Factor m	Capacity Range
Aluminum (from alumina)	Metric	0.76	20M-200M
Ammonia (by steam-methane reforming)	Tons/day	0.72	100-3M
Carbon Black	Tons/day	0.53	1-150
Ethylene	Tons/yr	0.72	20M-800M
Hydrogen (from refinery gases)	Cu ft/day	0.64	500M-10MM
Methanol	Gal/yr	0.83	5MM-100MM
Oxygen	Tons/day	0.72	1-1.5M
Power plants, coal, nuclear	Mw(elec) Mw(elec)	0.88 0.68	100-1M 100-4M
Styrene	Tons/yr	0.68	4M-200M
Sulfuric acid (100%)	Tons/day	0.67	100-1M
Water Treatment	Mgpd	0.67	1-100

M = million dollars

Deterministic project cost estimating

- **Estimating cost rates**:
  - **Cost Capacity Factor Estimates (cont'd):**
    - Cost Capacity Factors might also be used for equipment.

Example (from DOZZI, S. P. & AbouRizk S. M. "PROJECT MANAGEMENT Techniques in Planning and Controlling Construction Projects" John Wiley & Sons, Inc., 1994.)

Equipment	Item	Capacity Exponent (m)
Horizontal	Pressure vessel	0.65
Centrifugal	Pumps and driver	*0.52*
Industrial	Boilers	0.5

Example: According to past jobs, production cost of an industrial boiler with the capacity of 200 kcal/h is 10MT, we can estimate the cost of our 300 kcal/h industrial boiler as: C2= 10MT *(300/200)^0.5=12.25 MT

Deterministic estimating- In class practice

The construction cost of a Carbon Black plant with the capacity of 20 tons/ day is equal to 50 M$, estimate the cost of Carbon Black plant with a capacity of 130 tons/ day!

Process	Unit	Cost-Capacity Factor m	Capacity Range
Aluminum (from alumina)	Metric	0.76	20M-200M
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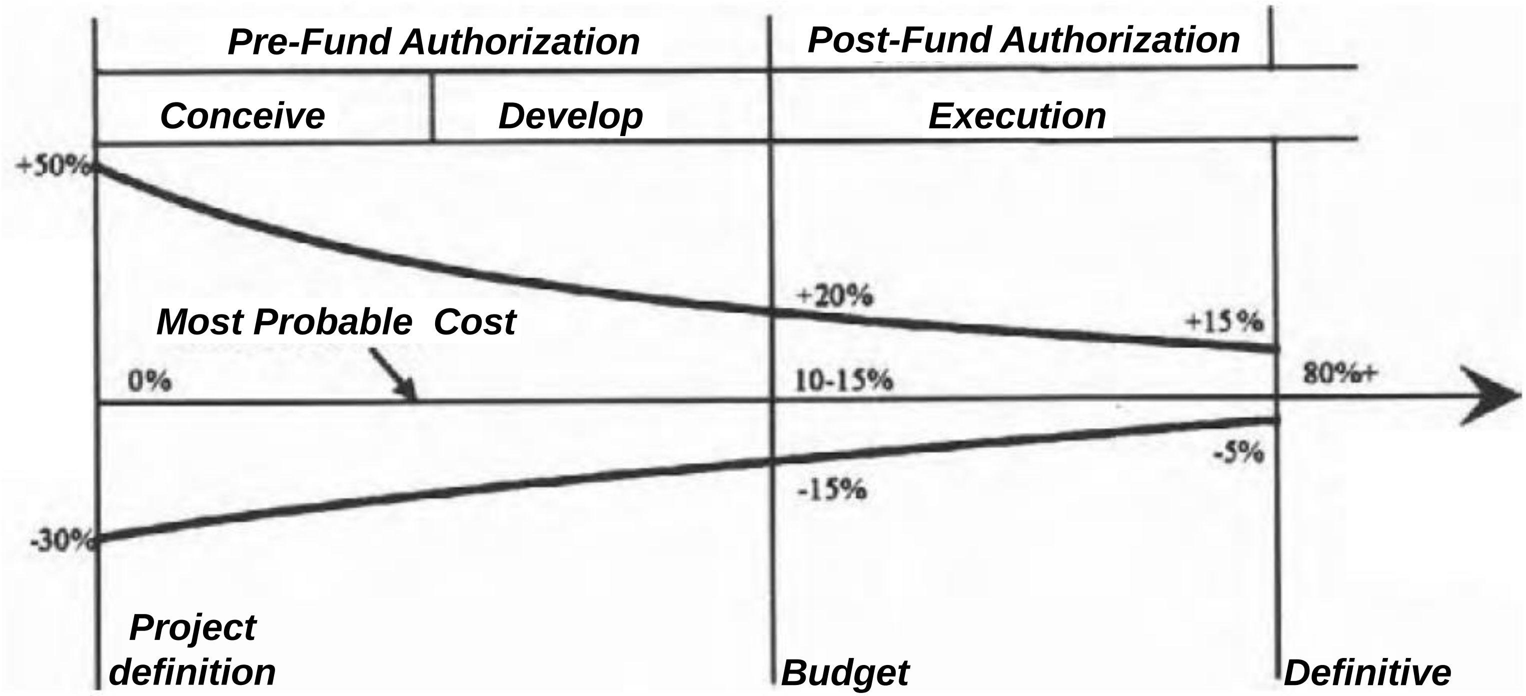
Water Treatment	Mgpd	0.67	1-100

M = million dollars

Stochastic estimation

- Activity costs are subject to changes as results of prolonged activities or variations in the number or cost rate of resources.
- It is common that project activity list gets changed during the project execution; new activities might be added; some activities might be removed from the project list!
- The earlier the project estimation is developed, the more variation in the estimation is expected.



Stochastic estimation

- **Cost Contingency** is the amount of money we reserve for the uncertain (stochastic) costs that will probably occur!!!!!!
- Use of contingencies in cost estimation is a very common practice in most projects including construction projects.

Question: How much cost contingency should we consider for a project?

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Business norms:** In many business sectors according there is an accepted percentage for cost contingencies (e.g., 20%). This percentage has been determined according to different project's outcomes in this sector.

    Example: The expected direct project cost of a project is calculated at 320 MT. If the business norm for the cost contingency is 15%, the company has a mark up rate of 10% and the legal deductions (including tax, insurance and environmental fees) are 25%. What is the total estimated cost of the project announced by the company?

Cost contingency = 320 * 15% = 48 MT

Total direct cost = 320 + 48 = 368 MT

Total cost = direct cost + indirect cost = 368 + 10% * 368 = 404.8 MT

Cost to be announced = Total cost + Legal cost   Cost to be announced * 75%

Legal cost = Cost to be announced * 25%   = Total cost

=> Cost to be announced = Total cost / 75% = 404.8 / 75% **= 539.7 MT**

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Company's past experiences**: According to the company's previous project's outcomes, a percentage has been determined for the cost contingency of projects.
  - **PERT calculation**: PERT calculations for cost estimation can be followed as in below:
    - Identify probability functions of activities with uncertain costs (use one of the methods explained in lecture 8)
    - The cost estimation of the project (C) with the activity cost of of A1, A2, ..., An has a normal distribution with the mean of μc and standard deviation of σc as follows:

$$\mu_c = \sum_{i=1}^{n} \mu_{Ai} \quad \text{and} \quad \sigma_c^2 = \sum_{i=1}^{n} \sigma_{Ai}^2$$

    - The probability of project gets completed within a certain cost (Cs) is determined by:

1) Calculating Z function (or standard normal distribution value) for Cs:

$$Zs = (Cs-\mu) / \sigma$$

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **PERT calculation (cont'd)**:

    2) Finding the probability of Zs in the Z Table of the cumulative standard normal distribution (Z-table is presented on the next page)

    - The estimation of project completion cost (of Cs) with a certain confidence level (of Ps) is determined by:

      1) Finding value of Zs base on the given probability value of Ps from Z-table

      2) Calculating value of Cs base on Zs as:

      $$Zs = (Cs-\mu) / \sigma \Rightarrow Cs = \mu + Zs \cdot \sigma$$

    - The cost contingency (CC) is calculated as the difference between expected cost (*μ) and the cost estimation.*

      $$CC = Cs - \mu$$

Stochastic estimation- In class practice

- With the stochastic project activities listed below, what is the estimated cost and cost contingency of the project with the 85% percent of the confidence level? What is the cost estimation and contingency of the project with 75% of the confidence level?

Project Cost (TT)			
Activity	Optimistic Cost (Co)	Most Likely Cost (Cm)	Pessimistic Cost (Cp)
A	5000	7000	10000
B	2000	3000	4000
C	1000	5000	6000
D	500	1000	2000

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:

    Monte Carlo simulation is computational algorithm used for recognizing stochastic phenomena in which using analytical calculations are very sophisticated or even impossible.

    In Monte-Carlo simulation we follow the actual structure of the complicated phenomena (or simulate it) by relying on repeated random sampling!

    ***Example**: Calculating π value using Monte Carlo simulation:*

    Assumptions:

    A circle area is πr^2

    Circle function centered at the origin(0,0) : $x^2 + y^2 = r^2$

Stochastic estimation

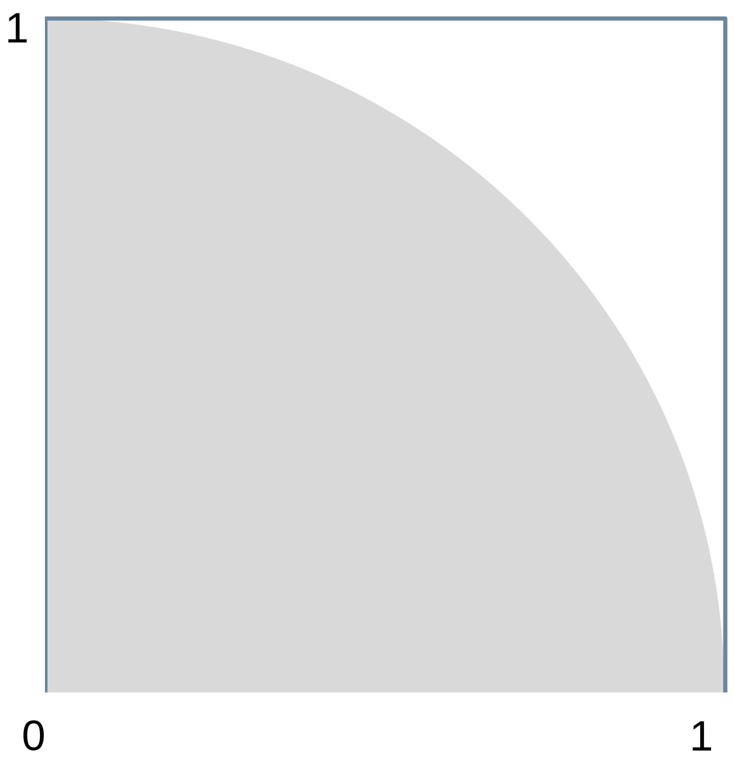
- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:

Example (cont'd):

– The area of a quarter-circle with radius of 1 unit is: *π/4*

– By throwing a random dart inside the square with the side of 1 unit, the chance that dart goes inside the embedded quarter-circle is: *π/4*

– *In Monte-Carlo simulation for calculating the value of π/4 we sample random number (0,1) for x and y.*



Stochastic estimation

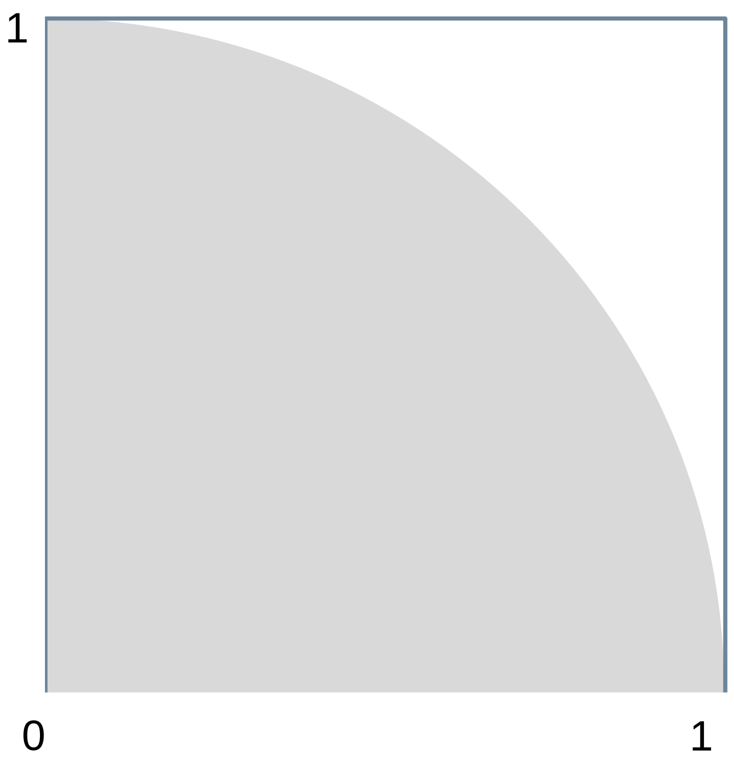
- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:

Example (cont'd):

– If $x^2 + y^2 <= 1$ we count the sample as inside quarter-circle.

– Value of *π/4 is calculated by dividing number of inside samples divided by total number of samples*

– The more the number of samples, the more accurate *π/4 is calculated*



Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:

Example (cont'd):

– If $x^2 + y^2 <= 1$ we count the sample as inside quarter-circle.

– Value of *π/4 is calculated by dividing number of inside samples divided by total number of samples*

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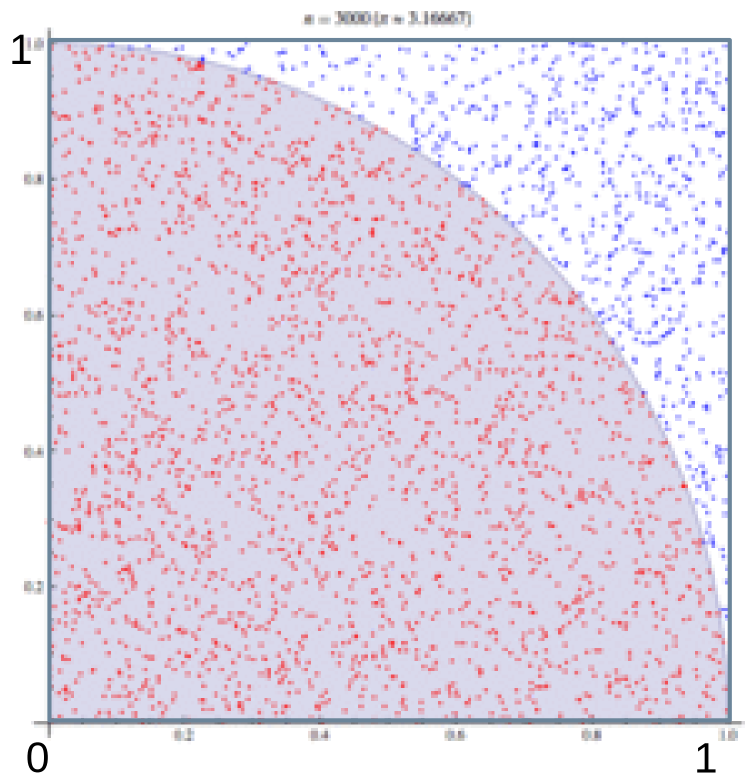


Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Main steps for using Monte-Carlo simulation in cost estimation:**
      1. Identify the distribution function of all stochastic activities (work packages or project phases) (e.g., uniform distribution, triangular distribution, normal distributions)

$$F(x \mid a, b, c) = \begin{cases} 0 & \text{for } x < a, \\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{for } a \le x \le c, \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{for } c < x \le b, \\ 1 & \text{for } b < x. \end{cases}$$

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a, b) \\ 1 & \text{for } x \ge b \end{cases}$$

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Main steps for using Monte-Carlo simulation in cost estimation:**

2. Inverse functions:

   CDFs identified for the activities cost estimations receive cost estimations (x) as inputs and give the probabilities (F(x) or P) as their output. What we need is the *inverse* of CDFs ($F^{-1}(p)$) which receive the probabilities (between 0 and 1) as their inputs and give us cost estimations as their outputs. So we should calculate the inverse functions of all cumulative distribution functions identified for activity cost estimations.

   For example inverse CDF of the uniform distribution becomes:

$$F^{-1}(p) = a + p(b - a) \quad \text{for } 0 < p < 1$$

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Main steps for using Monte-Carlo simulation in cost estimation:**

2. Inverse functions (cont'd):

   Calculations of many CDF functions are impossible!

   Who can calculate inverse function of Normal CDF?

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$

   Many statistical or even general computer programs provide inverse functions of famous statistical distributions, e.g., MS Excel!

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Main steps for using Monte-Carlo simulation in cost estimation (cont'd):**

3. Use random numbers (between 0 and 1) as inputs of inverse-CDFs and calculate the sample cost estimations of the stochastic activities through the inverse-CDFs.

Example: If we have an inverse-CDF of $F^{-1}(p) = 100 + 200p$ for an activity cost in TT, with a random number of *0.38794* the sample cost estimation for this activity will be:

$$F^{-1}(0.38794) = 100 + 200 * 0.38794 = 177.6 \text{ TT}$$

Note: There are different ways for creating random numbers, such as directly using random number generation formula, using books with tables of random number printed on them, and using computer programs (e.g., spreadsheets programs and programming language packages such as MS Excel).

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Main steps for using Monte-Carlo simulation in cost estimation (cont'd):**

4. Calculate the sample cost estimation of the project by summing up the sample cost estimation for stochastic activities and the cost of deterministic activities. Record the project cost estimation sample to be used later on!
5. Continue steps 3 and 4 until you have enough number of samples recorded! (10s, 100s, 1,000s or even 10,000s).
6. According to the *central limit theorem* sample cost estimations have normal distribution with:

   Sample mean = $\mu = \bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$ *Sample variance* = $\sigma^2 = s^2 = \frac{1}{n-1}\sum_{i}(x_i - \bar{x})^2$

   where: xi is cost estimation sample number i and n is total number of samples

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Main steps for using Monte-Carlo simulation in cost estimation (cont'd):**

7. The probability of project gets completed within a certain cost (Cs) is determined by:

   1) Calculating Z function (or standard normal distribution value) for Cs: $Zs = (Cs - \mu) / \sigma$

   2) Finding the probability of Zs in the Z Table of the cumulative standard normal distribution (Z-table is presented on the next page)

8. The estimation of project completion cost (of Cs) with a certain confidence level (of Ps) is determined by:

   1) Finding value of Zs base on the given probability value of Ps from Z-table

   2) Calculating value of Cs base on Zs as: $Zs = (Cs - \mu) / \sigma \Rightarrow Cs = \mu + Zs \cdot \sigma$

9. The cost contingency (CC) is calculated as the difference between expected cost (μ) and the cost estimation: $CC = Cs - \mu = Zs \cdot \sigma$

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Monte-Carlo Simulation Vs PERT**
      - Monte-Carlo simulation and PERT are rival methods for stochastic cost estimation calculations.
      - Monte Carlo simulation and PERT share the same statistical basis, i.e., central limit theorem, which is applied on aggregation of large number of stochastic variables.
      - Monte Carlo simulation can get more accurate since its number of samples can get quite large (e.g., 1,000s or 10,000s) compared to PERT in which number of variables participating in the summation is limited to the number of stochastic activities within the project!
      - We tend to use PERT when we can not identify statistical distribution of cost estimations for all activities and PERT formula (which is an empirical formula with more inaccuracy involved in it).

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Example:** Calculate the cost contingency required for the project in below when we are interested to accept a risk not more than 10% (i.e., the confidence level of 90%)

Project Cost (TT)			
Activity	Minimum (C_{min})	Most Likely (C_{ml})	Maximum (C_{max})
A	4000	7000	10000
B	1000	3000	4000
C	3000		6000
D	500	1100	2000

Based on the values provided we can estimate cost of activities A, B and D in triangular distribution and activity C in uniform distribution!

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Example (cont'd):** Inverse function of uniform distribution is already presented in a past slide. For inverse-CDF of triangular distribution we have:

$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & (a \le x < c) \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & (c \le x \le b) \end{cases} \quad \Rightarrow \quad F^{-1}(P) = \begin{cases} \sqrt{P(c-a)(b-a)} + a & \left(0 \le P < \frac{c-a}{b-a}\right) \\ -\sqrt{(1-P)(b-c)(b-a)} + b & \left(1 \ge P \ge \frac{c-a}{b-a}\right) \end{cases}$$

So the distribution function and inverse-CDF or each activity will become:

Project Cost (TT)		
Activity	**Distribution**	**Inverse-CDF**
A	Triangular (4000, 10000, 7000)	4000+√(P * 3000 * 6000) if (0<P<0.5) 10000-√[(1-P) * 3000 * 6000) if (0.5≤P≤1)
B	Triangular (1000, 4000, 3000)	1000+√(P * 2000 * 3000) if (0<P<0.67) 4000-√[(1-P) * 1000 * 3000) if (0.67≤P≤1)
C	Uniform(3000, 6000)	3000+ P * 3000
D	Triangular (500, 2000, 1100)	500+√(P * 600 * 1500) if (0<P<0.4) 2000-√[(1-P) * 900 * 1500) if (0.4≤P≤1)

Stochastic estimation

- Methods of estimating cost contingencies:
  - **Monte-Carlo Simulation**:
    - **Example (cont'd):** Monte Carlo simulation calculations are done in an excel sheet.

mean of 10	15437
Standard Dev of 10	1332
mean of 30	15239
Standard Dev of 30	1573
mean of 100	15339
Standard Dev of 100	1568
mean of 1000	15430
Standard Dev of 1000	1610
mean of 10000	15361
Standard Dev of 10000	1668

Sample Cost Estimates (TT)				
Project	A	B	C	D
15619	7920	2623	4202	875
12874	4360	2555	5377	581
17470	9586	2110	4387	1388
16453	8233	2137	4793	1290
16589	7881	3033	4840	835
15458	7568	3326	3200	1364
14733	5852	3061	4888	932
14980	8338	2087	3333	1222
14115	7777	1278	4232	827
16077	8853	1391	4915	917
13388	6497	1860	3267	1764
16060	7656	3406	4182	816
11315	5371	1554	3174	1217
14094	5895	1853	5023	1323
14587	7661	1714	4465	748

. . .

Random Nums			
0.7595803	0.43903	0.40061	0.15601
0.007205	0.4032	0.79248	0.00736
0.9904565	0.2053	0.46219	0.72271
0.8264573	0.21537	0.59777	0.62691
0.7506505	0.68811	0.61346	0.12466
0.6713732	0.84865	0.06658	0.70066
0.1905868	0.70593	0.62934	0.20753
0.8464973	0.197	0.11089	0.55158
0.7254936	0.01291	0.41077	0.11905
0.9269227	0.02554	0.63844	0.19303
0.3463983	0.12317	0.08901	0.95876
0.6947532	0.88243	0.3941	0.11092
0.1043732	0.05116	0.05801	0.5454
0.1995221	0.12124	0.67445	0.66044
0.6959864	0.08492	0.48822	0.06834
0.5924909	0.71539	0.54066	0.98026

. . .

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Example (cont'd):** With $Z_{0.90}$ =1.28 we have (by using the most accurate result, i.e., for 10,000 samples):

      $C_{0.90}$ = 15361 + 1.28 * 1668 = 17496 TT

      $CC_{0.90}$ = 1.28 * 1668 = 2135 TT

      If we base our calculations on 10 samples we would have:

      $C_{0.90}$ = 15437 + 1.28 * 1332 = 17141 TT

      $CC_{0.90}$ = 1.28 * 1332 = 1704 TT

      Question: What if we choose PERT calculations instead of Monte Carlo simulation?

      for uniform distribution we have: *mean (μ):* $\frac{1}{2}(a+b)$

      Variance (σ^2): $\frac{1}{12}(b-a)^2$

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Example (cont'd):**

      for triangular distribution we have:

$$Mean(\mu): \frac{a+b+c}{3}$$

$$Variance(\sigma^2): \frac{a^2+b^2+c^2-ab-ac-bc}{18}$$

Project Cost (TT)-PERT calculations

Activity	Distribution	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	Triangular (4000, 10000, 7000)	7000	1500000	1225
B	Triangular (1000, 4000, 3000)	2667	388889	624
C	Uniform(3000, 6000)	4500	750000	866
D	Triangular (500, 2000, 1100)	1200	95000	308
Project		**15367**	**2733889**	**1653**

Total cost and the contingency cost will be: $C_{0.90}$ = 15367 + 1.28 * 1653= 17483 TT

$CC_{0.90}$ = 1.28 * 1653 = 2116 TT

In this case PERT result is very close to Monte Carlo since aggregation of triangular distribution and uniform distribution very quickly converge to the normal distribution!

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **Inverse function issue:**

      Inverse functions of some CDFs are difficult to calculate; there are no CDF-inverse for many distributions. In case of no inverse function we need to stick to the heuristic methods or computationally calculate the values and build up tables.

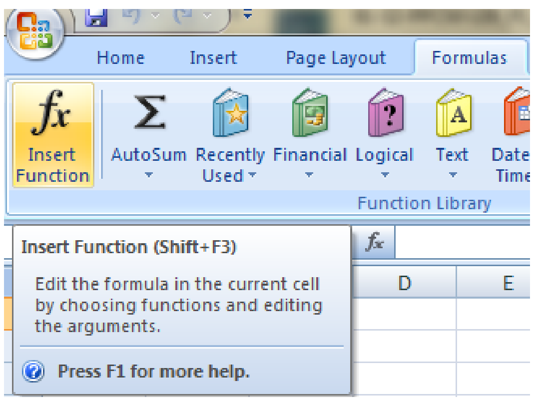
      For example there is not CDF-inverse function for normal distribution:

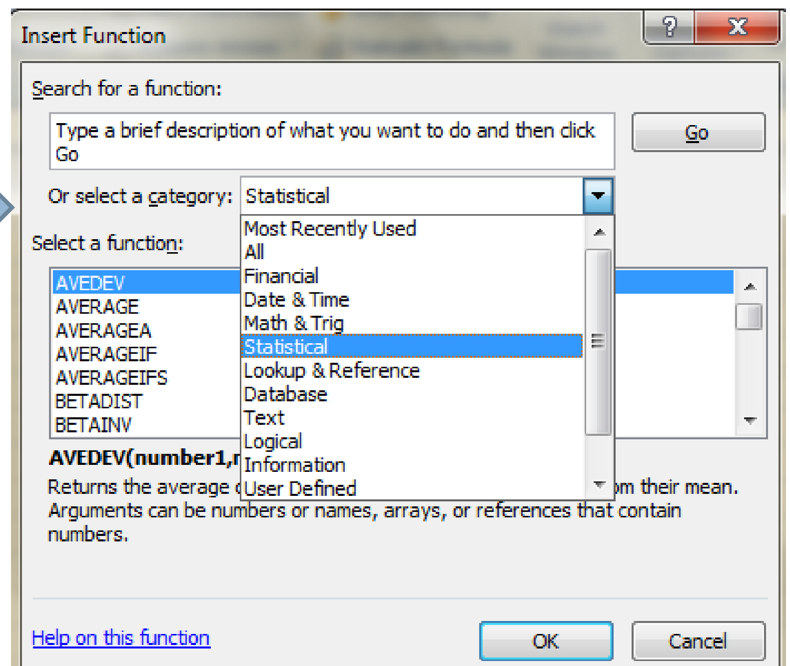
      Normal CDF $= F(x \mid \mu, \sigma) = \dfrac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(t-\mu)^2}{2\sigma^2}} \, dt$

      There is a heuristic method called *Box–Muller transform* which is used for creating normally distributed values.

Stochastic estimation

- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - Several computer programs provide specific Monte Carlo simulation features (e.g., @Risk, XLSim, MCSim and Simphony) and you can use them for developing your stochastic cost estimations.
    - Excel has embedded inverse functions for many distributions which you can use:





Find on the list:

-BETAINV

-CHIINV

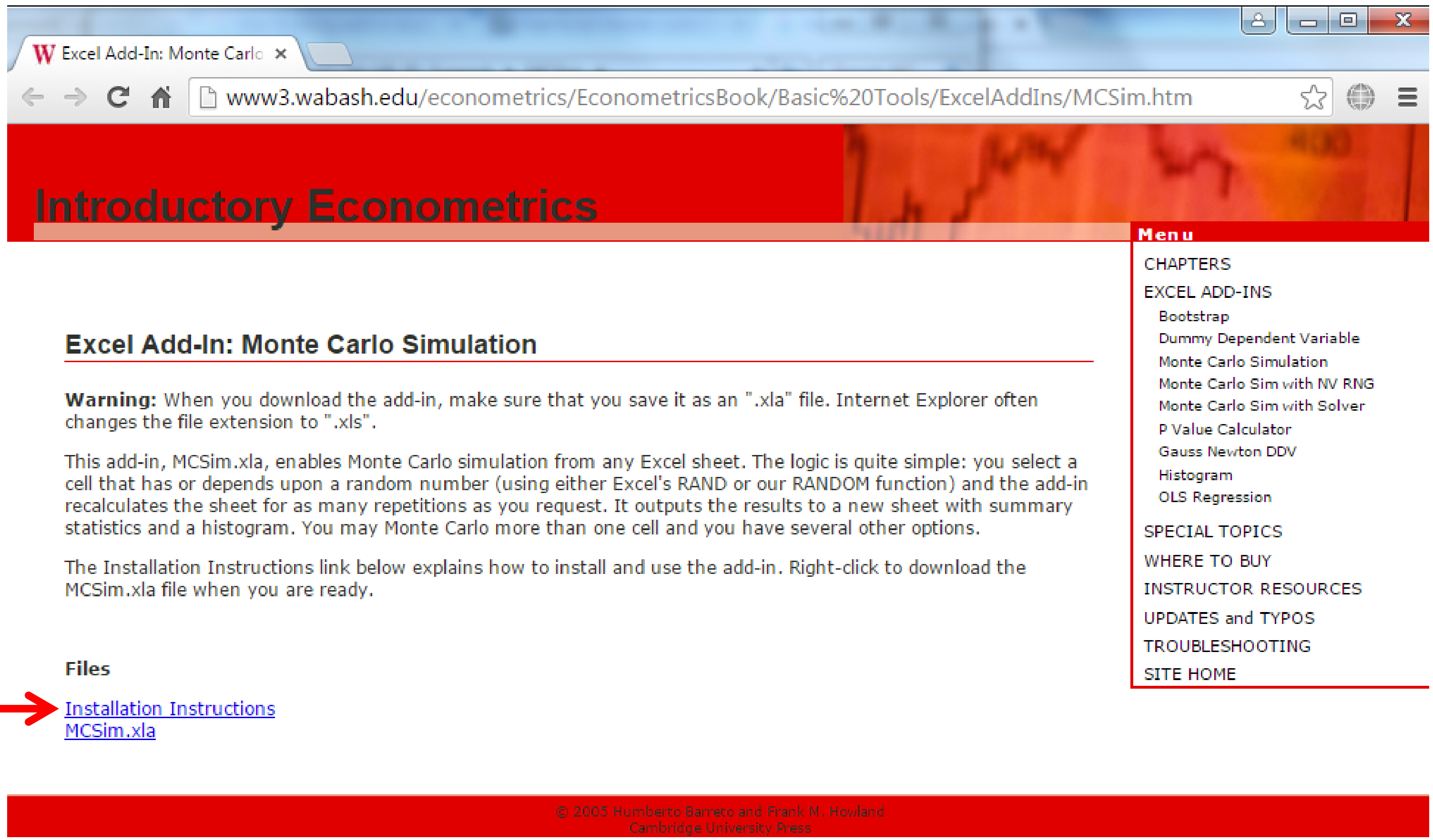
-GAMMAINV

-NORMINV (for normal distribution)

-TINV (for Student's t distribution)

Stochastic estimation

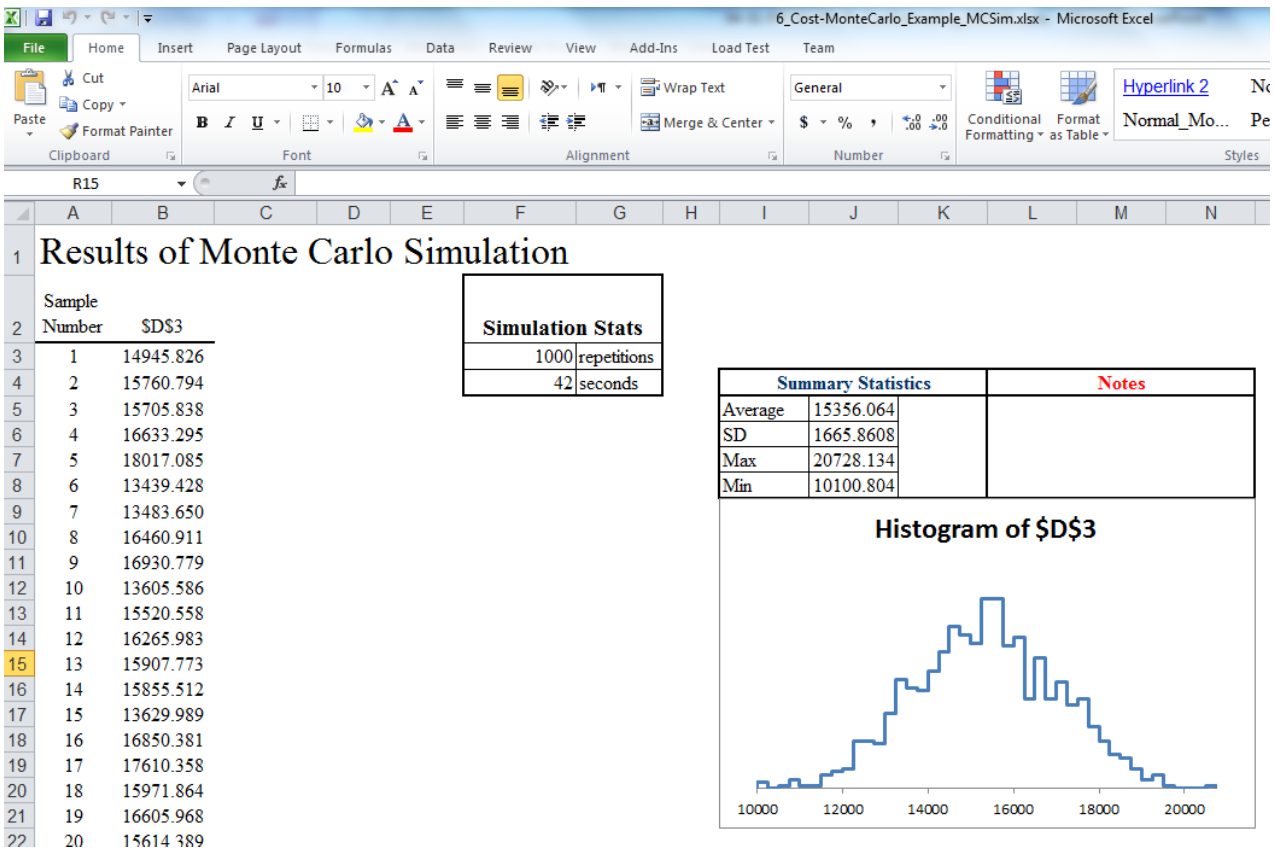
- **Methods of estimating cost contingencies:**
  - **Monte-Carlo Simulation**:
    - **MCSim** (like @Risk) is an add on software to MS Excel and is used for educational purposes!



Read the instruction to install and use it!
It is very simple

Estimating project cost

- **Stochastic estimation**:
  - **Methods of estimating cost contingencies:**
    - **Monte-Carlo Simulation**:
      - **MCSim** (like @Risk) is an add on software to MS Excel and is used for educational purposes!



Sample Number	D3
1	14945.826
2	15760.794
3	15705.838
4	16633.295
5	18017.085
6	13439.428
7	13483.650
8	16460.911
9	16930.779
10	13605.586
11	15520.558
12	16265.983
13	15907.773
14	15855.512
15	13629.989
16	16850.381
17	17610.358
18	15971.864
19	16605.968
20	15614.389

Estimating project cost

- **Methods of estimating cost contingencies:**
  - **Note**: Too many decimal digits in our numbers makes the numbers hard to read and too low number of decimal digits makes the number inaccurate. In general, in our calculations in this course for decimal place of the calculated numbers we stop at three meaningful digit level. This means:
    - In cases we have a number with three or more integer digits we do not present any decimal digits. Example: 872.47 $\rightarrow$ 872; 19654.73 $\rightarrow$ 19655
    - In cases we have a number with one or two integer digits we add two or one decimal digits to make the total digit numbers equal to three. Example: 73.57 $\rightarrow$ 73.6; 1.2 $\rightarrow$ 1.20
    - In decimal numbers (less than one), we do not consider left side zero digits as meaningful digits. So, after passing left side zero digits we will show three decimal digit numbers. Example:0.0057 $\rightarrow$ 0.00570; 0.013454$\rightarrow$ 0.0135

Estimating in practice

- **Cost Estimating Templates**
  - Most construction estimating is still performed manually, usually supported by spreadsheets and *templates* to facilitate calculation and organization. Simple Template Example.
  - There is a large number of estimating software packages such as Tadkar, Taksa, Tadbir, *ICE Interactive Cost Estimating Package*, *Timberline Software Corporation Construction Estimating Package,* Construction *Estimating Lite*, *Bid Master*, and many more, we can use for project estimating and budgeting.

Home assignment 8- Estimation

- Cost information of a project activity are given as in below table:

Activity	Cost Distribution ($)
A	Uniform (7000, 12000)
B	Triangular (2500, 4000, 6000)
C	Triangular (1000, 1500, 3000)
D	Normal(4000,500)
E	Triangular (500, 2000, 5500)
F	Exponential(mean=4000)

Calculate total cost contingency of the project with 90% level of confidence using:

1) PERT method (30 marks)

2) Mote Carlo simulation (of 1000 samples) directly using MS Excel formulas (40 marks)

3) Mote Carlo simulation (of 1000 samples) using MCSim add on to MS Excel (30 marks)

(Due: one week)

Thank you!