Project Planning and Control Methods

Lecture #8

Stochastic Duration

Amin Alvanchi, PhD

Construction Engineering and Management

LinkedIn Instagram WebPage

Outline

- Randomness in activity duration
- Distribution function - reminder
- Identifying duration distribution
- Program evaluation and review technique (PERT)

Randomness in activity duration

- All of our planning efforts so far were based on deterministic activity durations! We considered no variations in our activity durations!
- In reality, activity durations may vary from what we expect based on:
  - Random effective factors (e.g., change in the weather condition, accidents in the work, etc)
  - Unknown effective factors
  - Known, but complicated to capture, effective factors (incentives/ disincentives, skill levels, etc)
- So, we are better to be prepared for the variations in activities before start implementing our plan than blindly depending on our deterministically developed plan!
- With all these effective factors affecting activity duration, how can we still depend on deterministic planning?
- How can we show the probability in our activity durations?

Randomness in activity duration

- To be able to capture probability in an activity duration we need to assign a proper *probability distribution* Or probability function to it!
- **There are different ways for identifying activity duration distribution:**
  1) Using historical data
  2) Equipment specifications
  3) Expert judgement
  4) Sampling the activity
- Explaining all of these methods in detail is out of the scope of this course and is a topic in applied statistics or simulation courses. We are just going to briefly discuss each method as guidelines to be used when you faced similar problems in practice.

Distribution function - reminder

- A distribution function (e.g., f(x)) maps possible values of a random variable (e.g., x) against its respective probabilities (or density of probabilities) of occurrence,

- Probability distribution functions (PF) is defined for discrete variables where we can assign specific chance of occurrence to each value;

  Example: When chance that a worker shows up on the job (x=1) is 95% and chance of not showing on the job (x=0) is 5%, the probability distribution function is:

  $$f(x=1)= 0.95 \text{ and } f(x=0)=0.05$$

  - In probability distribution functions we have sum(f(x=xi); for each xi) = 1

Distribution function - reminder

- Probability density distribution functions (PDF) is defined for continuous variables where we can not assign specific chance of occurrence to each value but relational density (or weight) of occurrence is known!

  Example: When chance that an activity might be done uniformly every time between 10 to 20 hours of work, its density distribution function is:

  $$f(x) = 1/ (20-10)$$

  In this case the chance that the activity exactly gets done on an exact duration time (e.g., 15h) is 0, probability is defined on ranges of durations (e.g., from 10 to 16 hours).

Distribution function - reminder

- Cumulative distribution functions (CDF), represented as F(x), are defined as a complementary functions to probability distribution functions and probability density distribution functions defined as chance or receiving values equal or less than x;
- There are many famous distribution functions recognized in the in real world which are used mapping different stochastic phenomena (e.g., Uniform, Normal, Beta, Triangular, Exponential, etc.)
- A mean (or average or μ) value is defined for every distribution. It represents the expected value of the sum of all values observed divided by the number of observations when observations are repeated under the distribution for a large number.

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \quad ; N \to \infty$$

Distribution function - reminder

- A variance (σ^2) is another value defined for every distribution. It represents sum of the power two of deviations occurred from the mean divided by number of observations for large number of observations.

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(\mu - x_i)^2; \quad N \to \infty$$

- Square root of variance is called standard deviation (σ).
- For many distributions functions, mean and variance are calculated based on distribution parameters (examples are discussed later on in the lecture!)

Identifying duration distribution

- **Using historical data**
  - Duration records of similar activities are valuable sources for estimating activity duration distribution. Efforts done to identify best suited probability distribution/ function are called "Goodness of fit". There are several methods used for identifying activity duration distribution/ function from historical data:
    - **Data visualization:** In this method historical data are presented in a form of graph to see whether data follows a recognized statistical distribution to fit! This method is usually used in conjunction with other type of methods.

      Example:

      A construction company has historical data of its past excavation jobs as follows:

Identifying duration distribution

- Using historical data
  - Data visualization:

  Example (cont'd):

Job#	Volume (m3)	Duration (w)
90-5	15800	6
90-9	40300	15
90-10	81600	33
90-12	34200	14
90-13	45600	47
90-15	88600	110
90-17	75900	43
91-4	6700	2
91-5	87900	83
91-7	29500	9
91-8	47900	34
91-10	32800	13
91-12	32800	35
91-13	52400	23

Job#	Volume (m3)	Duration (w)
91-15	27900	9
91-19	19900	10
91-20	63600	27
91-21	4100	1
91-22	46100	32
91-23	12700	12
91-24	38000	12
91-26	51800	25
92-1	81900	31
92-2	87800	43
92-3	54600	27
92-4	99100	113
92-5	34000	63

Identifying duration distribution

- **Using historical data**
  - **Data visualization:**

    Example (cont'd): To be able to use historical data for new excavation activities estimation, we need to calculate duration rates!

Job#	Volume (m3)	Duration (w)	Duration rate (m3/w)
90-5	15800	6	**2633**
90-9	40300	15	**2687**
90-10	81600	33	**2473**
90-12	34200	14	**2443**
90-13	45600	47	**970**
90-15	88600	110	**805**
90-17	75900	43	**1765**
91-4	6700	2	**3350**
91-5	87900	83	**1059**
91-7	29500	9	**3278**
91-8	47900	34	**1409**
91-10	32800	13	**2523**
91-12	32800	35	**937**
91-13	52400	23	**2278**

Job#	Volume (m3)	Duration (w)	Duration rate (m3/w)
91-15	27900	9	**3100**
91-19	19900	10	**1990**
91-20	63600	27	**2356**
91-21	4100	1	**4100**
91-22	46100	32	**1441**
91-23	12700	12	**1058**
91-24	38000	12	**3167**
91-26	51800	25	**2072**
92-1	81900	31	**2642**
92-2	87800	43	**2042**
92-3	54600	27	**2022**
92-4	99100	113	**877**
92-5	34000	63	**540**

Identifying duration distribution

- **Using historical data**
  - **Data visualization:**

    Example (cont'd):

    - Before visualizing historical data, we need to categorize data into different categories.

    - Divide the data range (between minimum and maximum) into the same length data categories

    Note 1: Pick number of sub-ranges in a way that the *expected* sample number in each sub-range does not go below 5.

    Note 2: A good result in drawing histogram is expected when we have number of sub-ranges and expected number of samples within each sub-range are close! So for number of sub-ranges use a close integer number to the square root of the total number of historical data, the expected number of samples in each category then will be close to the number of sub-ranges!

Identifying duration distribution

- **Using historical data**
  - **Data visualization:**

Example (cont'd):

With total number of 27 historical data! We have:

Number of data subranges = Sqrt(*27) = 5.2 => 5 sub ranges*

Expected number of samples in each group = 27/ 5 = 5.4

We are going to divide the range into 5 sub-ranges with the following ranges:

Min(duration rate)= 540 m3/w Max(duration rate)= 4100 m3/w

Sub-range length = (4100 - 540) / 5 = 3560 / 5 = 712

The ranges are going to be:

540-1252	**1252-1964**	**1964-2676**	**2676-3388**	**3388-4100**

Identifying duration distribution

- **Using historical data**
  - **Data visualization:**

    Example (cont'd):

    Historical data are sorted and divided into categories!

Job#	Volume (m3)	Duration (w)	Duration rate (m3/w)
92-5	34000	63	540
90-15	88600	110	805
92-4	99100	113	877
91-12	32800	35	937
90-13	45600	47	970
91-23	12700	12	1058
91-5	87900	83	1059
91-8	47900	34	1409
91-22	46100	32	1441
90-17	75900	43	1765
91-19	19900	10	1990
92-3	54600	27	2022
92-2	87800	43	2042
91-26	51800	25	2072
91-13	52400	23	2278
91-20	63600	27	2356
90-12	34200	14	2443
90-10	81600	33	2473
91-10	32800	13	2523
90-5	15800	6	2633
92-1	81900	31	2642
90-9	40300	15	2687
91-15	27900	9	3100
91-24	38000	12	3167
91-7	29500	9	3278
91-4	6700	2	3350
91-21	4100	1	4100

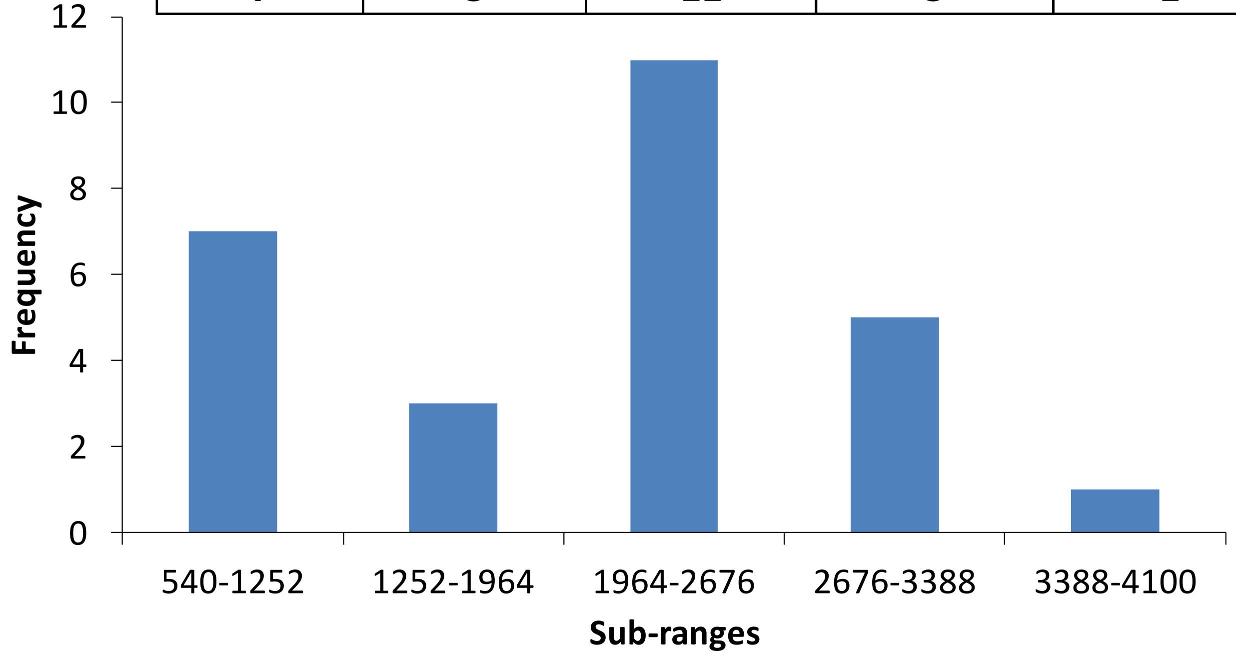
540-1252	1252-1964	1964-2676	2676-3388	3388-4100
7	3	11	5	1

Identifying duration distribution

- **Using historical data**
  - **Data visualization:**

    Example (cont'd):

540-1252	1252-1964	1964-2676	2676-3388	3388-4100
7	3	11	5	1



Can we estimate the distribution directly from the chart?

Identifying duration distribution

- **Using historical data**
  - **Fit of distributions:** In this method historical data are mathematically tested against different statistical distributions. There are different test methods; the most common methods used are *Kolmogorov–Smirnov test* and *Chi Square test.*
  - **Regression analysis:** In this method we are going to estimate the relationships among historical data by identifying a custom-built distribution function.
  - **Use of statistical computer programs for identifying the distribution:** Statistical based computer program provide a variety of features for doing *goodness of fit* on our historical data using improved mathematical methods. Statistical programs such as: SPSS, SAS, R, Stata, EasyFit, BestFit

Identifying duration distribution

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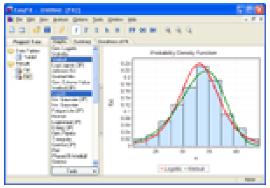

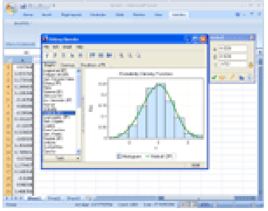

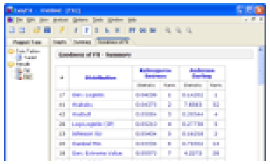

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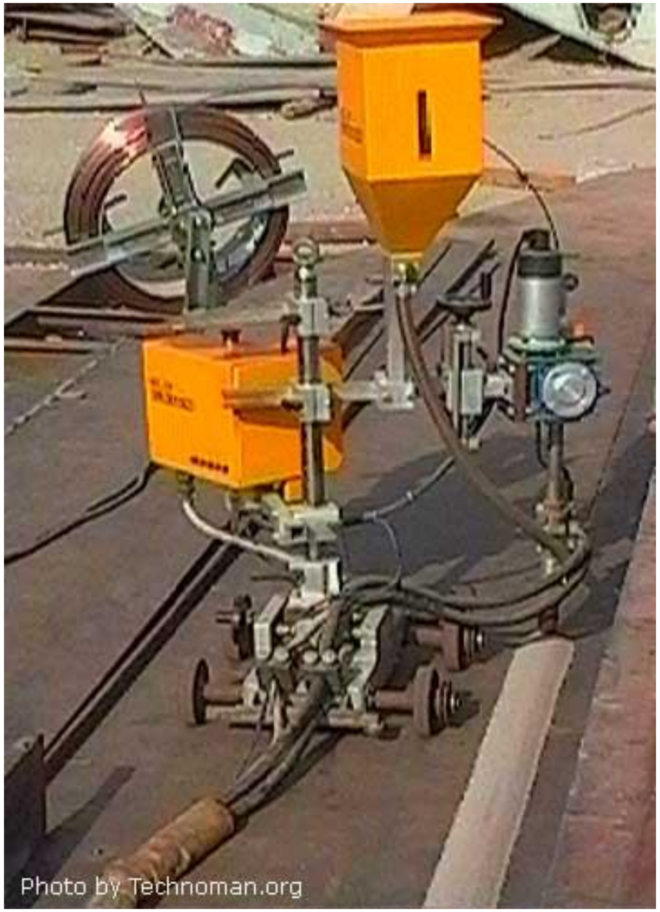
Identifying duration distribution

- **Equipment specification**
  - When an activity duration is driven by equipment, manufacturer's published capacity is the reference for our duration estimation. There are some guidelines to be follows in such cases:
    - Variation in equipment work-capacities usually follow normal distribution. In many cases we can estimate duration of equipment driven tasks with normal distribution!
    - Variations in the equipment work-capacity provided by equipment manufacturer usually are presented as $\mu \pm 3\sigma$ or "capacity average ± 3 times of standard deviation".

      Note: 3σ is an important value in the normal distribution because almost all (or 99.73% of) random values following a normal distribution will fall under $\pm 3\sigma$ range.
    - When work-capacity of equipment follows normal distribution (i.e., it is presented as $\mu \pm 3\sigma$) we can estimate duration distribution of equipment driven activity with normal distribution. In this case duration average is calculated base on average work-capacity of the equipment. For calculating standard deviation, we can consider the difference between *upper* and *lower* bound as 6σ.

Identifying duration distribution

- **Equipment specification (cont'd)**
  - **Example 1**: According to the manufacturer's published specification a sub-arch welding machine can weld 1cm of 1in plates in 1minute±10%.
    What is the duration distribution for our activity of "welding 1in plates" with total welding length of 100 meter?
    Capacity has a normal distribution with the average of 1 minute and standard deviation of 2 seconds



Note: $X = \sum X_i$; $i = 1, 2, \ldots, n$ and $X_i \sim Normal(\mu_{i,} \sigma_i{}^2)$

$$X \sim Normal(\sum \mu_i , \sum \sigma_i{}^2)$$

Identifying duration distribution

- **Equipment specification (cont'd)**
  - **Example 1 (cont'd)**:

    For 100 m or 10,000 cm of the plate length we are going to have a duration with normal distribution with the average of (10,000 * 1 minute =) 10,000 minutes (or 167 hours) and standard deviation of ($\sqrt{10{,}000}$ * 2 seconds = 100 * 2 seconds =) 200 seconds.
  - **Example 2**: According to the manufacturer's published specification a sub-arch welding machine has a speed of 2 cm±15% per minute for our ½ in plates. What is the duration distribution for our activity of "welding ½ in plates" with total welding length of 500 m?

    Since variations in speed has been given, we need first to calculate the duration variation per specific amount of progress then multiply our length to it.

Lower bound time for welding 2 cm of the plate is: $\dfrac{2\text{cm}}{(2+15\%)\ \text{(cm/minute)}} = (2 / 2.3)\ \text{minute} = 0.87\text{minute}$

Identifying duration distribution

- **Equipment specification (cont'd)**
  - **Example 2 (cont'd)**:

Upper bound time for welding 1 cm of the plate is: $\dfrac{2\text{cm}}{(2\text{-}15\%)\ \text{(cm/minute)}}$ = (2 / 1.7) minute = 1.18minute

Average time for welding 1 cm of the plate is(μ): $\dfrac{2\text{cm}}{2\ \text{(cm/minute)}}$ = 1 minute

Standard deviation: (upper bound – lower bound) / 6=(1.18 -0.87) / 6 minute =0.051 minute

We can calculate our 500 m or 50,000 cm welding activity duration with normal distribution:

Average: 25000 *1 minute=25000 minute = **417 hour**

Standard deviation: $\sqrt{25000}$ * 0. 051 minute = 158 * 0.051 minute = **8.09 minute**

Note: You may need to add overhead time to the equipment operation duration when estimating activity durations!

In class practice - equipment specification

- According to the manufacturer’s published specification an excavator can excavate 100lit mud from the ditch during every excavation cycle. Every excavation cycle has a duration of 150s±25%. What is the duration distribution for digging our ditch with the length of 100m, width of 50cm and depth of 1m?

Identifying duration distribution

- **Expert judgement**
  - In a lack of historical data, use of expert judgement is a method to be used for estimating activity duration distribution.
  - Use of expert judgement in estimating activity durations can result in three types of distributions: - *Uniform distribution* - *Triangular distribution* and – *PERT Formula*
  - **Uniform distribution** is used for duration estimation when activity experts give a duration range (with a minimum of *a* and a maximum of *b*) for the activity duration. In this case activity duration distribution will be estimated as a uniform distribution:

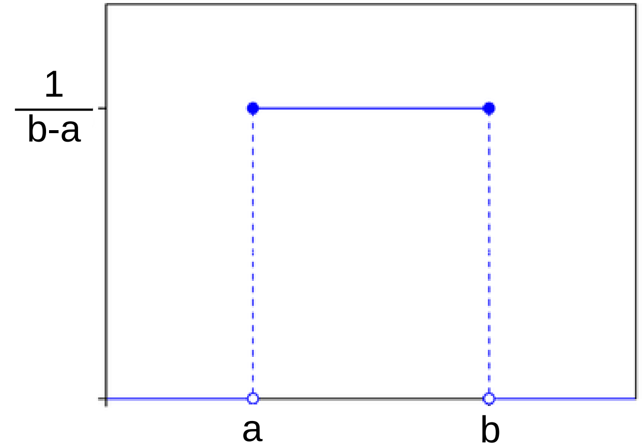
$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a, b) \\ 1 & \text{for } x \ge b \end{cases}$$

mean (μ): $\frac{1}{2}(a+b)$

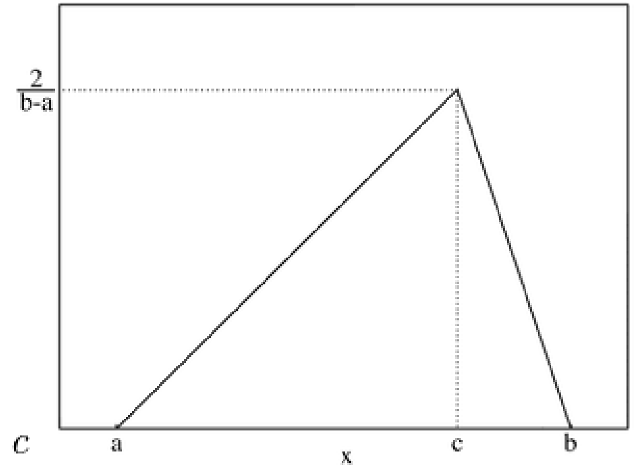
Variance (σ^2): $\frac{1}{12}(b-a)^2$

σ: $\frac{1}{\sqrt{12}}(b-a)$



Identifying duration distribution

- **Expert judgement (cont'd)**
  - **Triangular distribution** is used for duration estimation when an activity experts give a duration range (with a minimum of *a* and a maximum of *b*) for the activity duration and a most likely value (distribution mode of *c*). In this case activity duration distribution will be estimated as triangular distribution :



$$f(x|a,b,c) = \begin{cases} 0 & \text{for } x < a, \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \le x \le c, \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \le b, \\ 0 & \text{for } b < x, \end{cases}$$

$$F(x|\ a,\ b,\ \ c) = \begin{cases} 0 & \text{for } x < a, \\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{for } a \le x \le c, \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{for } c < x \le b, \\ 1 & \text{for } b < x. \end{cases}$$

Mean(μ): $\frac{a+b+c}{3}$

Variance(σ²): $\frac{a^2+b^2+c^2-ab-ac-bc}{18}$

σ: $\sqrt{\frac{a^2+b^2+c^2-ab-ac-bc}{18}}$

Identifying duration distribution

- **Expert judgement (cont'd)**
  - **Triangular distribution**

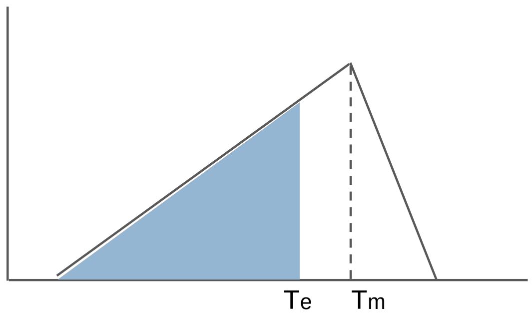
Question: What is difference between most likely and expected (mean or average)?

From a linguistic viewpoint, most likely and expected may be thought of as similar.

However, statistically they are different.

Most likely duration (T_m) is the duration that we believe has more likelihood of happening

Expected duration (T_e or μ_x) *the weighted average of all possible values that a random variable can take on*. $\mu_x = \int_{-\infty}^{\infty} x f(x)\, \mathrm{d}\, x.$

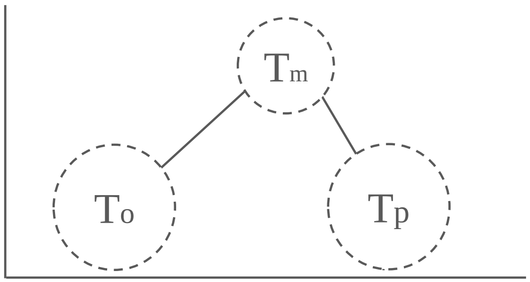


Identifying duration distribution

- **Expert judgement (cont'd)**
  - **PERT (program evaluation and review technique) formula** is used for duration estimation when activity experts give optimistic duration (T_o), pessimistic duration (T_p) and most likely duration (T_m). PERT formula is one the simplest but with reasonable accuracy that we can get experimentally. In this case activity duration distribution will be estimated as:

$$Te\ (\mu): \frac{To + 4Tm + Tp}{6}$$

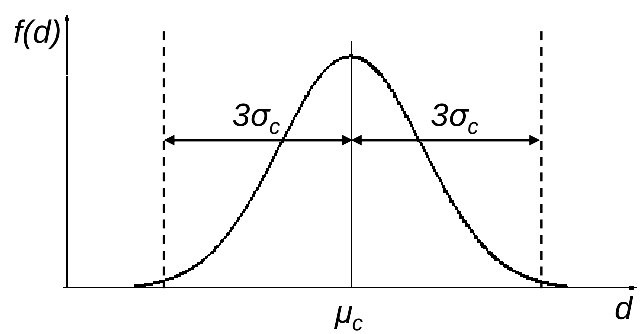
$$\sigma: \frac{Tp - To}{6}$$



- **Sampling the activity:** When we are dealing with a repetitive activities sampling is a good method; usually if historical data is not available. After sampling a proper number of samples, the "goodness of fit" will be done for identifying the distribution of the activity similar to what we explained for historical data.

Program evaluation and review technique (PERT)

- PERT was developed in 1950s by US navy to predict project duration with probabilistic activity durations.
- Use PERT network analysis technique for estimating project duration (or the duration of specific chains of activities) when activity durations are ***uncertain*** or ***probabilistic***.
- PERT analysis is used as a *complementary* technique to CPM (not a replacement)!
- **PERT calculation for a chain of activities:**
  - Identify the distribution (or probability function) of every activity duration for the activities on your desire activity-chain
  - The duration of the activity chain (C) with the activity duration of A1, A2, ..., An on the chain has a ***normal distribution*** with the mean of μ_c and standard deviation of σ_c as follows:
    - $\mu_c = \sum_{i=1}^{n} \mu_{Ai}$ and $\sigma_c^2 = \sum_{i=1}^{n} \sigma_{Ai}^2$



Program evaluation and review technique (PERT)

- **PERT calculation for a chain of activities (cont'd):**

  Note: The basis on why summation of a set of random variables with *different* probability distributions (e.g., uniform, triangular, PERT function, etc) converges to the normal distribution with the mentioned mean and standard deviation has been provided on the *Central limit theorem* available in the main statistical references.

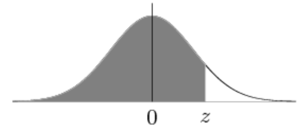
  - The probability of activity-chain gets complete within a certain duration (of Ts) is determined by:
    - Calculating Z function (or standard normal distribution value) for Ts:

      $$Zs = (Ts - \mu) / \sigma$$

    - Finding the probability of Zs in the Z Table of the cumulative standard normal distribution (Z-table is presented on the next page)

Cumulative Normal Distribution Table

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-s^2/2}\, ds$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Program evaluation and review technique (PERT)

- **PERT calculation for a chain of activities (cont'd):**
  - **Example 1:** We have a chain of activities (ABCDE) as follows and we want to estimate the chance that the chain gets complete before 35 days.

Activity	Duration (Days) Optimistic (T_o)	Most Likely (T_m)	Pessimistic (T_p)
A	2	4	7
B	5	8	14
C	4	6	8
D	2	2	2
E	7	10	21

  - Calculating activity distribution for example for activity A we have:

$$\mu_A = \frac{To + 4Tm + Tp}{6} = \frac{2 + 4 \times 4 + 7}{6} = 4.167 \qquad \sigma_A = \frac{Tp - To}{6} = \frac{7-2}{6} = 0.833$$

Program evaluation and review technique (PERT)

- **PERT calculation for a chain of activities (cont'd):**
  - **Example 1 (cont'd):** Estimating *μ (Te) and* σ of the rest of activities we are have:

Activity	Duration: Optimistic (T_o)	Most Likely (T_m)	Pessimistic (T_p)	Expected Duration (T_e)	Standard Deviation (σ_e)	Variance $V_e = \sigma_e^2$
A	2	4	7	4.167	0.833	0.694
B	5	8	14	8.500	1.500	2.250
C	4	6	8	6.000	0.667	0.444
D	2	2	2	2.000	0	0
E	7	10	21	11.333	2.333	5.444

$\mu = \Sigma\, \mu_{Ai} = 32,\qquad \sigma^2 = \Sigma\, \sigma_{Ai}^2 = 8.833 \quad and \quad \sigma = 2.972$

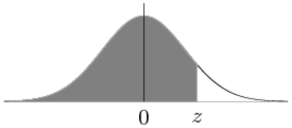
For Ts = 35 we have $Zs = (Ts - \mu) / \sigma = (35 - 32) / 2.972 = 3 / 2.972 = 1.01$

Looking into the Z-Table we have the probability estimated as:

$P(Z<Zs=1.01) = P(T<Ts=35) = 0.8438 = 84.38\%$

Cumulative Normal Distribution Table

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-s^2/2}\, ds$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Program evaluation and review technique (PERT)

- **PERT calculation for a chain of activities (cont'd):**
  - **Example 1 (cont'd):**

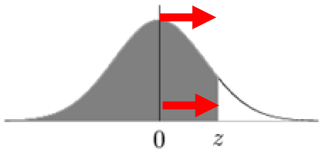
What is the chance of completing the activity chain within 29 days?

For $T_s = 29$ *we have* $Z_s = (T_s - \mu_c) / \sigma = (29 - 32) / 2.972 = -3 / 2.972 = \mathbf{-1.01}$

Looking into the Z-table we can not find any negative values. In our Z-table (as is indicated on the table graph at the top the table), only positive values of the *standard* normal distribution, which represent values above mean in normal distributions, are presented.

CUMULATIVE NORMAL DISTRIBUTION TABLE

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-s^2/2}\, ds$$



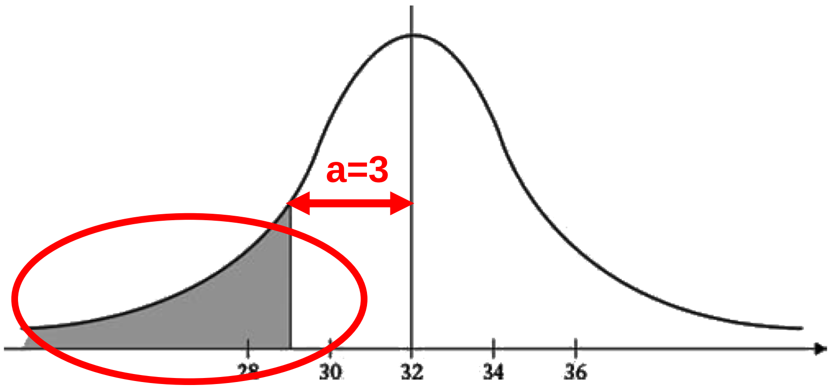
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224

Program evaluation and review technique (PERT)

- **PERT calculation for a chain of activities (cont'd):**
  - **Example 1 (cont'd):**

However our Ts value of 29 is below the mean value of 32 and results in the negative Zs value of -1.01.

Since normal distribution is a symmetric distribution around the mean, we are going to use the absolute value of the Zs to find the probability of Ts.



As a result of symmetry in the normal distribution we have:

P(T<Ts= μ-a) = P(T> μ+a) = 1-P(T< μ+a) therefore for standard normal distribution with mean value of 0 we have: P(Z<-Zs) = P(Z>Zs) = 1-P(Z<Zs)

Program evaluation and review technique (PERT)

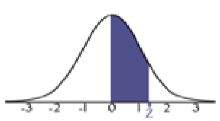
- **PERT calculation for a chain of activities (cont'd):**
  - **Example 1 (cont'd):**

So for calculating the probability of Zs= -1.01 we have:

P(T<Ts=29) = P(Z<Zs= -1.01) =1- P(Z<1.01) = 1 – 0.8438 = **0.1562**

Note: Z-tables may come in different formats. They usually present Z-values for the one side of the mean. Regardless of the presentation format they are following you can always extract probability of the both positive and negative values following the symmetry concept.

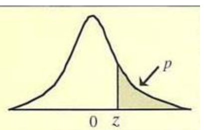
Some examples of the Z-tables are as in below:



STANDARD NORMAL TABLE (Z)

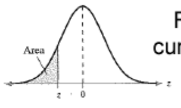
Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852



	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121

Standard Normal Distribution



Remember that these areas are the cumulative totals to the ***LEFT*** of, or less than, the z score.

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047

Program evaluation and review technique (PERT)

- **PERT calculation for a chain of activities (cont'd):**
  - We might want to find the activity-chain completion duration (of Ts) with a certain confidence level (of Ps) :
    - Finding value of Zs base on the given probability value of Ps from Z-table
    - Calculating value of Ts base on Zs as:

      $Zs = (Ts-\mu) / \sigma \Rightarrow Ts = \mu + Zs \cdot \sigma$

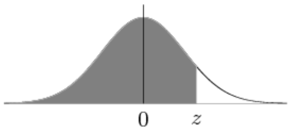
    Example 2: With the same set of activities as previous example (example 1) suppose that client asks us to announce the activity-chain completion duration with confidence level of 90%.

    Finding corresponding Z value to 90% in Z-table we have Zs = *1.28* (see the next slide)

    Ts can be calculated as Ts = 32 + 1.28 * 2.972 = *35.8 days*

Cumulative Normal Distribution Table

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-s^2/2}\, ds$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	$\boxed{0.08}$	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
$\boxed{1.2}$	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	$\boxed{.8997}$	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

PERT - In class exercise

- For the activity chain of ABCDE presented in below, what is the chance of completing the activity chain within 54 days? What is the activity chain duration with the confidence level of 95%?

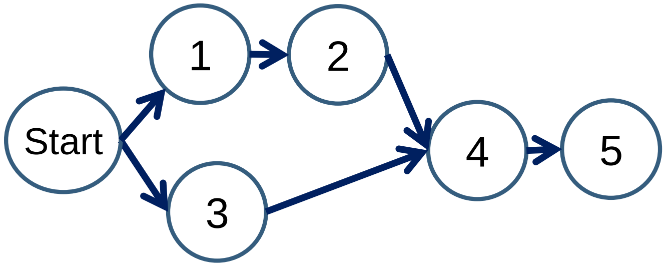
	Duration (Days)		
Activity	Optimistic (T_o)	Most Likely (T_m)	Pessimistic (T_p)
A	4	5	6
B	10	12	14
C	14	17	22
D	10	12	15
E	8	9	10

Program evaluation and review technique (PERT)

- **Guidelines for PERT calculation in a *Project***
  - So far what we have discussed was about an activity-chain not a Project!
  - For calculating the project duration with uncertain activity durations,
    1) Like calculations in a activity chain we first need to identify activity duration distribution
    2) Do CPM calculations on the project network base on the expected durations (μ) of the activities
    3) Variations in the project duration will be led by critical chain(s). So you need to follow all steps mentioned for activity-chain, but for the critical chain(s)!
    4) In cases we have several critical chains (i.e., critical chains with the same expected duration). For your PERT calculations pick the critical chain with *higher variance*!
  - By doing PERT analysis on a project we get a better understanding on the project and its path forward.
  - PERT analysis helps us to adjust our milestones in the project including the project's completion date!

Home assignment 7 - PERT

- Referring to our “Prepare foundation form work” work package with the following dependencies:
  1) Extract foundation sizes from drawings (1 engineer)
  2) Order form sheets (1 purchaser)
  3) Hire form-workers for the job(1 HR-person)
  4) Size form sheets (2 form-worker)
  5) Install form sheets in place (2 form-worker)



- Activity durations are provided according to the information collected:

Activity	Duration
1	2 days
2	Min=4 h; Max= 7 h
3	Min= 6 h; Max= 9h; Most likely = 7 h
4	Optimistic = 10 m2/ManH; Pessimistic = 5 m2/ManH; Most likely = 8 m2/ManH
5	Refer to the historical data from past jobs

Home assignment 7 - PERT

Suppose we have a 3000 m2 foundation job and we are going to hire total 3 form worker for the job! The starting date is Dec 6th and our working week is from Saturday to Wednesday, 8 hours in each day. Site is closed on statutory holidays.

1- Visualize historical data provided for activity 5 (Install form-sheets in place)! (20% mark)

2- Calculate mean and standard deviation of the historical data! (10% mark)

3- Do CPM calculations! (15% mark)

4- Present the schedule on MSP! (15% mark)

5- With 90% level of confidence calculate the work package duration. (20% mark)

6- What is the chance that this work package is done before January ends? (20% mark)

(Due: 1 week)

Thank you!