Chapter 6
Viscous and
Viscoelastic Dampers

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Major References

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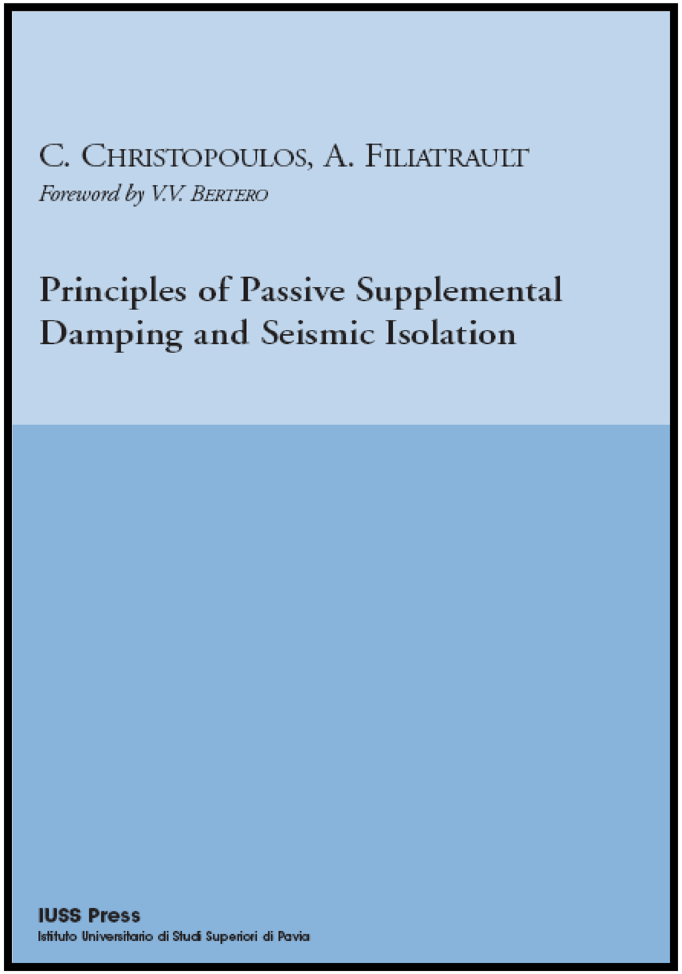
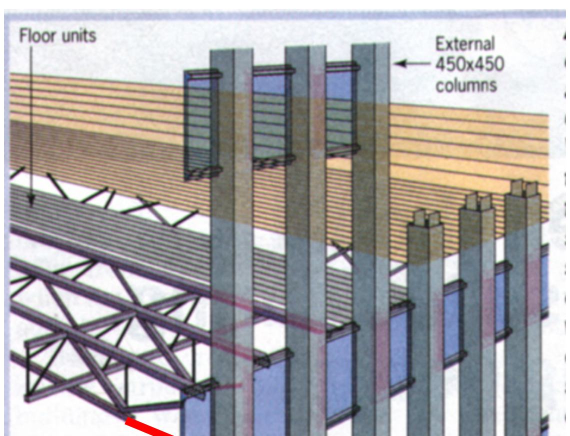
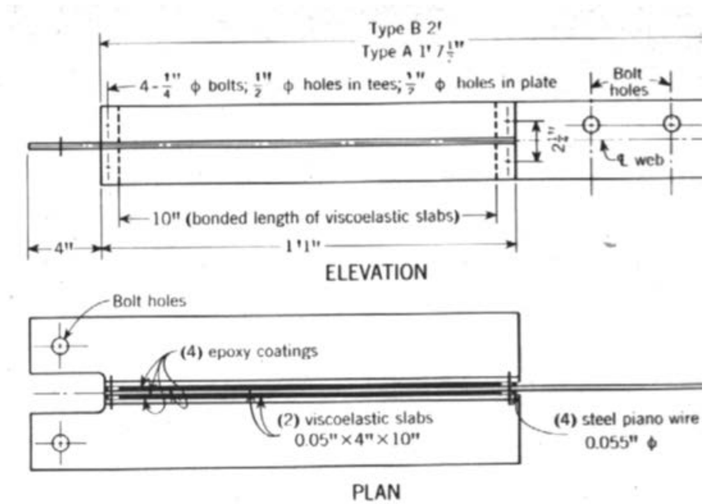


1. Introduction

- Control of vibrations by viscous and/or viscoelastic materials used for several decades on aircrafts and aerospace structures.
- First use of viscoelastic dampers Civil Engineering structures:
  - 10 000 viscoelastic dampers installed in each twin towers of late World Trade Center in New York (1969).
  - dampers designed to reduce wind vibrations.



•Damper Location



1. Introduction

- Damped Outrigger Concept to mitigate wind vibrations in tall buildings -The St. Francis Shangri-La Place, Mandaluyong City, Philippines.
- Viscous dampers used to damp relative motion between outriggers attached to central core and perimeter structure.
- Similar or higher damping levels (up to 10% critical) than Tuned Mass Dampers (TMD) can be achieved without adding weight.

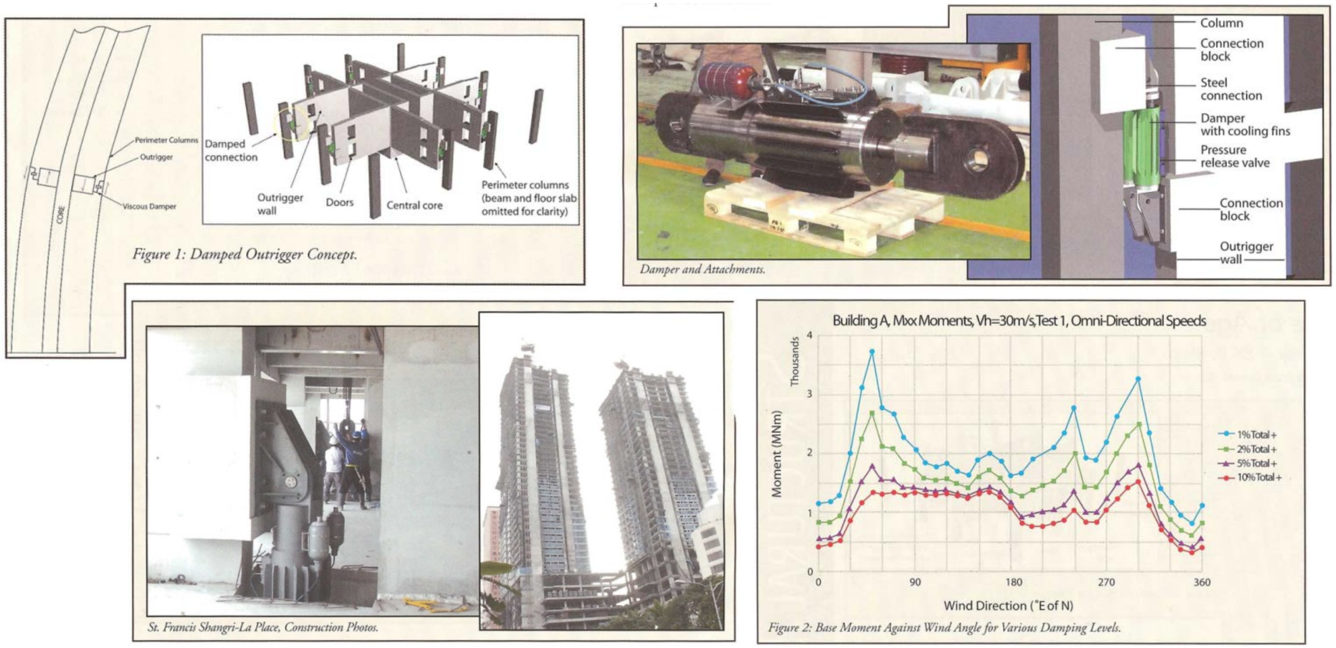


Figure 1: Damped Outrigger Concept.

Damper and Attachments.

St. Francis Shangri-La Place, Construction Photos.

Figure 2: Base Moment Against Wind Angle for Various Damping Levels.

1. Introduction

- Only in last two decades dampers incorporating viscous and/or viscoelastic materials used in seismic applications.
- Chapter discusses behaviour of structures equipped with viscous or viscoelastic dampers under earthquake ground motions.

2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Linear Viscous Dampers

Consider first a pure viscous element subjected to a time-varying relative axial displacement history $x(t)$ given by:

$$x(t) = X_0 \sin \omega t \tag{6.1}$$

where X_0 is the displacement amplitude between the two ends of the element and ω is the circular forcing frequency. The axial force $F(t)$ induced in the element is linearly proportional to the relative velocity between its two ends:

$$F(t) = C_L \dot{x}(t) \tag{6.2}$$

where C_L is the linear viscous damping constant.

Substituting Equation (6.1) into Equation (6.2) yields:

$$F(t) = C_L \omega X_0 \cos \omega t \tag{6.3}$$

From basic trigonometry:

$$\cos \omega t = \pm\sqrt{1 - \sin^2 \omega t} \tag{6.4}$$

2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

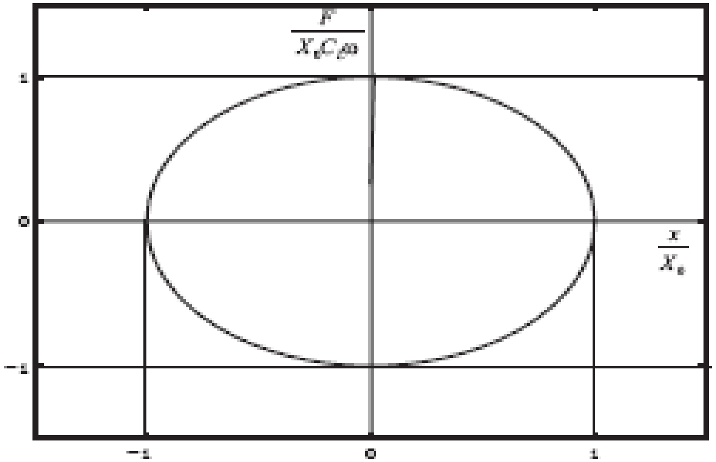
- Linear Viscous Dampers

Substituting Equation (6.4) into Equation (6.3) yields the force-displacement relationship for the linear viscous damper:

$$F(t) = \pm C_L \omega \sqrt{X_0^2 - x^2(t)} \tag{6.5}$$

or:

$$\frac{F(t)}{X_0 C_L \omega} = \pm \sqrt{1 - \left(\frac{x(t)}{X_0}\right)^2} \tag{6.6}$$



2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Linear Viscous Dampers

The energy dissipated by the linear viscous damper in each cycle E_{vd} is the area under the force-displacement relationship:

$$E_{vd} = \int_0^{2\pi/\omega} F(t)\dot{x}(t)dt = C_L\omega^2 X_0^2 \int_0^{2\pi/\omega} \cos^2\omega t \, dt \qquad (6.7)$$

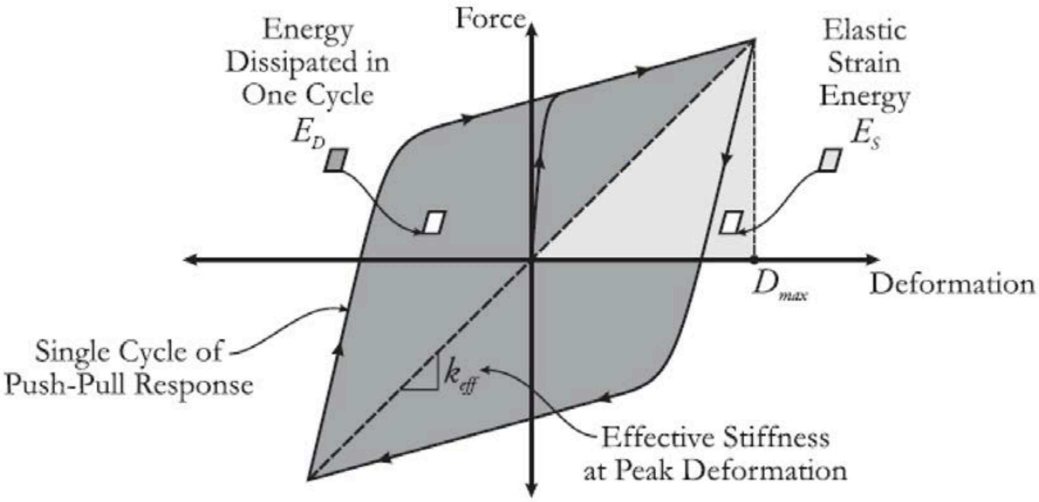
which yields:

$$E_{vd} = C_L\omega^2 X_0^2 \left[\frac{t}{2} + \frac{1}{4\omega}\sin 2\omega t\right]\Bigg|_0^{2\pi/\omega} = C_L\pi\ \omega X_0^2 \qquad (6.8)$$

2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

– Derivation of equivalent viscous damping formula

- Hysteretic response of any system



$$\text{Make}: E_{vd} = C_L \pi \omega D_{\max}^2 = E_D$$

$$C_L = 2\zeta_{eq}\omega m = \frac{E_D}{\pi \omega D_{\max}^2}$$

$$\zeta_{eq} = \frac{E_D}{2\pi m \omega^2 D_{\max}^2} = \frac{E_D}{2\pi k_{eff} D_{\max}^2}$$

$$\zeta_{eq} = \frac{E_D}{4\pi \frac{1}{2} k_{eff} D_{\max}^2} = \frac{E_D}{4\pi E_s}$$

- Note: ω is assumed equal to the fundamental frequency of the system

2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Nonlinear Viscous Dampers
  - Fluid type dampers can be designed to behave as nonlinear viscous elements by adjusting their silicone oil and orificing characteristics.
  - Main advantage of nonlinear viscous dampers is that in the event of a velocity spike, the force in the viscous damper is controlled to avoid overloading the damper or the bracing system to which it is connected.

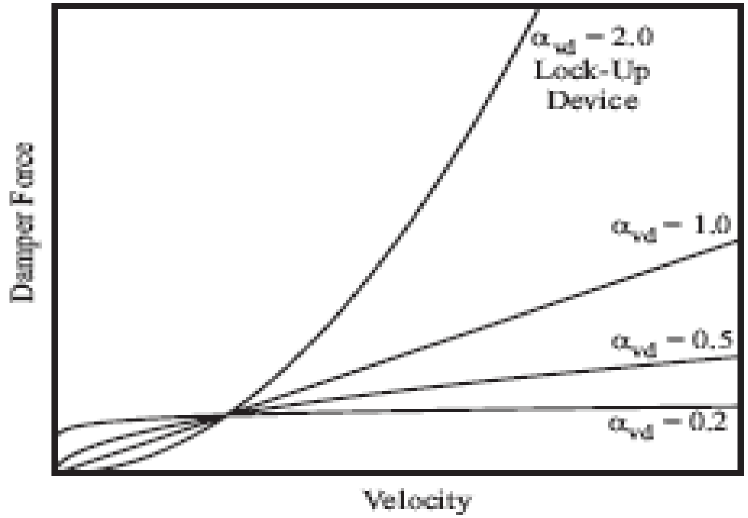
2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Nonlinear Viscous Dampers

The axial force developed by a nonlinear viscous damper $F(t)$ is expressed by:

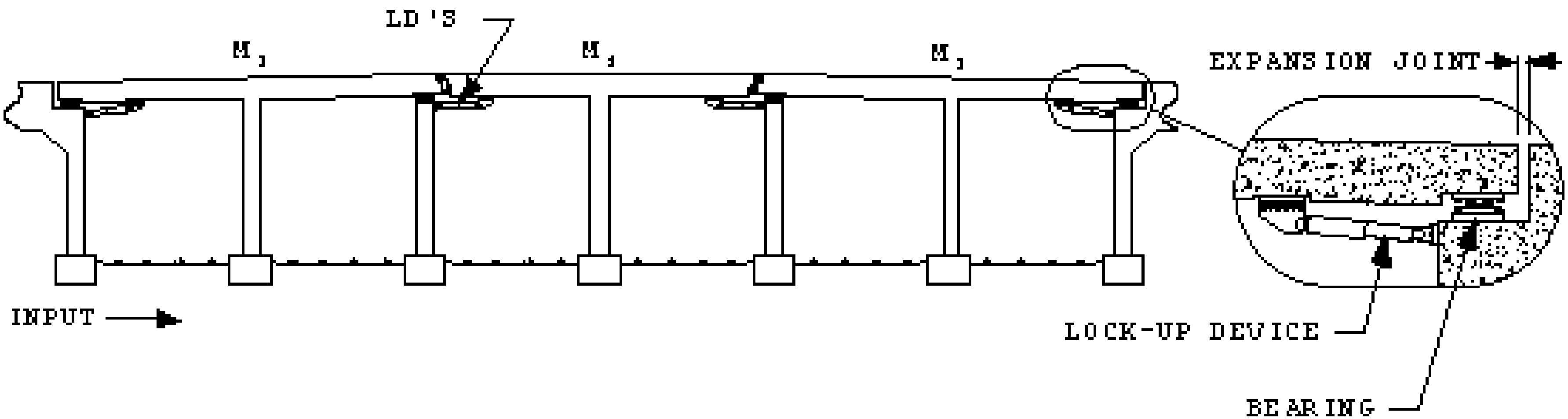
$$F(t) = C_{NL}\,\mathrm{sgn}(\dot{x}(t))\left|\dot{x}(t)\right|^{\alpha_{vd}} \tag{6.9}$$

where C_{NL} is the nonlinear viscous damping constant and α_{vd} is a predetermined velocity coefficient in the range of 0.2 to 1



2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Nonlinear Viscous Dampers



- Performance-Based Design Example of Lock-up Devices ($\alpha_{vd} > 1$)

2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Nonlinear Viscous Dampers

Consider a nonlinear viscous damper subjected to a harmonic relative displacement time-history between its ends $x(t)$ given by:

$$x(t) = X_0 \sin \omega t \tag{6.10}$$

Substituting Equation (6.10) into Equation (6.9) yields:

$$F(t) = C_{NL} \operatorname{sgn}(\cos \omega t) \left| \omega X_0 \cos \omega t \right|^{\alpha_{vd}} \tag{6.11}$$

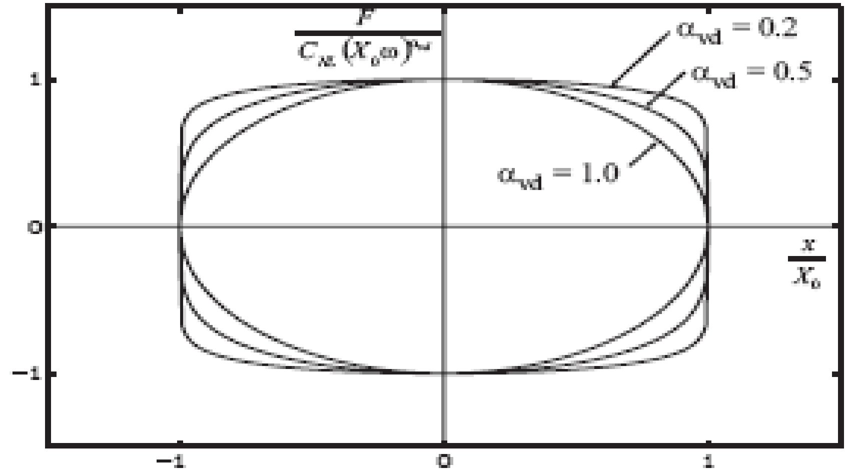
2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Nonlinear Viscous Dampers

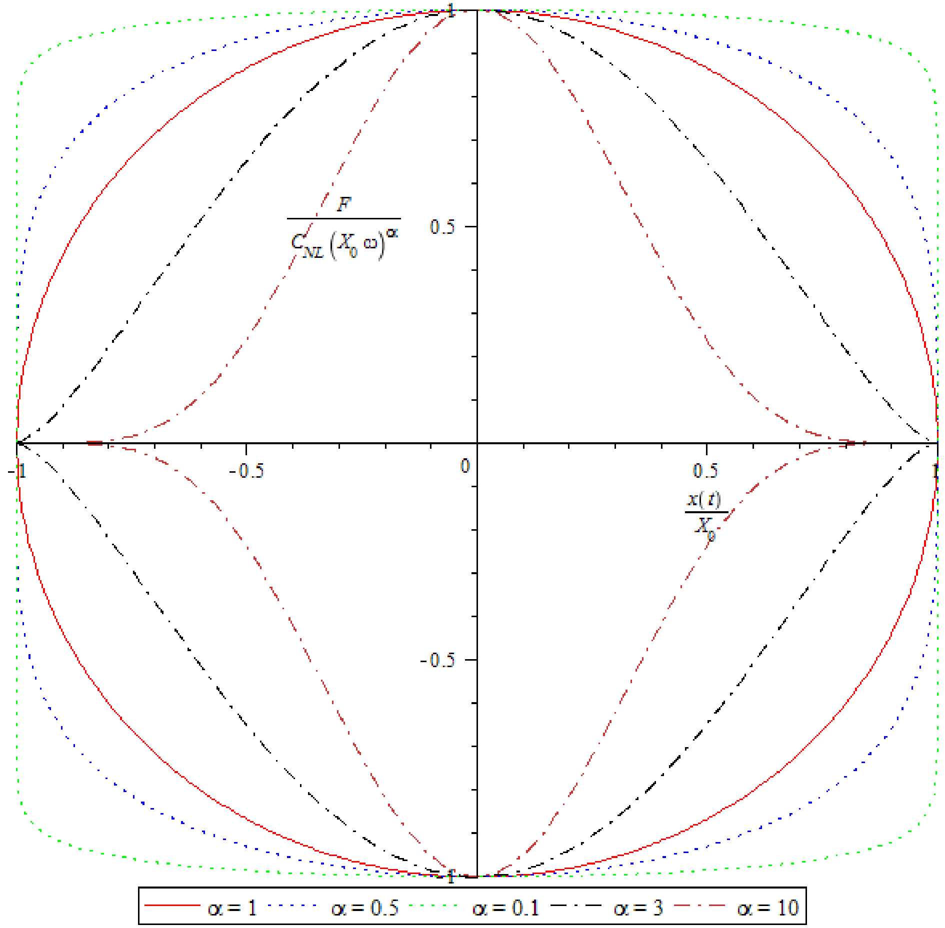
Substituting Equation (6.4) into Equation (6.11) yields the force-displacement relationship for the nonlinear viscous damper:

$$\cos\omega t = \pm\sqrt{1-\sin^2\omega t}$$

$$\frac{F}{C_{NL}(X_0\omega)^{\alpha_{vd}}} = \pm\left(1-\left(\frac{x(t)}{X_0}\right)^2\right)^{\frac{\alpha_{vd}}{2}} \qquad (6.12)$$



2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers



2. Hysteretic Behaviour of Linear and Nonlinear Viscous Dampers

- Nonlinear Viscous Dampers

The energy dissipated by a nonlinear viscous damper in each cycle E_{vd} is the area under the force-displacement relationship:

$$E_{vd} = \int_0^{2\pi/\omega} F(t)\dot{x}(t)dt = 4C_{NL}(X_0\omega)^{\alpha_{vd}+1}\int_0^{\pi/2\omega} \cos^{\alpha_{vd}+1}\omega t \; dt \qquad (6.13)$$

Evaluating the integral on the right-hand-side of Equation (6.13), we get:

$$E_{vd} = 4C_{NL}(X_0\omega)^{\alpha_{vd}+1}\frac{\sqrt{\pi}}{2\omega}\frac{\Gamma(1+\alpha_{vd}/2)}{\Gamma(3/2+\alpha_{vd}/2)} \qquad (6.14)$$

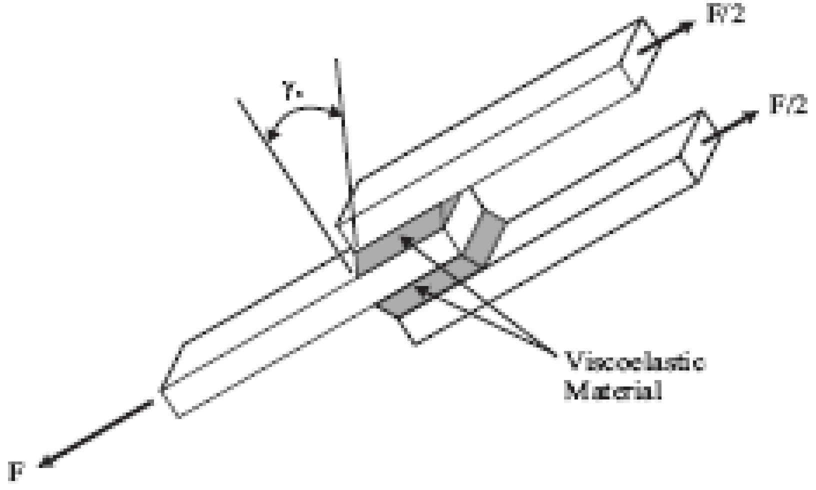
which can be rearranged to yield:

$$E_{vd} = 2\sqrt{\pi}C_{NL}(X_0)^{\alpha_{vd}+1}\omega^{\alpha_{vd}}\frac{\Gamma(1+\alpha_{vd}/2)}{\Gamma(3/2+\alpha_{vd}/2)} \qquad (6.15)$$

where Γ is the gamma function.

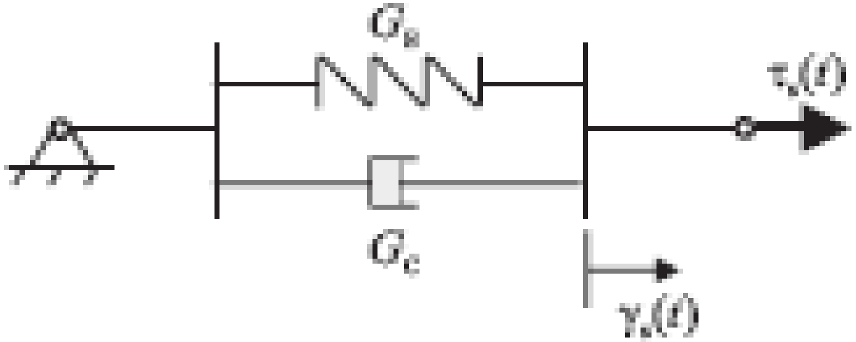
3. Hysteretic Behaviour of Viscoelastic Dampers

- Viscoelastic dampers provide both a velocity dependent force and a displacement-dependent elastic restoring force.
- Typically made of copolymers or glassy substances.
- Often incorporated in bracing members and dissipate seismic energy through shear deformations of viscoelastic material.

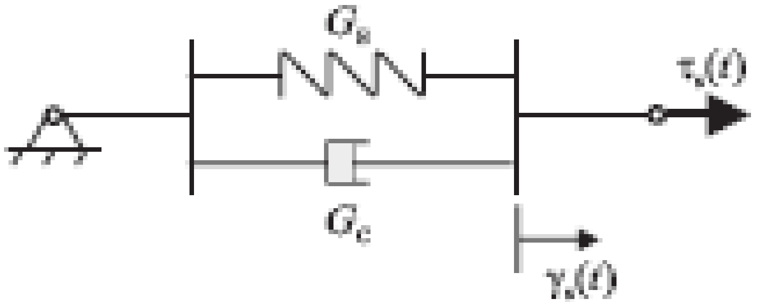


3. Hysteretic Behaviour of Viscoelastic Dampers

- Kelvin solid is simplest model that can represent behaviour of viscoelastic dampers.
- Assume that Kelvin solid is of unit height and unit area: displacements can be expressed as strains, and forces can be expressed as stresses.



3. Hysteretic Behaviour of Viscoelastic Dampers



By equilibrium:

$$\tau_E(t) + \tau_c(t) = \tau_s(t) \tag{6.16}$$

where $\tau_E(t)$ is the shear stress carried by the elastic component of the material at time t and $\tau_c(t)$ is the shear stress carried by the viscous component of the material at time t.

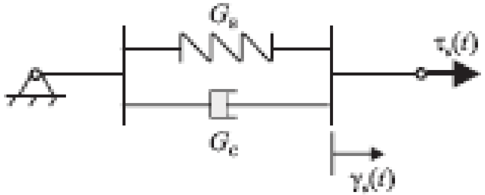
By shear strain compatibility:

$$\gamma_E(t) = \gamma_c(t) = \gamma_s(t) \tag{6.17}$$

where $\gamma_E(t)$ is the elastic shear strain at time t and $\gamma_c(t)$ is the viscous shear strain at time t. The shear strain rate across the element can be obtained by differentiating Equation (6.17) with respect to time:

$$\dot{\gamma}_E(t) = \dot{\gamma}_c(t) = \dot{\gamma}_s(t) \tag{6.18}$$

3. Hysteretic Behaviour of Viscoelastic Dampers



Replacing the constitutive relations for the elastic and viscous components in Equation (6.18) yields:

$$\frac{\dot{\tau}_E(t)}{G_E} = \frac{\tau_C(t)}{G_C} = \dot{\gamma}_s(t) \qquad (6.19)$$

or

$$\begin{aligned} \tau_E(t) &= G_E \gamma_s(t) \\ \tau_c(t) &= G_C \dot{\gamma}_s(t) \end{aligned} \qquad (6.20)$$

Substituting Equation (6.20) into Equation (6.16) leads to the shear constitutive relationship for the Kelvin solid:

$$\tau_s(t) = G_E \gamma_s(t) + G_C \dot{\gamma}_s(t) \qquad (6.21)$$

3. Hysteretic Behaviour of Viscoelastic Dampers

If the viscoelastic material has a shear thickness h_s and a shear area A_s, Equation (6.21) can be transformed into a force-displacement relationship $(F(t) - x(t))$:

$$F(t) = \bar{k}\, x(t) + \bar{c}\, \dot{x}(t) \qquad (6.22)$$

where:

$$\bar{k} = \frac{G_E\, A_s}{h} \quad \text{and} \quad \bar{c} = \frac{G_c\, A_s}{h} \qquad (6.23)$$

3. Hysteretic Behaviour of Viscoelastic Dampers

Now assume that the Kelvin element is subjected to the time-varying relative axial displacement history $x(t)$:

$$x(t) = X_0 \sin \omega t \quad (6.24)$$

where X_0 is the displacement amplitude between the two ends of the element and ω is the circular forcing frequency. The axial force induced in the viscoelastic damper $F(t)$ is obtained by substituting Equation (6.24) into Equation (6.22):

$$F(t) = \bar{k} X_0 \sin \omega t + \bar{c} X_0 \omega \cos \omega t \quad (6.25)$$

Substituting Equation (6.4) into Equation (6.25) yields: $\cos \omega t = \pm \sqrt{1 - \sin^2 \omega t}$

$$F(t) = \bar{k} X_0 \sin \omega t \pm \bar{c} X_0 \omega \sqrt{1 - \sin^2 \omega t} \quad (6.26)$$

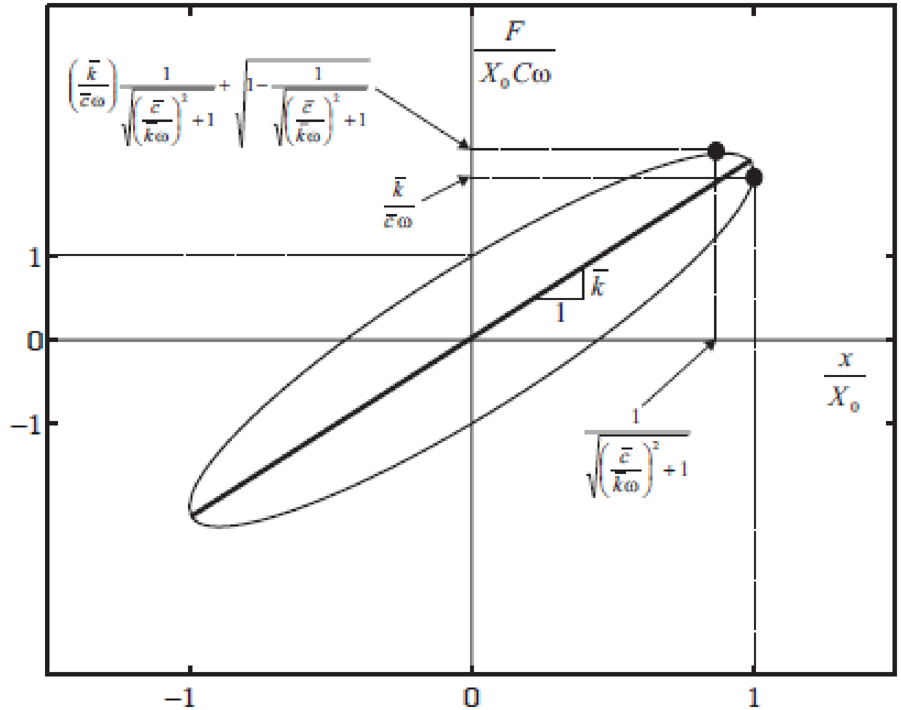
or:

$$F(t) = \bar{k}\, x(t) \pm \bar{c} \omega \sqrt{X_0 - x^2(t)} \quad (6.27)$$

3. Hysteretic Behaviour of Viscoelastic Dampers

Rearranging Equation (6.27):

$$\frac{F(t)}{X_0 \bar{c}\omega} = \frac{\bar{k}}{\bar{c}\omega}\left(\frac{x(t)}{X_0}\right) \pm \sqrt{1-\left(\frac{x(t)}{X_0}\right)^2} \qquad (6.28)$$



3. Hysteretic Behaviour of Viscoelastic Dampers

Again, the energy dissipated by the Kelvin solid element in each cycle E_{ved} is the area under the force-displacement relationship.

$$E_{ved} = \int_0^{2\pi/\omega} F(t)\dot{x}(t)dt = \bar{k}\, X_0^2\, \omega \int_0^{2\pi/\omega} \sin\omega t\, \cos\omega t dt + \bar{c}\, X_0^2\, \omega^2 \int_0^{2\pi/\omega} \cos^2\omega t\, dt \quad (6.29)$$

which yields:

$$E_{ved} = \bar{k}\, X_0^2\, \omega \left[\frac{1}{2\omega}\sin^2\omega t\right]\Bigg|_0^{2\pi/\omega} + \bar{c}\, X_0{}^2 \omega^2 \left[\frac{t}{2} + \frac{1}{4\omega}\sin 2\omega t\right]\Bigg|_0^{2\pi/\omega} \quad (6.30)$$

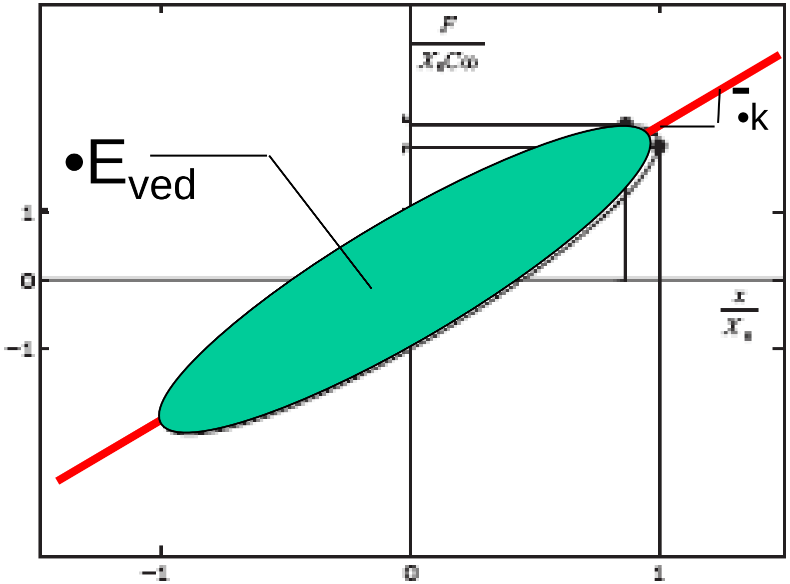
and finally:

$$\boxed{E_{ved} = \bar{c}\pi\omega X_0{}^2} \quad (6.31)$$

3. Hysteretic Behaviour of Viscoelastic Dampers

- Material properties of Kelvin element can easily be obtained from displacement- controlled sinusoidal tests at various excitation frequencies

$$\bar{c} = \frac{E_{ved}}{\pi \omega X_o^2}$$



3. Hysteretic Behaviour of Viscoelastic Dampers

The equivalent viscous damping ratio of a viscoelastic damper represented by a Kelvin element $\bar{\xi}$ can be obtained from first principles:

$$\bar{\xi} = \frac{\bar{c}}{2\bar{\omega}m} \tag{6.32}$$

where $\bar{\omega}$ is the oscillating circular frequency of the element and m is the mass connected to its ends. Equation (6.32) can be re-written as:

$$\bar{\xi} = \frac{\bar{c}\bar{\omega}^2}{2\bar{\omega}\bar{k}} = \frac{\bar{c}\bar{\omega}}{2\bar{k}} = \frac{G_c\bar{\omega}}{2G_E} \tag{6.33}$$

In the theory of viscoelasticity, G_E is defined as the *shear storage modulus* of the viscoelastic material, which is a measure of the energy stored and recovered per cycle; $G_C\,\bar{\omega}$ is defined as the *shear loss modulus*, which gives a measure of the

per cycle

3. Hysteretic Behaviour of Viscoelastic Dampers

Another measure of the energy dissipation capacity of the viscoelastic material is given by the *loss factor* η defined as:

$$\eta = \frac{G_C \, \bar{\omega}}{G_E} = 2\bar{\xi} \tag{6.34}$$

Note that by combining Equations (6.33) and (6.34), the damping constant of the viscoelastic damper $\bar{c}$ can also be expressed in terms of the loss factor, η :

$$\bar{c} = \frac{\eta \bar{k}}{\bar{\omega}} \tag{6.35}$$

4. Variation of Shear Storage and Shear Loss Moduli of Viscoelastic Materials

- Shear storage modulus and shear loss modulus, or shear storage modulus and loss factor determine dynamic response in shear of viscoelastic material, modeled as a Kelvin solid, under displacement-controlled harmonic excitation.
- Moduli depend on several parameters:
  - excitation frequency,
  - ambient temperature,
  - shear strain level, and
  - variation of internal temperature within the material during operation.

4. Variation of Shear Storage and Shear Loss Moduli of Viscoelastic Materials

Table 6-1: Geometry of Viscoelastic Dampers Tested (after Chang et al. 1993a, b)

Material Type	Shear Area (mm^2)	Thickness (mm)	Volume (mm^3)
A	968	5.08	4917
B	1936	7.62	14752
C	11613	3.81	44246

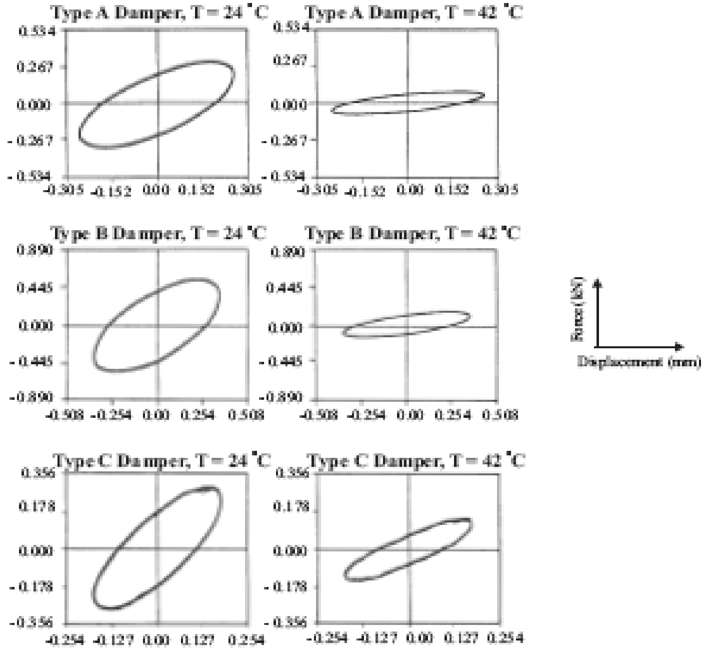


Figure 6.7 Hysteretic Responses of Viscoelastic Dampers, at 5% Shear Strain and at 3.5 Hz Frequency (after Chang et al. 1993b)

4. Variation of Shear Storage and Shear Loss Moduli of Viscoelastic Materials

Table 6-2: Variations of Viscoelastic Damper Moduli, 5% Shear Strain, 3.5Hz (after Chang et al. 1993b)

Material Type	Temperature (^{o}C)	G_E (MPa)	$G_C\ \overline{\omega}$ (MPa)	$\eta = \frac{G_C\ \overline{\omega}}{G_E}$
A	21	2.78	3.01	1.08
	24	2.10	2.38	1.13
	28	1.57	1.90	1.21
	32	1.17	1.37	1.17
	36	0.83	0.90	1.08
	40	0.63	0.63	1.00
B	25	1.73	2.08	1.20
	30	1.29	1.54	1.19
	34	0.94	1.11	1.18
	38	0.76	0.84	1.10
	42	0.62	0.65	1.05
C	25	0.19	0.17	0.90
	30	0.16	0.12	0.75
	34	0.14	0.10	0.71
	38	0.12	0.08	0.67
	42	0.11	0.07	0.64

4. Variation of Shear Storage and Shear Loss Moduli of Viscoelastic Materials

Table 6-3: Variations of Moduli for Viscoelastic Damper Type B (after Chang et al. 1993b).

Temperature (^{o}C)	Excitation Frequency (Hz)	Shear Strain Level (%)	G_E (MPa)	$G_C\ \overline{\omega}$ (MPa)	$\eta = \frac{G_C\ \overline{\omega}}{G_E}$
24	1.0	5	0.98	1.17	1.19
		20	0.96	1.15	1.20
	3.0	5	1.88	2.23	1.19
		20	1.77	2.11	1.19
36	1.0	5	0.41	0.46	1.12
		20	0.40	0.45	1.13
	3.0	5	0.74	0.82	1.11
		20	0.71	0.77	1.09

5. Dynamic Analysis of Structures Incorporating Viscous and Viscoelastic Dampers

- Single Storey Frame

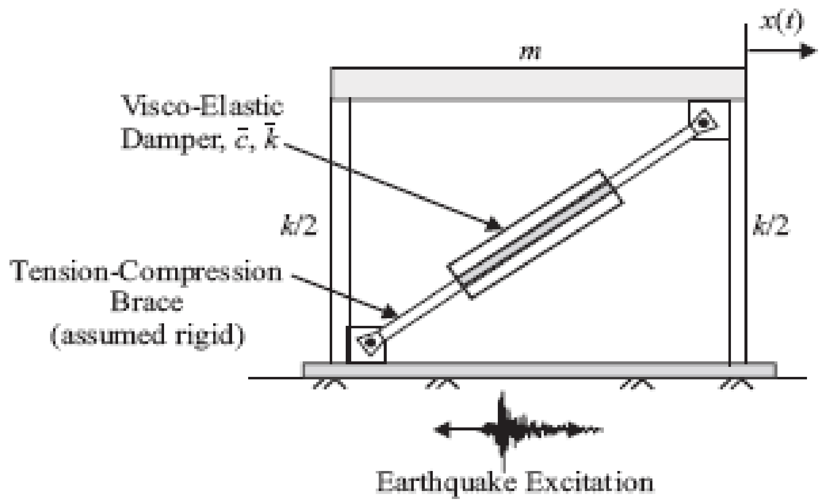
The equation of motion can be written as:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + F_{Ved}(t) = -m\ddot{x}_g(t) \tag{6.36}$$

where F_{Ved} is the lateral force provided by the viscoelastic damper and c is the equivalent viscous damping constant of the unbraced structure. The axial force in the bracing element $F(t)$ corresponding to the shear force across the damping material is given by Equation (6.22):

$$F(t) = \bar{k}\,\Delta(t) + \bar{c}\,\dot{\Delta}(t) \tag{6.37}$$

where $\Delta(t)$ is the displacement between the two ends of (and parallel to) the bracing member.



5. Dynamic Analysis of Structures Incorporating Viscous and Viscoelastic Dampers

- Single Storey Frame

By equilibrium:

$$F_{ved} = F(t)\cos\gamma \tag{6.38}$$

By compatibility:

$$\Delta(t) = x(t)\cos\gamma \tag{6.39}$$

Substituting Equations (6.38) and (6.39) into Equation (6.37) and the resulting expression into Equation (6.36) yields:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + (\bar{k}x(t) + \bar{c}\dot{x}(t))\cos^2\gamma = -m\ddot{x}_g(t) \tag{6.40}$$

or:

$$m\ddot{x}(t) + (c + \bar{c}\cos^2\gamma)\dot{x}(t) + (k + \bar{k}\cos^2\gamma)x(t) = -m\ddot{x}_g(t) \tag{6.41}$$

- # Extension to Multi-Storey Structures

Assuming that the structure remains elastic when equipped with viscoelastic dampers, the equations of motion can be written as:

$$[[M]+[\overline{m}]]\{\ddot{x}(t)\}+[[C]+[\overline{c}]]\{\dot{x}(t)\}+[[K]+[\overline{k}]]\{x(t)\} = -[M]\{r\}\ddot{x}_g(t) \quad (6.42)$$

where $[\overline{m}]$ is the mass matrix corresponding to the added dampers, $[\overline{c}]$ is the added viscous damping matrix attributed to the viscoelastic dampers and $[\overline{k}]$ is the global stiffness matrix attributed to the added viscoelastic dampers.

If the damping matrix corresponding to the added dampers $[\overline{c}]$ is assumed to have the same orthogonality properties as the original mass and stiffness matrices of the structure, standard modal analysis can be used to uncouple Equation (6.42). The equation of motion for the i^{th} mode can be written as:

$$\tilde{M}_i\ddot{u}_i(t)+\tilde{C}_i\dot{u}_i(t)+\tilde{K}_i u_i(t) = \tilde{P}_i(t) \quad (6.43)$$

where:

$\tilde{M}_i = \{A^{(i)}\}^T[[M]+[\overline{m}]]\{A^{(i)}\}$ is the generalized mass in mode i

$\tilde{C}_i = \{A^{(i)}\}^T[[C]+[\overline{c}]]\{A^{(i)}\}$ is the generalized damping coefficient in mode i

$\tilde{K}_i = \{A^{(i)}\}^T[[K]+[\overline{k}]]\{A^{(i)}\}$ is the generalized stiffness coefficient in mode i

$\tilde{P}_i(t) = -\{A^{(i)}\}^T[M]\{r\}\ddot{x}_g(t)$ is the generalized dynamic loading in mode i

5. Dynamic Analysis of Structures Incorporating Viscous and Viscoelastic Dampers

- Extension to Multi-Storey Structures

The analysis can be simplified by assuming, conservatively, that the inherent damping of the structure is negligible compared to the supplemental damping provided by the added dampers. The mass of the dampers can also be omitted since it is usually negligible compared to the mass of the structure.

The modal response in the i^{th} mode of vibration $u_i(t)$ can be obtained by Duhamel's integral:

$$u_i(t) = \frac{1}{\tilde{M}_i \tilde{\omega}_{di}} \int_0^t \tilde{P}_i(\tau) e^{\tilde{\xi}_i \tilde{\omega}_i (t-\tau)} \sin \tilde{\omega}_{di}(t-\tau) d\tau \qquad (6.44)$$

where $\tilde{\omega}_i$ is the i^{th} undamped modal frequency of the structure with added viscoelastic dampers, $\tilde{\omega}_{di}$ is the i^{th} damped modal frequency and $\tilde{\xi}_i$ is the i^{th} modal damping ratio due to the added viscoelastic dampers. These quantities are given by:

5. Dynamic Analysis of Structures Incorporating Viscous and Viscoelastic Dampers

- Extension to Multi-Storey Structures

$$\tilde{\omega}_i^2 = \frac{\tilde{K}_i}{\tilde{M}_i}$$

$$\tilde{\omega}_{di}^2 = \tilde{\omega}_i^2 \sqrt{1 - \tilde{\xi}_i^2} \qquad (6.45)$$

$$\tilde{\xi}_i = \frac{\tilde{C}_i}{2\tilde{\omega}_i \tilde{M}_i}$$

5. Dynamic Analysis of Structures Incorporating Viscous and Viscoelastic Dampers

- Extension to Multi-Storey Structures

The i^{th} modal damping ratio $\tilde{\xi}_i$ can be expressed in terms of the properties of the viscoelastic material. For this purpose, one realizes that the elements of $[\bar{c}]$ are a linear combination of the damping constants of the individual viscoelastic dampers in the structure. Assuming that only one viscoelastic material is used, these elements of the $[\bar{c}]$ matrix $\bar{c}_{jk}$ can be related to the elements of the $[\bar{k}]$ matrix $\bar{k}_{jk}$ using Equation (6.35):

$$\bar{c}_{jk} = \frac{2\tilde{\xi}_i \bar{k}_{jk}}{\tilde{\omega}_i} = \frac{\eta_i \bar{k}_{jk}}{\tilde{\omega}_i} \qquad \bar{c} = \frac{\eta \bar{k}}{\tilde{\omega}} \tag{6.46}$$

where η_i is the loss factor of the viscoelastic material at a frequency $\tilde{\omega}_i$.
From Equation (6.46), the $[\bar{c}]$ matrix can be expressed in terms of the $[\bar{k}]$ matrix:

$$[\bar{c}] = \frac{\eta_i}{\tilde{\omega}_i}[\bar{k}] \tag{6.47}$$

- Extension to Multi-Storey Structures

Furthermore, the generalized damping coefficient in the i^{th} mode can be expressed in terms of the basic properties of the viscoelastic material (Equations (6.33) and (6.34)):

$$\tilde{C}_i = \{A^{(i)}\}^T[\bar{c}]\{A^{(i)}\} = \frac{\eta_i \bar{K}_i}{\tilde{\omega}_i} \tag{6.48}$$

where $\bar{K}_i = \{A^{(i)}\}^T[\bar{k}]\{A^{(i)}\}$ is the generalized stiffness coefficient in mode i corresponding only to the added viscoelastic dampers.

From Equation (6.45), the i^{th} modal damping ratio can be obtained by:

$$\tilde{\xi}_i = \frac{\eta_i \bar{K}_i}{2\tilde{\omega}_i^2 \tilde{M}_i} = \frac{\eta_i \bar{K}_i}{2\tilde{K}_i} = \frac{\eta_i}{2}\left(1 - \frac{\{A^{(i)}\}^T[K]\{A^{(i)}\}}{\{A^{(i)}\}^T[[K]+[\bar{k}]]\{A^{(i)}\}}\right) = \frac{\eta_i}{2}\left(1 - \frac{\omega_i^2}{\tilde{\omega}_i^2}\right) \tag{6.49}$$

where ω_i is the i^{th} modal frequency of the original structure before the viscoelastic dampers are added.

5. Dynamic Analysis of Structures Incorporating Viscous and Viscoelastic Dampers

- Extension to Multi-Storey Structures
  - As an alternative to modal analysis: time-history analysis.
  - Each individual damper is inserted into the structure with own mechanical properties ($\bar{k}$ and $\bar{c}$).
  - Properties are assumed constant for all modal frequencies.

6. Existing Viscous and Viscoelastic Dampers

– Despite significant research effort, viscoelastic dampers have not wide application in North America and Europe.

– Purely viscous fluid systems, on the other hand, are now widely used and are the focus of this section.

6. Existing Viscous and Viscoelastic Dampers

- Typical fluid dampers incorporate a stainless steel piston with a bronze orifice head.
- Device filled with silicone oil.
- Piston head utilizes specially shaped orifices that alter flow characteristics with fluid relative velocity.
- Force produced by damper is generated by the pressure differential across piston head.

•Photo: Courtesy of M. Constantinou

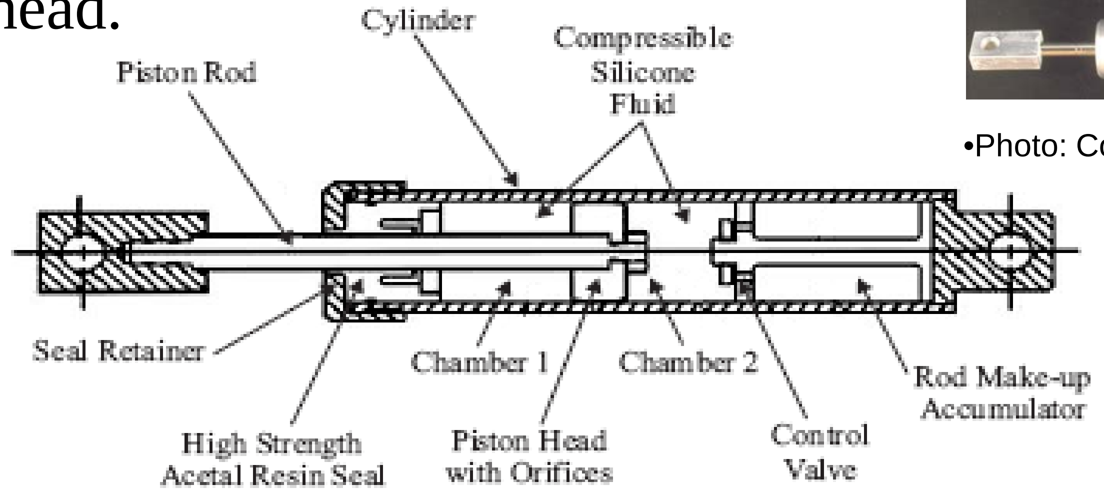


Figure 6.9 Viscous Fluid Device (after Lee 2003)

6. Existing Viscous and Viscoelastic Dampers

- Various structural models, with and without fluid dampers manufactured by Taylor Devices Inc., tested on the shake table at the University at Buffalo from 1991 to 1995.
- e.g. 1/4 scale 3-storey test structure (Constantinou et al. 1993).
- Model had weights = 28.5 kN distributed equally on the three floors.

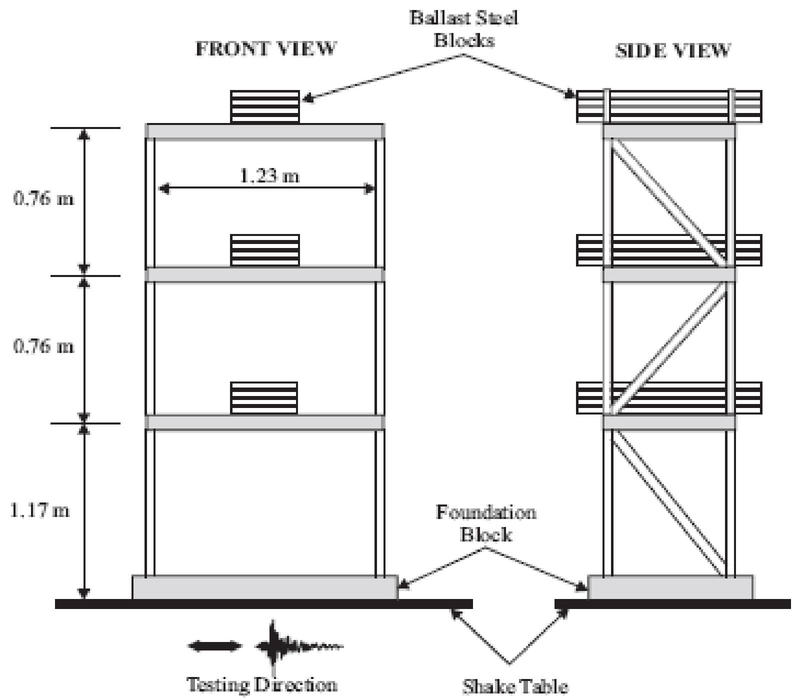


Figure 6.10 Three-Storey Steel Frame Tested With and Without Taylor Devices (after Constantinou et al. 1993)

6. Existing Viscous and Viscoelastic Dampers

Table 6-4: Dynamic Characteristics of Three-Storey Steel Frame (after Constantinou et al. 1993)

Mode		No Dampers	4 Dampers	6 Dampers
1	Frequency (Hz)	2.00	2.11	2.03
	Damping Ratio (%)	1.8	17.7	19.4
2	Frequency (Hz)	6.60	7.52	7.64
	Damping Ratio (%)	0.8	31.9	44.7
3	Frequency (Hz)	12.20	12.16	16.99
	Damping Ratio (%)	0.3	11.3	38.0

6. Existing Viscous and Viscoelastic Dampers

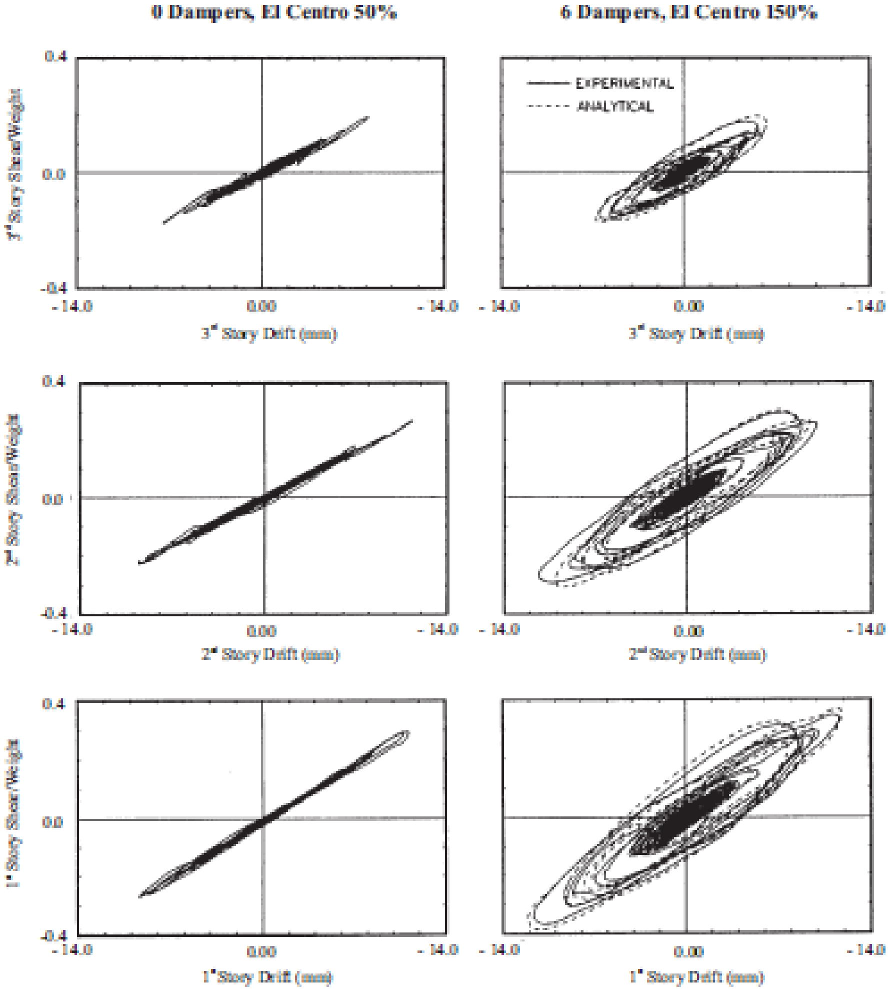


Figure 6.11 Hysteretic Behaviour of 3-Storey Steel Frame with and without Fluid Dampers (after Constantinou et al. 1993)

6. Existing Viscous and Viscoelastic Dampers

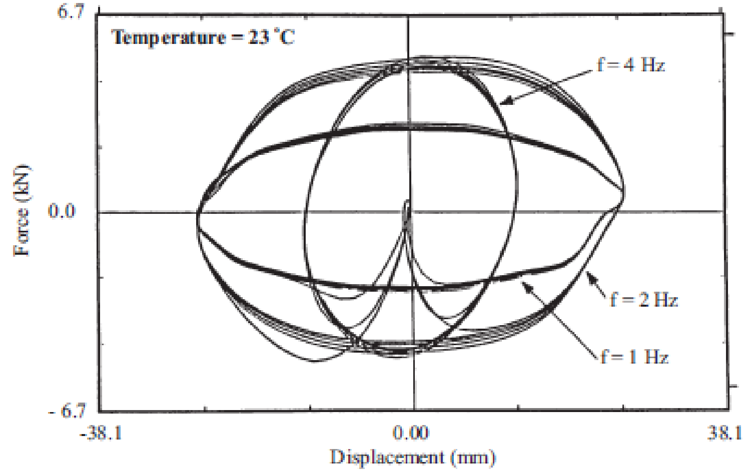


Figure 6.12 Experimental Force-Displacement Hysteresis Loops of Fluid Damper (after Constantinou et al. 1993)

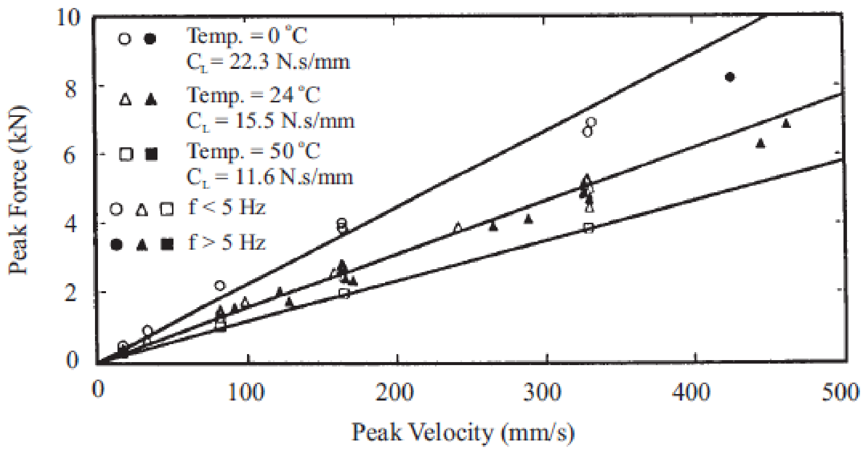


Figure 6.13 Evaluation of Damping Constant for Fluid Damper (after Constantinou et al. 1993)

Table 6-5: Typical Peak Response of Tested Three-Storey Steel Frame, the floor at which the peak response was recorded is indicated in ( ), (after Constantinou et al. 1993)

Excitation	Number of Dampers	Peak Floor Acceleration (g)	Base Shear Total Weight	Storey Drift Height (%)
El Centro 33%	0	0.417 (3)	0.220	1.069 (2)
El Centro 50%	0	0.585 (3)	0.295	1.498 (2)
Taft 100%	0	0.555 (3)	0.255	1.161 (1)
El Centro 50%	4	0.282 (3)	0.159	0.660 (2)
El Centro 100%	4	0.591 (3)	0.314	1.279 (2)
Taft 100%	4	0.246 (3)	0.130	0.638 (2)
El Centro 50%	6	0.205 (3)	0.138	0.510 (2)
El Centro 100%	6	0.368 (3)	0.261	0.998 (2)
El Centro 150%	6	0.534 (3)	0.368	1.492 (2)
Taft 100%	6	0.178 (3)	0.120	0.463 (2)
Taft 200%	6	0.348 (3)	0.235	0.921 (2)
Pacoima Dam 50%	6	0.376 (3)	0.275	1.003 (1)
Hachinohe 100%	6	0.334 (3)	0.256	0.963 (2)
Miyagiken 200%	6	0.342 (3)	0.254	0.963 (2)

- http://www.taylordevices.com/

7. Design of Structures Equipped with Viscoelastic Dampers

- Primary design parameters to be evaluated:
  - required viscous damping ratio; and
  - stiffness of damping system.
- Design methods based on elastic modeling of main structure.
- If required, design verification by nonlinear time-history dynamic analyses.
- Desired damping ratio in fundamental mode of vibration set by examining response spectra at various damping ratios and choosing ratio corresponding to desired response level.
- Once damping established, damper locations in building must be selected.

7. Design of Structures Equipped with Viscoelastic Dampers

The stiffness $\bar{k}$ and loss factor η of each viscoelastic damper must be selected based on the available viscoelastic material and the geometry of the damper. This could be a trial-and-error procedure. These parameters can also be determined based on the assumption that the added stiffness due to the viscoelastic dampers at a storey i, $\bar{k}_i$, be proportional to the i^{th} storey stiffness of the initial structure k_i. This is obtained by applying Equation (6.49) to each storey:

$$\tilde{\xi}_i = \frac{\eta_i \bar{K}_i}{2\tilde{K}_i}$$

$$\xi_1 = \frac{\eta \bar{k}_i}{2(k_i + \bar{k}_i)} \qquad (6.50)$$

or:

$$\bar{k}_i = \frac{2\xi_1 k_i}{\eta - 2\xi_1} \qquad (6.51)$$

7. Design of Structures Equipped with Viscoelastic Dampers

For a viscoelastic material with known properties G_E and G_C at the design frequency (usually assumed to be the fundamental natural frequency of the structure) and design temperature, the shear area of the damper A_s can be determined by Equation (6.23):

$$A_s = \frac{\bar{k} h_s}{G_E} \tag{6.52}$$

The thickness of the viscoelastic material h_s must be such that the maximum shear strain in the viscoelastic material is lower than the ultimate value.

7. Design of Structures Equipped with Viscoelastic Dampers

The modal damping ratio in each mode of vibration $\tilde{\xi}_i$ can then be estimated by Equation (6.49). Alternatively, the damping constant for each damper $\bar{c}$ can be determined by Equation (6.23).

$$\tilde{\xi}_i = \frac{\eta_i}{2}\left(1 - \frac{\omega_i^2}{\tilde{\omega}_i^2}\right) \qquad (6.49)$$

$$\bar{c} = \frac{G_e A_s}{h} \qquad (6.23)$$

Finally, a dynamic analysis (linear or nonlinear) needs to be performed on the structure with added viscoelastic dampers to evaluate if its seismic response is satisfactory.

8. Design of Structures Equipped with Viscous Dampers

- Conceptual Design with Linear Viscous Dampers

Conceptually, the design of structures incorporating linear viscous dampers ($\alpha_{vd} = 1$ in Equation (6.9)) is simple, since the well established method of modal superposition can be used to analyze the damped structure, as shown in Section 6.5. Since viscous dampers are usually inserted in bracing members between diaphragms of the structures (e.g. building floors or bridge decks), the global damping matrix generated by the linear viscous dampers $[\overline{C}_L]$ is proportional to the global stiffness matrix of the structure $[K]$ (Chopra 2001):

$$[\overline{C}_L] = \alpha_0[K] \tag{6.53}$$

where α_0 is a proportionality constant.

8. Design of Structures Equipped with Viscous Dampers

- Conceptual Design with Linear Viscous Dampers

From modal analysis, the generalized damping coefficient in the i^{th} mode of vibration $\overline{C}_i$ is given by:

$$\overline{C}_i = \{A^{(i)}\}^T[\overline{C}_L]\{A^{(i)}\} = \{A^{(i)}\}^T\alpha_0[K]\{A^{(i)}\} = \alpha_0 K_i = 2\xi_i\omega_i M_i \qquad (6.54)$$

where $\{A^{(i)}\}$ is the i^{th} mode of vibration, ξ_i is the damping ratio in the i^{th} mode, K_i is the generalized stiffness in the i^{th} mode of the original structure without damper, ω_i is the circular frequency associated with the i^{th} mode of vibration, and M_i is the generalized mass in the i^{th} mode of vibration.

From Equation (6.54), the proportionality constant α_0 can be easily obtained as:

$$\alpha_0 = \frac{2\xi_i}{\omega_i} \qquad (6.55)$$

8. Design of Structures Equipped with Viscous Dampers

- Conceptual Design with Linear Viscous Dampers
  - Conceptually, design process is simple.
  - Once a desired viscous damping ratio in a particular mode is established (usually first mode), proportionality constant can be computed [Equation (6.55)].
  - Resulting global damping matrix can then be obtained [Equation (6.53)].
  - Each element of global damping matrix expressed as a linear combination of damping constants of linear viscous dampers incorporated in structure. Knowing linear combinations, constant for each damper can be extracted.
  - Difficult to apply for large structural systems for which explicit form of global damping matrix may not be obtained easily.
  - In most practical design situations, damping constant for each damper obtained by trial-and-error.

8. Design of Structures Equipped with Viscous Dampers

- Practical Design with Linear Viscous Dampers

Considering that the introduction of linear viscous damping in typical structures yields a stiffness proportional damping matrix, a practical design procedure for estimating the damping constants of individual dampers can be derived. For this purpose, the damping constant C_L^n of the linear viscous damper that is introduced at storey n, is chosen to be proportional to the interstorey lateral stiffness k_n of storey n, in the initial structure:

$$C_L^n = \varepsilon \frac{T_1}{2\pi} k_n \tag{6.56}$$

where T_1 is the fundamental period of the original (unbraced) structure and ε is a constant. The approach described in the following paragraphs is a simple procedure for determining the value of ε (and ultimately the values of all C_L^n) that will yield the required amount of damping in the desired mode of vibration (usually the damping in the fundamental mode of vibration). For simplicity, the inherent damping of the original structure without dampers is neglected.

8. Design of Structures Equipped with Viscous Dampers

- Practical Design with Linear Viscous Dampers

A fictitious undamped braced structure is first defined by adding fictitious springs $\hat{k}_0$ at the proposed locations of the linear viscous dampers, and then by distributing them according to the interstorey lateral stiffness of the original structure:

$$\hat{k}_0 = \frac{2\pi}{T_1} C_L \tag{6.57}$$

With this distribution, the fundamental mode shape of this fictitiously braced structure will be the same as that of the unbraced structure, which ensures the existence of classical normal modes.

8. Design of Structures Equipped with Viscous Dampers

- Practical Design with Linear Viscous Dampers

The generalized stiffness coefficient in the first mode of the fictitiously braced structure $\hat{K}_1$ can be written as:

$$\hat{K}_1 = \{A^{(1)}\}^T[\hat{K}]\{A^{(1)}\} = \{A^{(1)}\}^T[K]\{A^{(1)}\} + \{A^{(1)}\}^T[\hat{k}]\{A^{(1)}\} = K_1 + \hat{k}_1 \quad (6.58)$$

where $[\hat{K}]$ is the global stiffness matrix of the braced structure, $[K]$ is the global stiffness matrix of the original (unbraced) structure, $[\hat{k}]$ is the global stiffness matrix corresponding to the fictitious springs alone ($\hat{k}_0$ defined in Equation (6.57)), $\{A^{(1)}\}$ is the first mode shape, K_1 is the generalized stiffness coefficient in the first mode of the original (unbraced) structure, and $\hat{k}_1$ is the generalized stiffness coefficient in the first mode of a structure composed of only fictitious springs.

8. Design of Structures Equipped with Viscous Dampers

- Practical Design with Linear Viscous Dampers

Equation (6.58) can be re-written as:

$$\hat{k}_1 = \hat{K}_1 - K_1 \tag{6.59}$$

and using Equation (6.57):

$$\hat{k}_1 = \frac{2\pi}{T_1}\{A^{(1)}\}^T[\overline{C_L}]\{A^{(1)}\} = \frac{2\pi}{T_1}\overline{C}_1 = \frac{2\pi}{T_1}\left(2\xi_1\frac{2\pi}{T_1}M_1\right) \tag{6.60}$$

where $[\overline{C}_L]$ is the global damping matrix arising from all the added viscous dampers with damping constant C_L, $\overline{C}_1$ is the generalized damping coefficient in the first mode of the original (unbraced) structure provided by the added viscous dampers, ξ_1 is the first mode viscous damping ratio resulting from the addition of the viscous dampers and M_1 is the generalized mass in the first mode of the original structure.

8. Design of Structures Equipped with Viscous Dampers

- Practical Design with Linear Viscous Dampers

Substituting Equation (6.60) into Equation (6.59) yields:

$$2\xi_1\left(\frac{2\pi}{T_1}\right)^2 = \frac{\hat{K}_1}{M_1} - \frac{K_1}{M_1} = \left(\frac{2\pi}{\hat{T}_1}\right)^2 - \left(\frac{2\pi}{T_1}\right)^2 \qquad (6.61)$$

which indicates that the desired first modal damping ratio can be related to the fundamental period of the original (unbraced) structure T_1 and to the fundamental period of the structure braced with the fictitious springs T_1 defined by Equation (6.57).

Equation (6.61) can be simplified as:

$$\xi_1 = \frac{1}{2}\left[\left(\frac{T_1}{\hat{T}_1}\right)^2 - 1\right] \qquad (6.62)$$

A simple design procedure for the required viscous damping constant of each damper C_L can then be established from equations Equations (6.57) and (6.62):

- Practical Design with Linear Viscous Dampers
    - Step 1

The properties of the initial unbraced structure are computed including its fundamental period T_1 .

- Step 2

The desired first mode viscous damping ratio to be supplied by the viscous dampers, ξ_1 must be selected. In practice, a maximum damping ratio of about 35% of critical can be achieved economically with currently available viscous dampers.

- Step 3

From Equation (6.62), the required fundamental period of the fictitiously braced structure $\hat{T}_1$ is computed.

$$\hat{T}_1 = \frac{T_1}{\sqrt{2\xi_1 + 1}} \tag{6.63}$$

- Practical Design with Linear Viscous Dampers

  - Step 4

A set of fictitious springs is then introduced at the proposed locations of the linear viscous dampers and distributed according to the lateral stiffness of the unbraced structure. The stiffness constants of these springs must yield a fundamental period of the fictitious braced structure equal to $\hat{T}_1$. Only one iteration is required since the variation of stiffness is linearly proportional to the square of the period. Therefore, the final constant $\hat{k}_0^n$ for the n th fictitious spring in the structure can be obtained from:

$$\hat{k}_0^n = \frac{\hat{k}_{0tr}^n}{\left(\frac{T_1^2 - \hat{T}_{1tr}^2}{T_1^2 - \hat{T}_1^2}\right)\left(\frac{\hat{T}_1^2}{\hat{T}_{1tr}^2}\right)} \tag{6.64}$$

where $\hat{k}_{0tr}^n$ is an initial trial value of the stiffness coefficient of the fictitious spring n and $\hat{T}_{1tr}$ is the corresponding trial value of the fundamental period of the fictitious braced structure obtained with these first trial spring constants.

8. Design of Structures Equipped with Viscous Dampers

- Practical Design with Linear Viscous Dampers

  - Step 5

Once the stiffness constants for the fictitious springs $\hat{k}_0^n$ are obtained, the required viscous damping coefficient of each viscous damper C_L can be computed using Equation (6.57). The initial structure is then fitted with dampers with the corresponding values of C_L^n.

$$\hat{k}_0 = \frac{2\pi}{T_1} C_L$$

  - Step 6

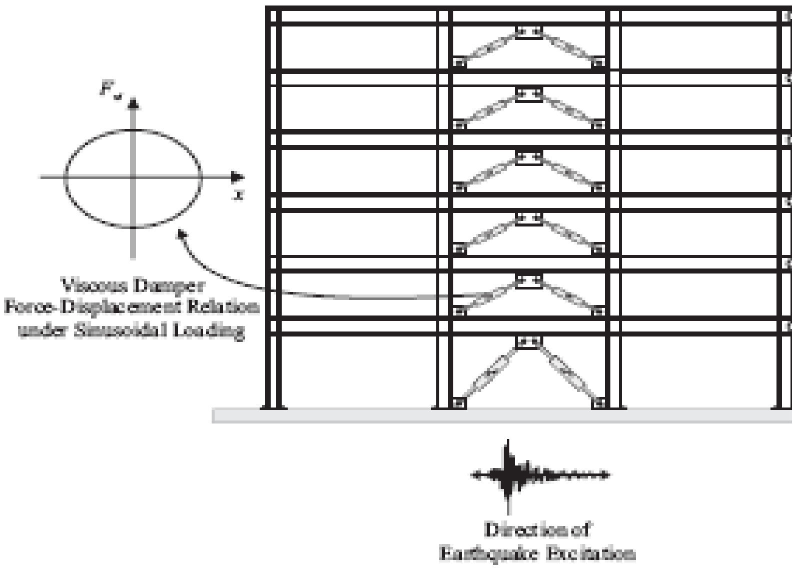
The final bracing member sections are then selected based on the anticipated maximum force in each viscous damper (see Section 4.5).

  - Step 7

The final design of each linear viscous damper is performed based on the required damping constant, anticipated maximum force and stroke. Nonlinear dynamic analyses using a representative set of design ground motion ensembles are typically used for estimating these anticipated values. Also, the final design of the dampers is usually left to manufacturers.

8. Design of Structures Equipped with Viscous Dampers

- Design Example for Linear Viscous Dampers
  - Same six-storey steel building structure discussed in Section 5.7.4.
  - Fundamental period of unbraced frame = 1.304 s.
  - Retrofit strategy consists of introducing a tubular chevron braced frame in the central bay.



• Design Example for Linear Viscous Dampers

Assuming that a damping ratio of 35% of critical is required in the first mode of vibration, Equation (6.63) is used to obtain the required fundamental period of the structure braced with the fictitious springs T_1:

$$\hat{T}_1 = \frac{T_1}{\sqrt{2\xi_1 + 1}} = \frac{1.304}{\sqrt{2(0.35) + 1}} = 1.000 \text{ s} \tag{6.65}$$

Table 6-6: Fictitious Spring Constants and Viscous Damping Constants for Six-Storey Steel Building Structure

Level	Fictitious Spring Constants (kN/mm)	Linear Viscous Dampers Constants (kN.s/mm)
6	70	15
5	68	14
4	89	19
3	98	20
2	110	23
1	80	17

8. Design of Structures Equipped with Viscous Dampers

- Design Example for Linear Viscous Dampers

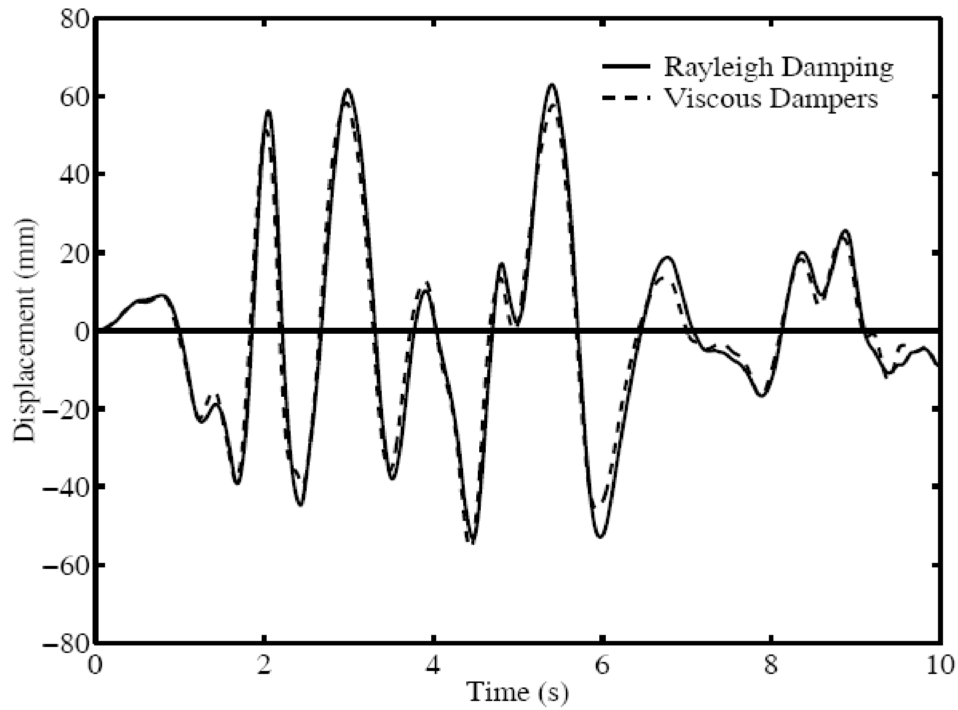


Figure 6.14 Top Floor Displacement Time-Histories of Elastic Six-Storey Steel Building Structure, S00E Component of 1940 El Centro Earthquake: Comparison Between Rayleigh Damping of 35% of Critical in First and Fifth Mode of Vibration and Viscous Dampers of Table 6-6

8. Design of Structures Equipped with Viscous Dampers

- Additional Design Considerations for Nonlinear Viscous Dampers
  - Advantage of nonlinear viscous dampers is reduction of damper forces at high velocity.
  - Although certain amount of trial-and-error is required for selecting appropriate values of damping constant and velocity coefficient, approximate design procedure can be established based on energy considerations.

• Additional Design Considerations for Nonlinear Viscous Dampers

The energy dissipated per cycle for a linear damper ($\alpha_{vd} = 1$) and for a nonlinear viscous damper ($\alpha_{vd} < 1$) have already been evaluated by Equations (6.8) and (6.15). The value of the damping constant for a nonlinear viscous damper C_{NL} that dissipates the same amount of energy per cycle as a linear viscous damper can be obtained as a function of the velocity coefficient α_{vd}, the displacement amplitude X_0 and the excitation frequency ω by equating these two equations:

$$\frac{C_{NL}}{C_L} = \frac{\sqrt{\pi}}{2}(\omega X_0)^{1-\alpha_{vd}} \frac{\Gamma(3/2 + \alpha_{vd}/2)}{\Gamma(1 + \alpha_{vd}/2)} \tag{6.66}$$

For the typical range of values of the velocity coefficient $0.2 \le \alpha_{vd} \le 1$, the ratio of gamma functions in Equation (6.66) is close to unity and the damping constant can be approximated as:

$$\frac{C_{NL}}{C_L} \approx \frac{\sqrt{\pi}}{2}(\omega X_0)^{1-\alpha_{vd}} \tag{6.67}$$

Note that consistent units must be used since Equations (6.66) and (6.67) are not dimensionally homogeneous.

8. Design of Structures Equipped with Viscous Dampers

- Additional Design Considerations for Nonlinear Viscous Dampers

$$\frac{C_{NL}}{C_L} \approx \frac{\sqrt{\pi}}{2}(\omega X_0)^{1-\alpha_{vd}} \qquad (6.67)$$

  - Equation (6.67) can provide initial estimates of nonlinear damping constants once linear damping constants have been established.
  - Excitation frequency ω can be taken as fundamental frequency of original structure without dampers.
  - X_0 can be taken as displacement in the dampers corresponding to a desired performance drift level.

8. Design of Structures Equipped with Viscous Dampers

- Optimal Distribution of Viscous Dampers
  - Design approach based on distribution of damping constants proportional to lateral stiffness of original structure may not be optimum from an economical point of view where same size dampers should be used as much as possible.
  - Constraint on maintaining classical normal modes is not required if nonlinear time-history dynamic analysis used in design process.
  - Optimum distribution of dampers in a structure can be cast in context of optimal control theory.
  - Several design methods for obtaining optimum distribution of dampers in a structure have been proposed.
  - Require advanced programming capabilities to implement.

8. Design of Structures Equipped with Viscous Dampers

- Optimal Distribution of Viscous Dampers
  - Sequential search algorithm developed by Zhang and Soong (1992) and modified by Lopez-Garcia (2001) simple to implement.
  - General approach for of any type of dampers.
  - Based on maximizing a given set of optimum location indices.
  - For linear viscous dampers, optimum location index is maximum inter-storey velocity.
  - Optimum location of dampers between two adjacent stories.

8. Design of Structures Equipped with Viscous Dampers

- Optimal Distribution of Viscous Dampers

For illustration purposes, a multi-storey structure in which N_d identical linear viscous dampers with constant C_L are to be introduced, is considered. From Equation (6.8) and assuming that the structure responds in its fundamental mode of vibration T_1, the energy dissipated per cycle for all dampers in the structure E_{vd} is given by:

$$E_{vd} = c\pi\omega X_0^2$$

$$E_{vd} = \sum_{i=1}^{N_d} \frac{2\pi^2 C_L \dot{\delta}_i^2 \cos^2\gamma_i}{T_1} = \frac{2\pi^2 C_L}{T_1} \sum_{i=1}^{N_d} \delta_i^2 \cos^2\gamma_i \qquad (6.68)$$

where δ_i is the interstorey drift at the storey where the i^{th} damper is located and γ_i is the inclination angle of the i^{th} damper.

8. Design of Structures Equipped with Viscous Dampers

- Optimal Distribution of Viscous Dampers

Assuming that the viscous dampers add no supplemental stiffness to the structure, the total recoverable elastic strain energy of the system E_{es} can be written as:

$$E_{es} = \frac{1}{2}\sum_{i=1}^{N_f} k_i \delta_i^2 \tag{6.69}$$

where k_i is the lateral stiffness of the i^{th} storey and N_f is the number of floors (or stories) in the structure.

8. Design of Structures Equipped with Viscous Dampers

- Optimal Distribution of Viscous Dampers

The first modal damping ratio ξ_1 provided by the viscous dampers can be obtained by:

$$\xi_1 = \frac{E_{vd}}{4\pi E_{es}} = \frac{\pi C_L \sum_{i=1}^{N_d} \delta_i^2 \cos^2\gamma_i}{T_1 \sum_{i=1}^{N_f} k_i \delta_i^2} \tag{6.70}$$

Re-arranging Equation (6.70), the required damping constant C_L required for all N_d dampers in order to achieve a given first modal damping ratio ξ_1 can be obtained from:

$$C_L = \frac{\xi_1 T_1 \sum_{i=1}^{N_f} k_i \delta_i^2}{\pi \sum_{i=1}^{N_d} \delta_i^2 \cos^2\gamma_i} \tag{6.71}$$

8. Design of Structures Equipped with Viscous Dampers

- Optimal Distribution of Viscous Dampers

For the particular case where all stories have the same height and the fundamental mode shape is approximated by a straight line, the interstorey drifts are all given by:

$$\delta_i = \frac{1}{N_f} \tag{6.72}$$

thereby further simplifying Equation (6.71):

$$C_L = \frac{\xi_1 T_1 \sum_{i=1}^{N_f} k_i}{\pi N_d \cos^2\gamma} \tag{6.73}$$

8. Design of Structures Equipped with Viscous Dampers

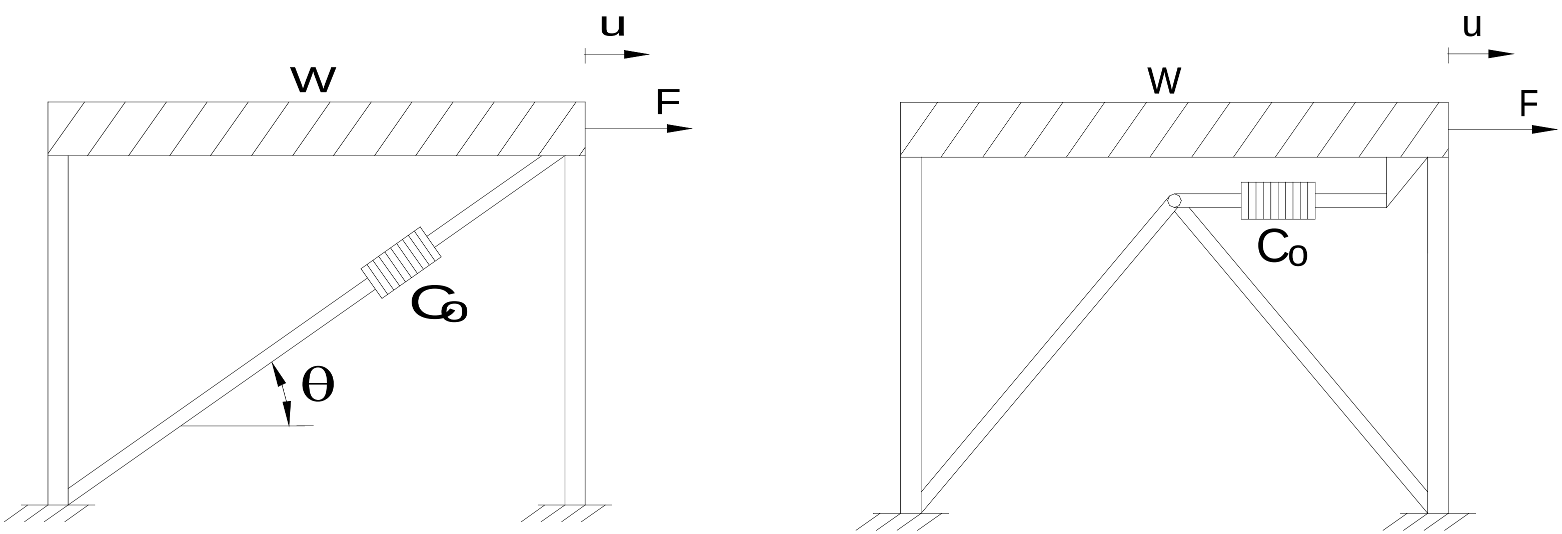
- Optimal Distribution of Viscous Dampers
  - Similar approach for nonlinear viscous dampers

$$\zeta_1 = \frac{E_{vd}}{4\pi E_s} = \frac{C_{NL}(2\pi)^{\alpha_{vd}}\left(\Gamma(1+\alpha_{vd}/2)/\Gamma(3/2+\alpha_{vd}/2)\right)\sum_{i=1}^{N_d}\delta_i^{\alpha_{vd}+1}(\cos\gamma_i)^{\alpha_{vd}+1}}{T_1^{\alpha_{vd}}\sqrt{\pi}\sum_{i=1}^{N_f}k_i\delta_i^2}$$

$$C_{NL} = \frac{\zeta_1 T_1^{\alpha_{vd}}\sqrt{\pi}\sum_{i=1}^{N_f}k_i\delta_i^2}{(2\pi)^{\alpha_{vd}}\left(\Gamma(1+\alpha_{vd}/2)/\Gamma(3/2+\alpha_{vd}/2)\right)\sum_{i=1}^{N_d}\delta_i^{\alpha_{vd}+1}(\cos\gamma_i)^{\alpha_{vd}+1}}$$

9. Geometrical Amplification of Damping

- Damper installed in-line with bracing element experiences displacement between its two ends less than inter-storey drift.
- Damper installed horizontally at apex of a chevron bracing system, displacement between two ends of the damper equals the inter-storey drift.
- Efficiency of supplemental damping systems can be improved by providing geometrical configuration of bracing system to amplify damper displacement for specified inter-storey drift.



9. Geometrical Amplification of Damping

- Toggle-brace configuration (Constantinou et al. 2001)

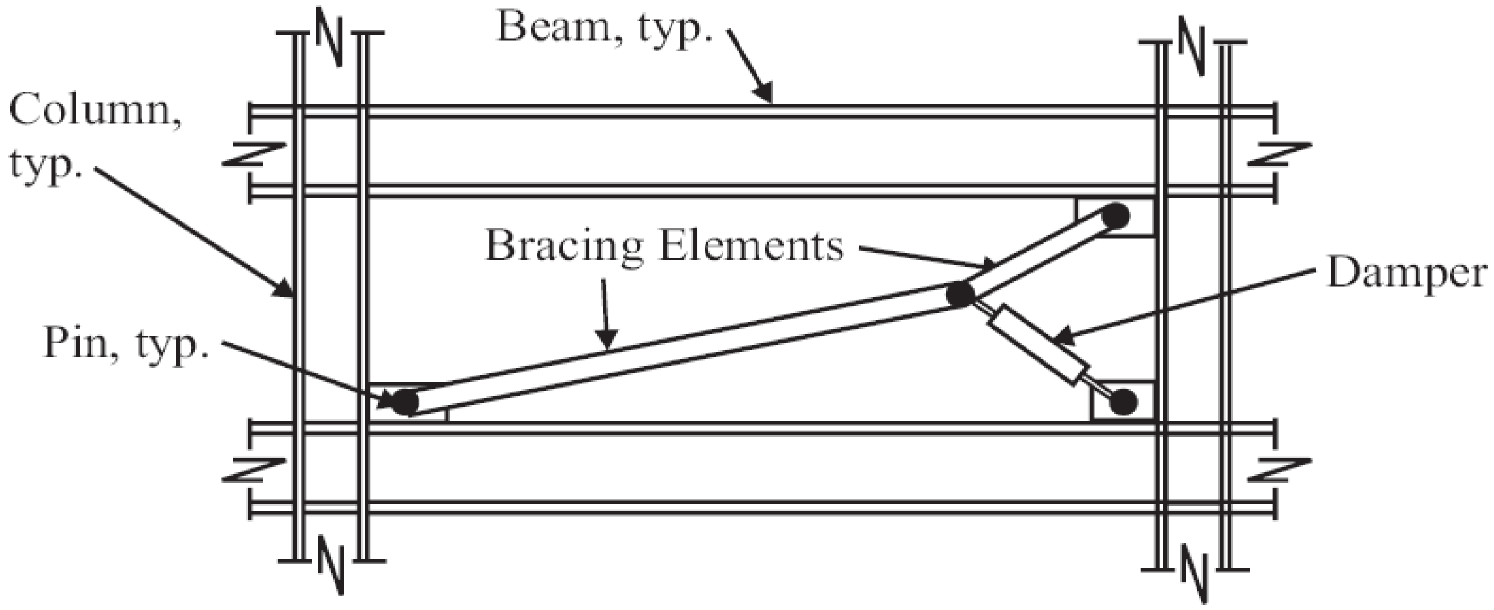


Figure 6.15 Geometrical Arrangement of Toggle-Brace Damping System (after Constantinou 2001)

9. Geometrical Amplification of Damping

- Toggle-brace configuration (Constantinou et al. 2001)

•Photo: Courtesy of M. Constantinou

9. Geometrical Amplification of Damping

- Scissor-jack configuration (Sigaher and Constantinou 2003).

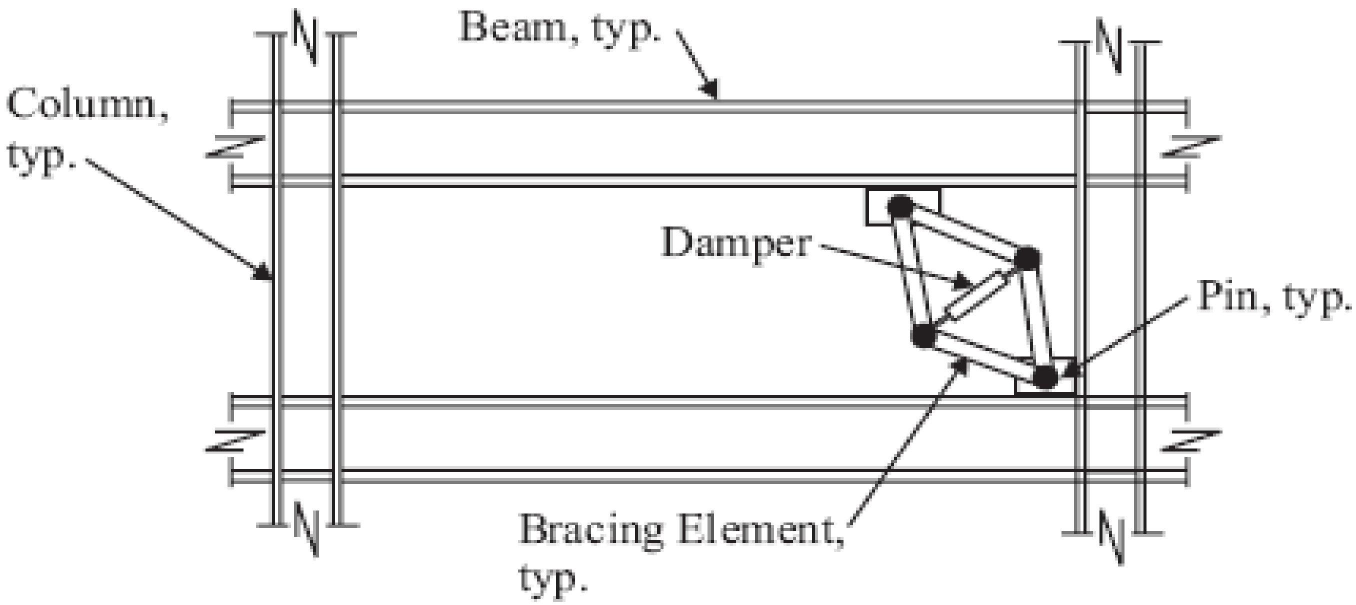


Figure 6.16 Geometrical Arrangement of Scissor-Jack Damping System (after Sigaher and Constantinou 2003)

9. Geometrical Amplification of Damping

- Scissor-jack configuration (Sigaher and Constantinou 2003).

•Photo: Courtesy of M. Constantinou

9. Geometrical Amplification of Damping

The damping amplification provided by the toggle-brace and the scissor jack systems can be established by re-writing general forms of the equilibrium and compatibilities relationships given by Equations (6.38) and (6.39) for a single-storey structure equipped with a linear viscous damper (see Figure 6.8).

$$F_{vd} = f\,F(t) \tag{6.74}$$

and

$$\Delta(t) = f\,x(t) \tag{6.75}$$

where f is a geometrical amplification factor equal to $\cos\gamma$ for a diagonal in-line configuration and 1.0 for a horizontal chevron bracing configuration. Re-calling the

9. Geometrical Amplification of Damping

Re-calling the constitutive equation for the fluid viscous damper:

$$F(t) = C_L \dot{\Delta}(t) \tag{6.76}$$

Substituting Equations (6.75) and (6.76) into Equation (6.68) leads to a final expression for the horizontal force provided by the damper assembly on the structure:

$$F_{vd}(t) = C_L f^2 \dot{x}(t) \tag{6.77}$$

Therefore, the equivalent viscous damping ratio provided by the damper assembly can be written as (Sigaher and Constantinou 2003):

$$\xi = \frac{C_L f^2 g T}{4\pi W} \tag{6.78}$$

where T is the natural period of the structure and W is the seismic weight. Equation (6.78) indicates that the damping ratio is amplified by the square of the geometrical amplification factor.

9. Geometrical Amplification of Damping

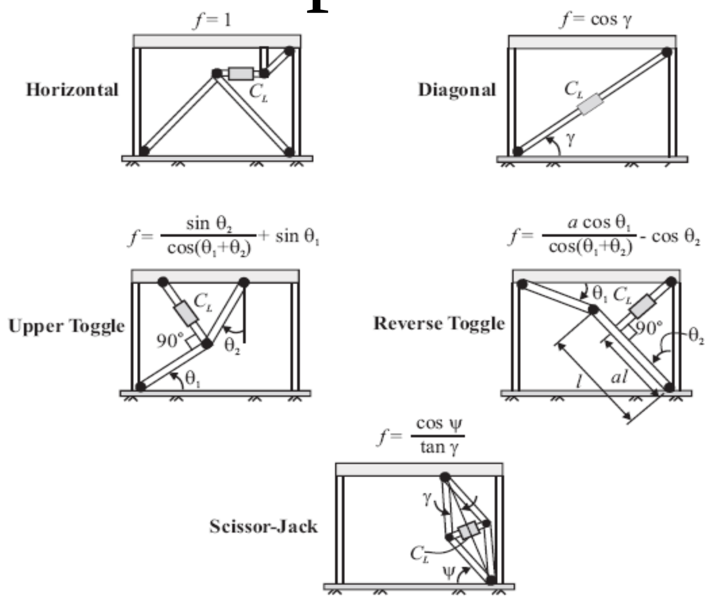


Table 6-7: Numerical Examples of Damping Provided by Same Viscous Damper in Different Configurations (see Figure 6.17)

Configuration	Geometric Parameters	f	ξ
Horizontal	-	1.00	0.05
Diagonal	$\gamma = 37°$	0.80	0.03
Upper Toggle	$\theta_1 = 31.9°,\ \theta_2 = 43.2°$	3.19	0.51
Reverse Toggle	$\theta_1 = 30°,\ \theta_2 = 49°,\ a = 0.7$	2.52	0.32
Scissor-Jack	$\gamma = 9°,\ \psi = 70°$	2.16	0.23

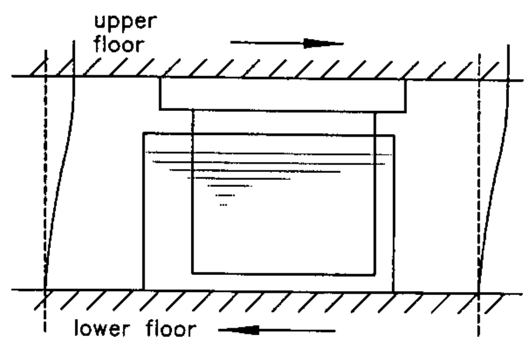
10. Structural Implementations

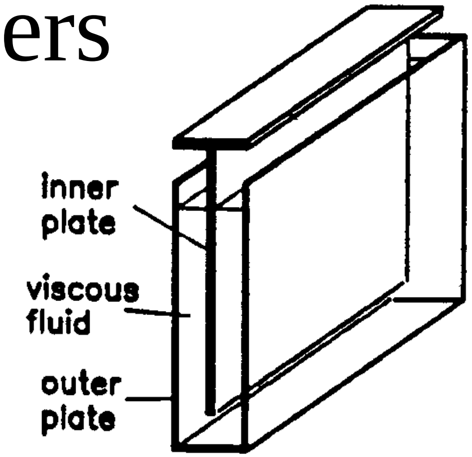
APPENDIX C: Implementation of Viscoelastic and Viscous Dampers in Structures. [N]: New Construction, [R]: Retrofit

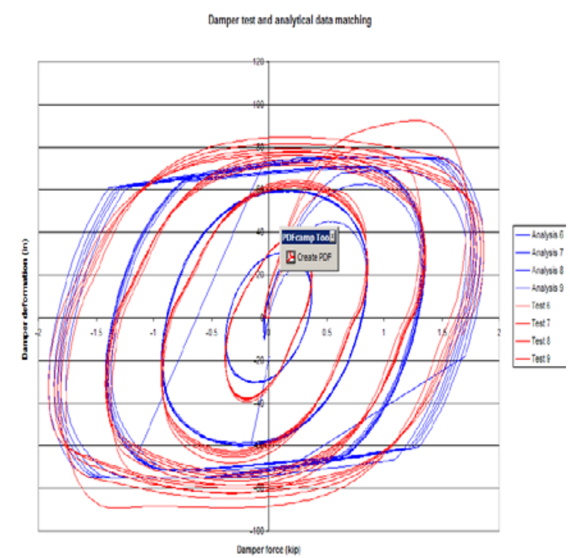
Structure	Location	Year	Damper Type	Number of Dampers	Reference
4-storey RC building [N]	Tsukuba Japan	1987	Viscous Damped Walls (Oiles and Sumitomo Corp.)	Unknown	Arima et al., 1988
15-storey building [N]	Shizuoka City Japan	1991	Viscous Damped Walls (Oiles and Sumitomo Corp.)	Unknown	Miyazaki and Mitsusaka, 1992
24-storey steel building [N]	Japan	1991	Viscoelastic Dampers (Shimizu Corp.)	Unknown	Yokota et al., 1992
School Building 2-stories [N]	Phoenix Arizona	1992	Viscoelastic beam-column connectors	Unknown	Aiken, 1997
Santa Clara County Civic Center, east Wing Building 13-storey steel frame, 51 m x 51 m plan, constructed in 1976 [R]	San Jose California	1993	Viscoelastic Dampers (3M)	96	Crosby et al., 1994
Pacific Bell North Area Operation Center 3-storey steel braced frame [N]	Sacramento California	1995	Fluid Viscous Dampers (Taylor)	62	Aiken, 1997
Science Building II, California State University, Sacramento 6-storey steel frame [N]	Sacramento California	1996	Fluid Viscous Dampers (Taylor)	40	Aiken, 1997

10. Structural Implementations
•Viscous Wall Dampers

- Steel box filled with viscous fluid with a vane dipped in it
- Fluid under atmospheric pressure
- Low-pressure, large area device
- Viscoelastic, synthetic rubber
  - 98% isobutylene with 2% isoprene







10. Structural Implementations

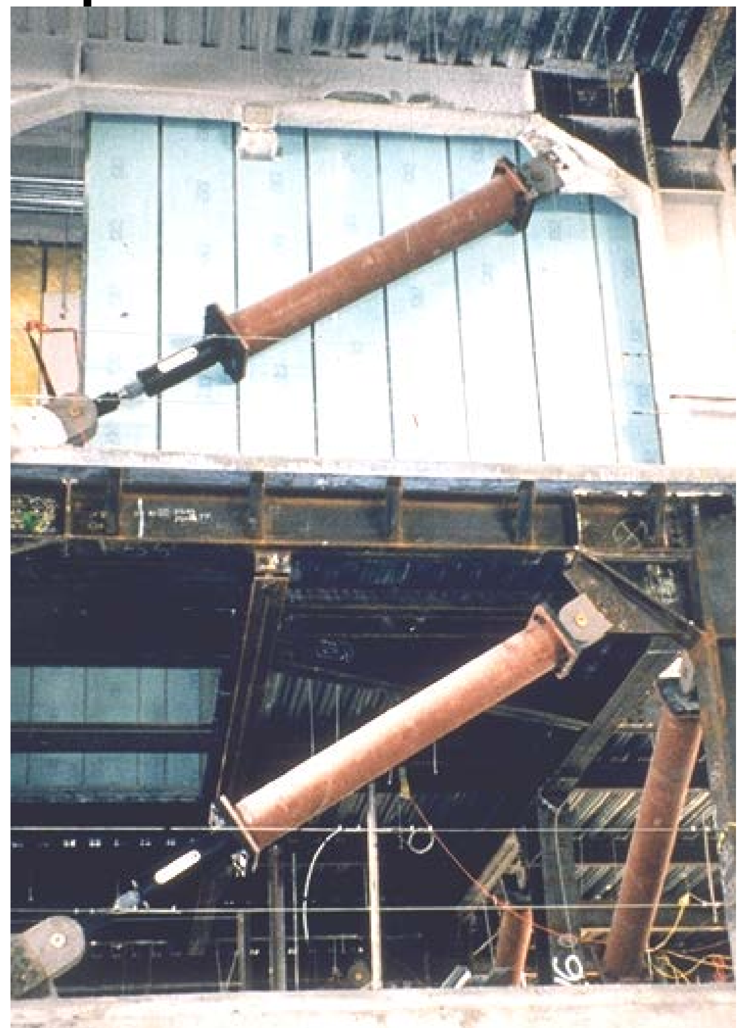
Woodland Hotel 4-storey non-ductile RC frame/shear wall constructed in 1927 [R]	Woodland California	1996	Fluid Viscous Dampers (Taylor)	16	Aiken, 1997
San Francisco Opera House [R]	San Francisco California	1996	Fluid Viscous Dampers (Enidine)	16	Aiken, 1997
Building 116, Naval Supply Facility 3-storey non-ductile RC wall flat-slab Structure 121 ft x 365ft plan. [R]	San Diego California	1996	Viscoelastic Dampers (3M)	64	Soong and Dargush, 1997
Rockwell Building 505 [R]	Newport Beach California	1997	Viscous Dampers (Taylor)	Unknown	Aiken, 1997
San Francisco Civic Center Building 15-storey steel frame [N]	San Francisco California	1997	Viscous Dampers (Taylor)	292	Aiken, 1997
The Money Store 11-storey steel frame [N]	Sacramento California	1997	Viscous Dampers (Taylor)	120	Aiken, 1997
Los Angeles Police Department Recruit Training Center 4-storey steel frame constructed in 1988 [R]	Los Angeles California	1997	Viscoelastic Dampers (3M)	Unknown	Aiken, 1997
San Mateo County Hall of Justice 8-storey steel frame with precast cladding, constructed mid-1960s [R]	Redwood City California	1997	Viscoelastic Dampers (3M)	Unknown	Aiken, 1997
Arrowhead Medical Center [N]	Colton California		Fluid Viscous for Base Isolation (Taylor)	186	Lee, 2003
Los Angeles City Hall [R]	Los Angeles California		Fluid Viscous for Base Isolation (Taylor)	52	Lee, 2003
Hayward City Halls	Hayward California		Fluid Viscous (Taylor)	15	Lee, 2003

10. Structural Implementations

- **Woodland Hotel, Woodland, CA**
- **Four-story reinforced concrete/shear wall building**
- **Constructed in 1927**
- **16 Taylor Dampers installed horizontally**
- **Capacity of each damper = 100 kips**

10. Structural Implementations

- San Francisco Civic Center
- 292 Fluid Viscous Dampers Installed In Line



•Courtesy of M. Constantinou

10. Structural Implementations

Tokyo-Rinkai Hospital	Japan		Fluid Viscous (Taylor)	45	Lee, 2003
Jimbo-Cho Office Building	Tokyo Japan		Fluid Viscous (Taylor)		Lee, 2003
San Francisco Oakland Bay Bridge [R]	San Francisco California		Fluid Viscous (Taylor)		Lee, 2003
Millennium Bridge [R]	U.K.		Fluid Viscous (Taylor)		Lee, 2003
Petronas Twin Towers	Malaysia		Fluid Viscous (Taylor)		Lee, 2003
28 State Street Office Building [N]	Boston Massachusetts		Fluid Viscous (Taylor)	40	Lee, 2003
Torre Mayor Office Building 57-storey steel frame [N]	Mexico City Mexico		Fluid Viscous (Taylor)	98	Lee, 2003
999 Sepulveda Building [N]	Los Angeles California		Fluid Viscous (Taylor)		Lee, 2003
Enron Field Stadium [N]	Houston Texas		Fluid Viscous (Taylor)	16	Lee, 2003
California State University – Administration Building [N]	Los Angeles California		Fluid Viscous (Taylor)	14	Lee, 2003
Computer Data Storage Center [N]	Northern California		Fluid Viscous (Taylor)	32	Lee, 2003
Kaiser Corona Data Center [N]	Corona California		Fluid Viscous (Taylor)	16	Lee, 2003
Money Store National Headquarters [N]	Sacramento California		Fluid Viscous (Taylor)	120	Lee, 2003
Novelty Bridge [N]	Seattle Washington	2000	Fluid Viscous (Taylor)	8	Lee, 2003
Bill Emerson Memorial Bridge [N]	Cape Girardeau Missouri		Fluid Viscous (Taylor)	16	Lee, 2003
SAFECO Field Stadium [N]	Seattle Washington		Fluid Viscous (Taylor)	44	Lee, 2003
Beijing Railway Station [N]	Beijing China		Fluid Viscous (Taylor)	32	Lee, 2003

10. Structural Implementations

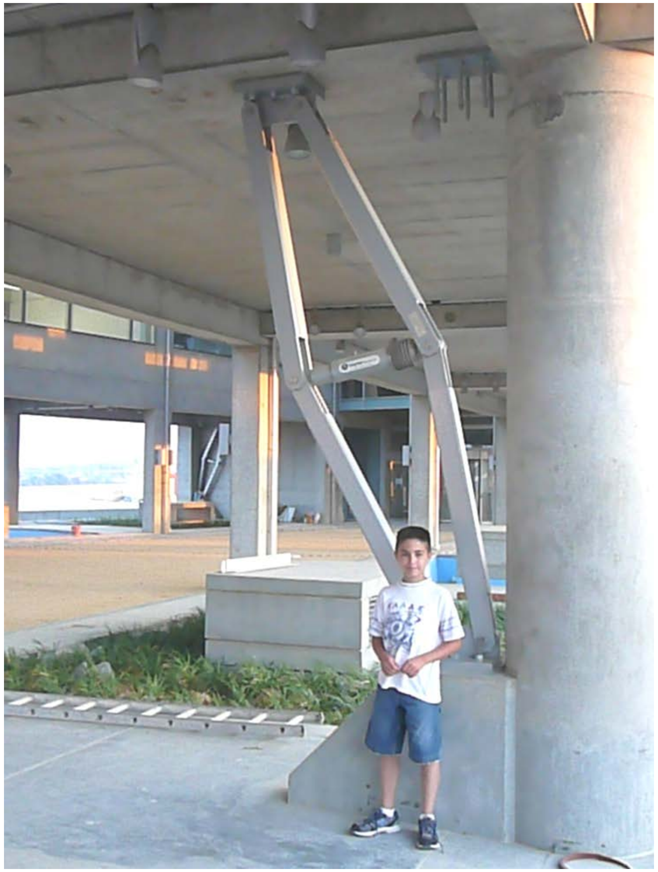
Park Hyatt Hotel [N]	Chicago Illinois	Fluid Viscous part of Tuned Mass Damper (Taylor)		Lee, 2003
Yerba Buena Tower / Four Seasons Hotel [N]	San Francisco California	Fluid Viscous (Taylor)	20	
British Columbia Electric Company Building [R]	Vancouver Canada	Fluid Viscous (Taylor)		Lee, 2003

10. Structural Implementations

- YERBA-BUENA TOWER, SAN FRANCISCO
- 37-STORY WITH REVERSE UPPER TOGGLE
- SYSTEM. UNDER CONSTRUCTION 2001.
- 20 FLUID DAMPERS IN UPPER STORIES.

•Courtesy of M. Constantinou

10. Structural Implementations



•OLYMPIC COMMITTEE BUILDING, CYPRUS

•3-STORY, V-SHAPED IN PLAN

•52 SCISSOR-JACK ASSEMBLIES

•COMPLETED JULY 2006

•Courtesy of M. Constantinou

11. Performance-Based Design Example

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Influence of passive supplemental damping systems on structural and nonstructural seismic fragilities of a steel building

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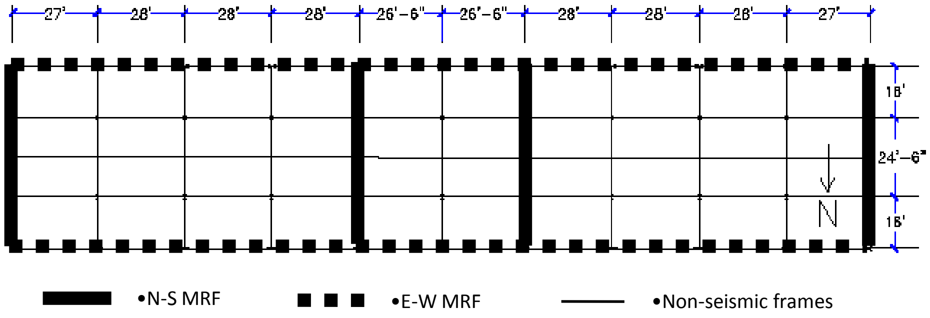
11. Performance-Based Design Example

- Objectives
  - Evaluate the performance of structural passive supplemental hysteretic and viscous damping systems for retrofitting a four-storey steel framed building containing generic rigidly anchored and vibration isolated secondary nonstructural components installed in various locations in the building.
  - Construct fragility curves based on Nonlinear Incremental Dynamic Analyses (IDA)for the structural system and nonstructural components based on various performance objectives in order to compare the influence of each passive supplemental damping system.
  - Illustrate how structural and nonstructural fragility data can be generated and used to support the decision process for the performance based seismic design or retrofit of a building.

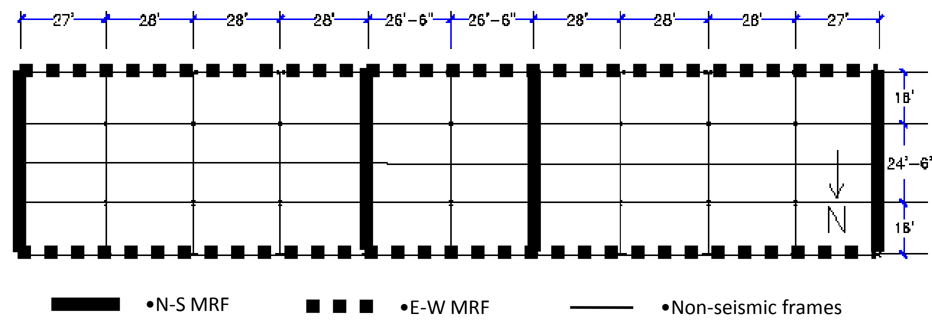
11. Performance-Based Design Example

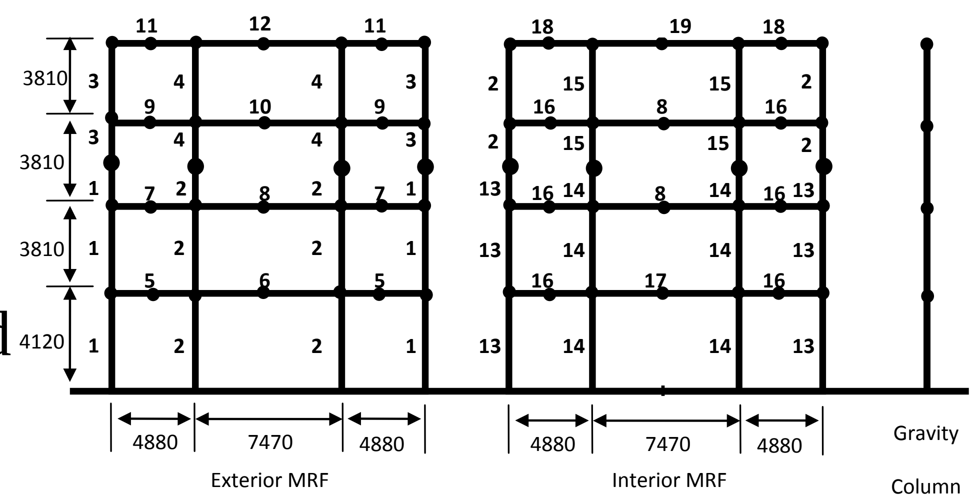
- Structural Modeling
  - Four-storey steel framed building model assumed to be part of an existing critical facility located in Southern California.
  - Designed according to UBC 1970.
  - Symmetrical building composed of four parallel seismic frames in transverse direction.



11. Performance-Based Design Example

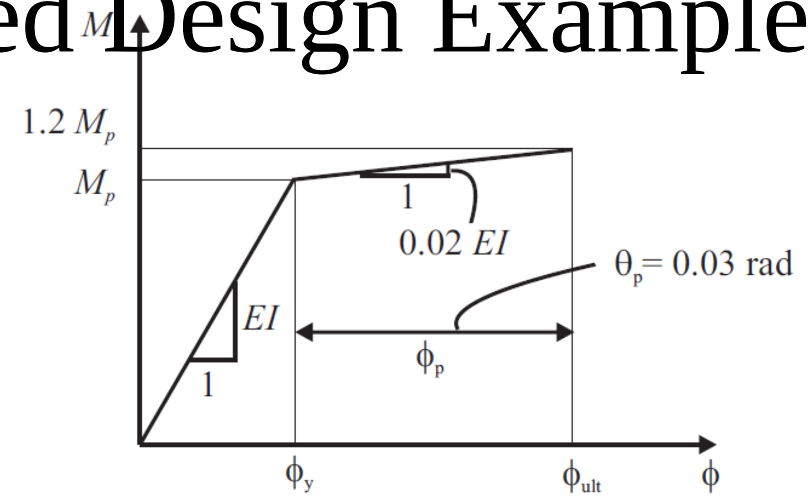
- Structural Modeling
  - 2D model of half of the building in transverse direction, including one interior and one exterior seismic frame.
  - RUAUMOKO program.
  - Pin-ended gravity column included to account for second order (P-Δ) effects from gravity frames.

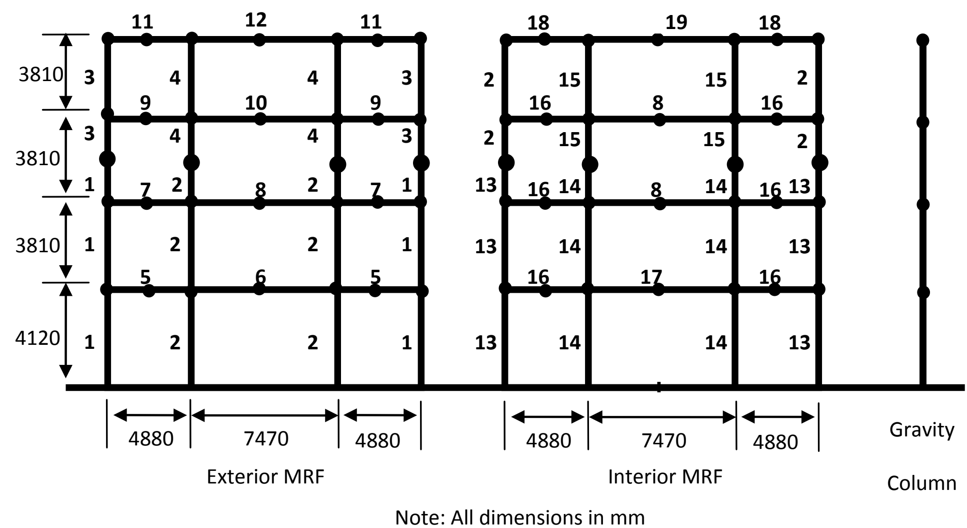




11. Performance-Based Design Example

- Structural Modeling
  - Bilinear moment-curvature hysteresis rule with 2% curvature hardening ratio assigned to all frame member ends in the model.
  - Plastic hinge length assumed equal to the depth of each member section.
  - Failure plastic rotation = 0.03 rad (curvature ductility = 11).
  - Seismic weights:
    - 5037 kN at each of first three floors.
    - 4830 kN at roof level.
  - 2% Rayleigh damping to first and third modes of vibration.





11. Performance-Based Design Example

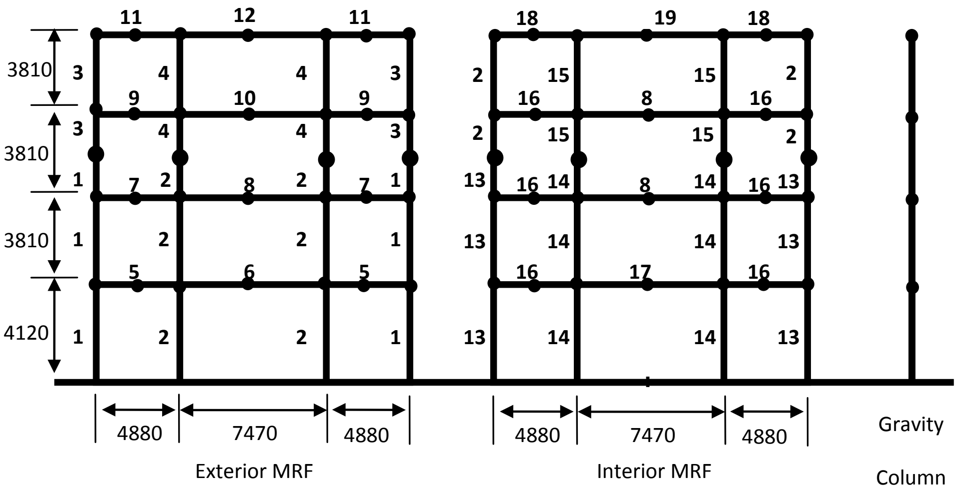
- Structural Modeling

List of member sections (see Fig. 1 for section locations)

Section no.	Designation	Section no.	Designation
1	W 14 × 193	11	W 24 × 68
2	W 14 × 342	12	W 24 × 104
3	W 14 × 159	13	W 14 × 398
4	W 14 × 257	14	W 14 × 455
5	W 24 × 146	15	W 14 × 370
6	W 33 × 221	16	W 24 × 162
7	W 24 × 131	17	W 33 × 241
8	W 30 × 211	18	W 24 × 94
9	W 24 × 103	19	W 30 × 173
10	W 30 × 211		

Modal properties of building model

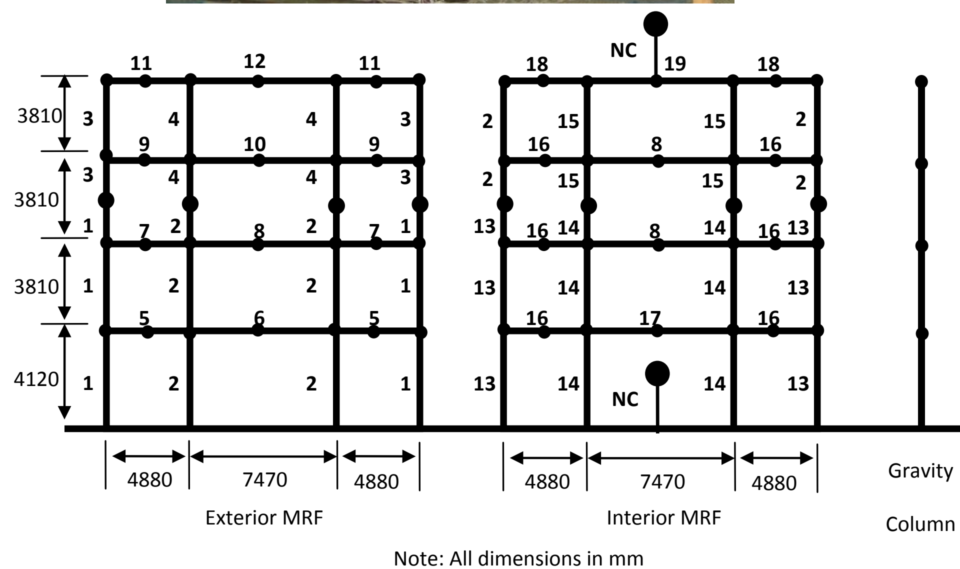
Mode no.	Period (s)	Cumulative mass (%)
1	0.76	85
2	0.26	96
3	0.15	99
4	0.10	100



Note: All dimensions in mm

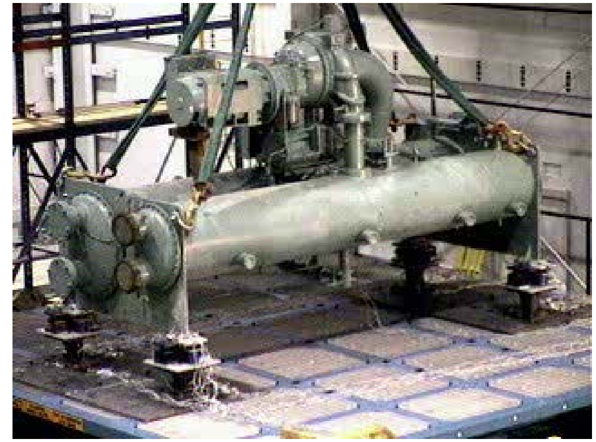
11. Performance-Based Design Example

- Nonstructural Modeling
  - Nonstructural Components (NC) representing large HVAC type equipment, (e.g. chiller or air handling unit) modeled as SDOF systems.
  - Seismic weight of each NC: 100 kN.
  - Damping of each NC: 1% of critical.

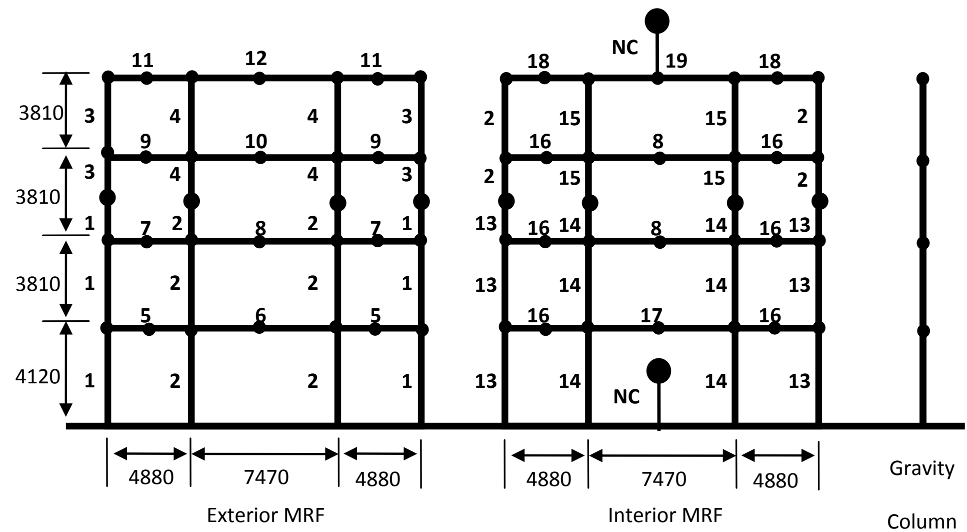


11. Performance-Based Design Example

- Nonstructural Modeling
  - Two different base supports for NC: rigid anchor and vibration isolation.
  - Natural periods of NC:
    - Rigidly anchored: 0.2 s
    - Vibration isolated: 1.0 s
  - Two locations for NC: roof and base level.



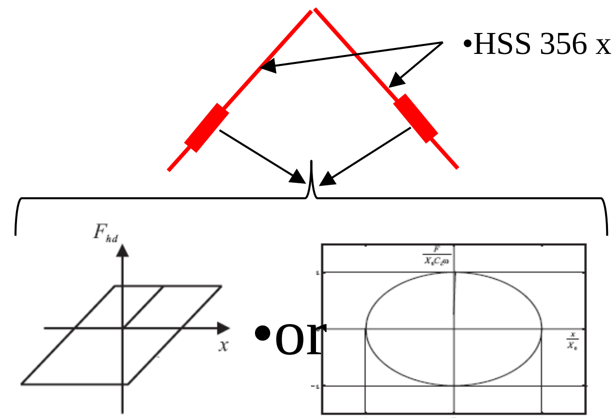
- Video 1
- Video 2

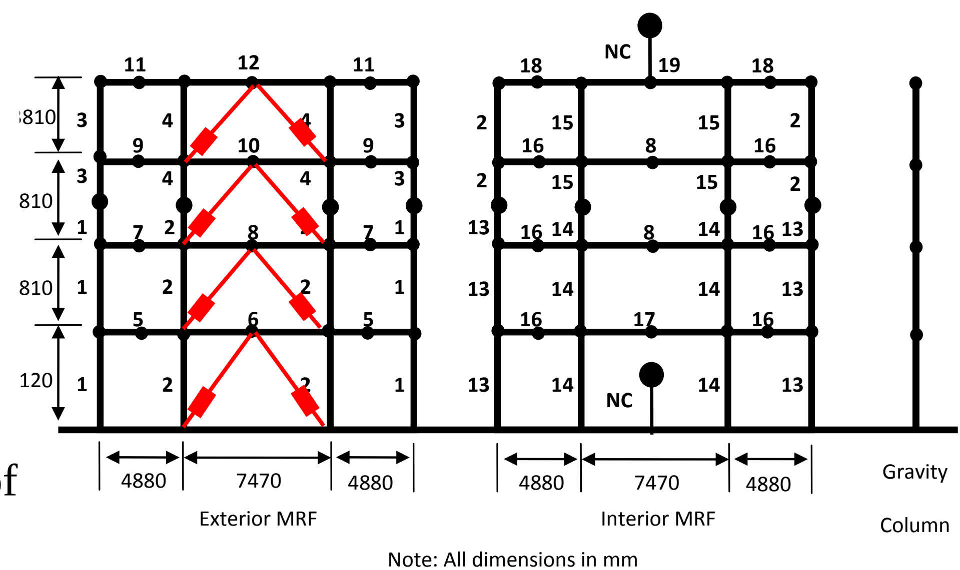


Note: All dimensions in mm

11. Performance-Based Design Example

- Structural Retrofit
  - Hysteretic and linear viscous dampers incorporated in-line with chevron bracing installed in central bay of exterior moment-resisting frame.
  - 356 x 356 x 16 mm tubular bracing.
  - Activation loads of hysteretic dampers:
    - 2322 kN for dampers in first story.
    - 2228 kN for dampers in other three stories.
  - Damping coefficients for viscous dampers:
    - 27.4, 26.8, 23,8 and 18.0 kN s/mm for dampers located on first, second, third and fourth story, respectively.
    - Equivalent first modal damping ratio of 30% of critical.

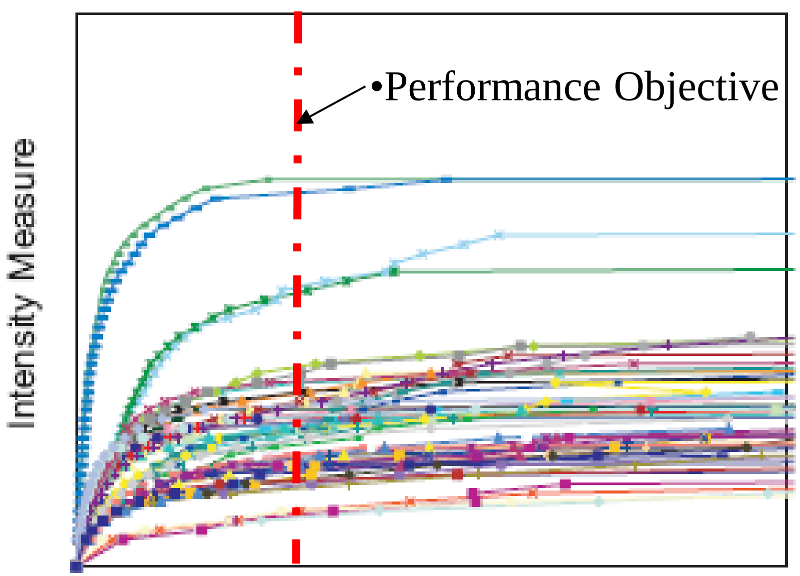




11. Performance-Based Design Example

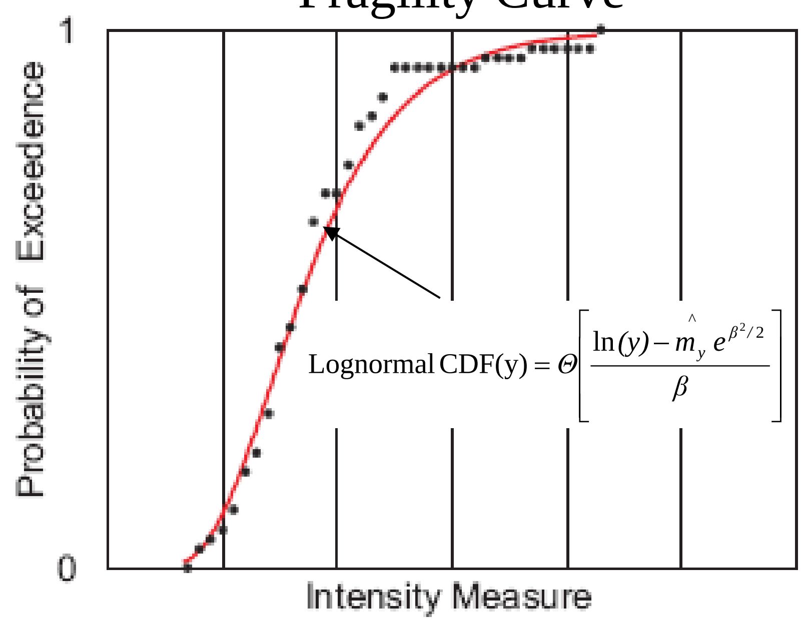
- Concept of Incremental Dynamic Analyses (IDA)

•IDA Curves



•Response Parameter

•Fragility Curve



11. Performance-Based Design Example

- Structural Performance Objectives
  - Based on Peak Story Drifts (NEHRP 2003)
    - 0.7%: Immediate Occupancy
    - 2.5%: Life Safety
    - 5%: Collapse Prevention
- Nonstructural Performance Objectives
  - Based on Peak Component Acceleration (ASHRAE Handbook)

American Society of Heating, Refrigerating and Air Conditioning Engineers

    - 2.0 g: Functional Damage (temporary interruption)
    - 4.0 g: Physical Damage (permanent interruption)

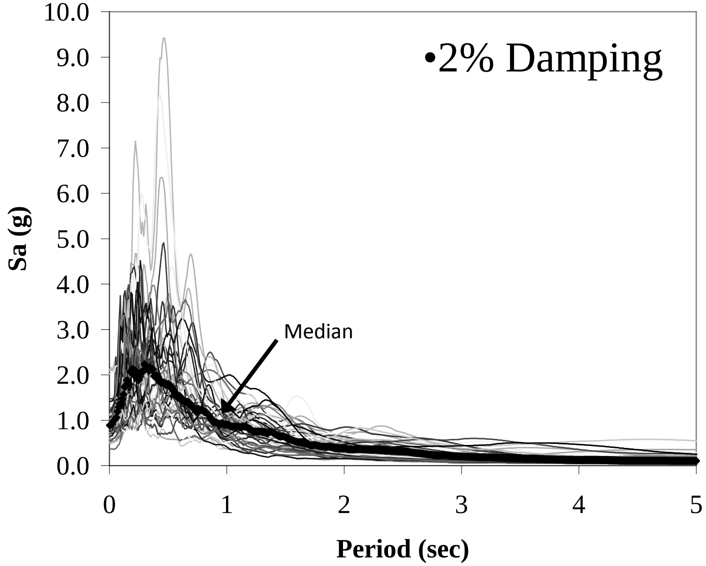
11. Performance-Based Design Example

- Earthquake Ground Motions
  - FEMA P695 Far-Field Ground Motion Set
  - 22 pairs of scaled historical ground motion records

Pair no.	Seismic events			Records		Amplitude scale factor
	Magnitude	Year	Name	Recording station	Epicentral distance (km)	
1	6.7	1994	Northridge	Beverly Hills–14145 Mulhol	13.3	0.755
2	6.7	1994	Northridge	Canyon Country-W Lost	26.5	0.832
3	7.1	1999	Duzce, Turkey	Bolu	41.3	0.629
4	7.1	1999	Hector Mine	Hector	26.5	1.092
5	6.5	1979	Imperial Valley	Delta	33.7	1.311
6	6.5	1979	Imperial Valley	El Centro Array #11	29.4	1.014
7	6.9	1995	Kobe, Japan	Nichi-Akashi	8.7	1.718
8	6.9	1995	Kobe, Japan	Shin-Osaka	46.0	1.099
9	7.5	1999	Kocaeli, Turkey	Duzce	98.2	0.688
10	7.5	1999	Kocaeli, Turkey	Arcelik	53.7	1.360
11	7.3	1992	Landers	Yermo Fire Station	86.0	0.987
12	7.3	1992	Landers	Coolwater	82.1	1.073
13	6.9	1989	Loma Prieta	Capitola	9.8	0.822
14	6.9	1989	Loma Prieta	Gilroy Array #3	31.4	0.880
15	7.4	1990	Manjil, Iran	Abbar	40.4	0.787
16	6.5	1987	Superstition Hills	El Centro Imp. Co. Cent	35.8	0.870
17	6.5	1987	Superstition Hills	Poe Road (temp)	11.2	1.362
18	7.0	1992	Cape Mendocino	Rio Dell Overpass-FF	22.7	1.516
19	7.6	1999	Chi-Chi, Taiwan	CHY101	32.0	0.636
20	7.6	1999	Chi-Chi, Taiwan	TCU045	77.5	0.563
21	6.6	1971	San Fernando	LA-Hollywood Stor FF	39.5	2.096
22	6.5	1976	Driuli, Italy	Tolmezza	20.2	1.440

11. Performance-Based Design Example

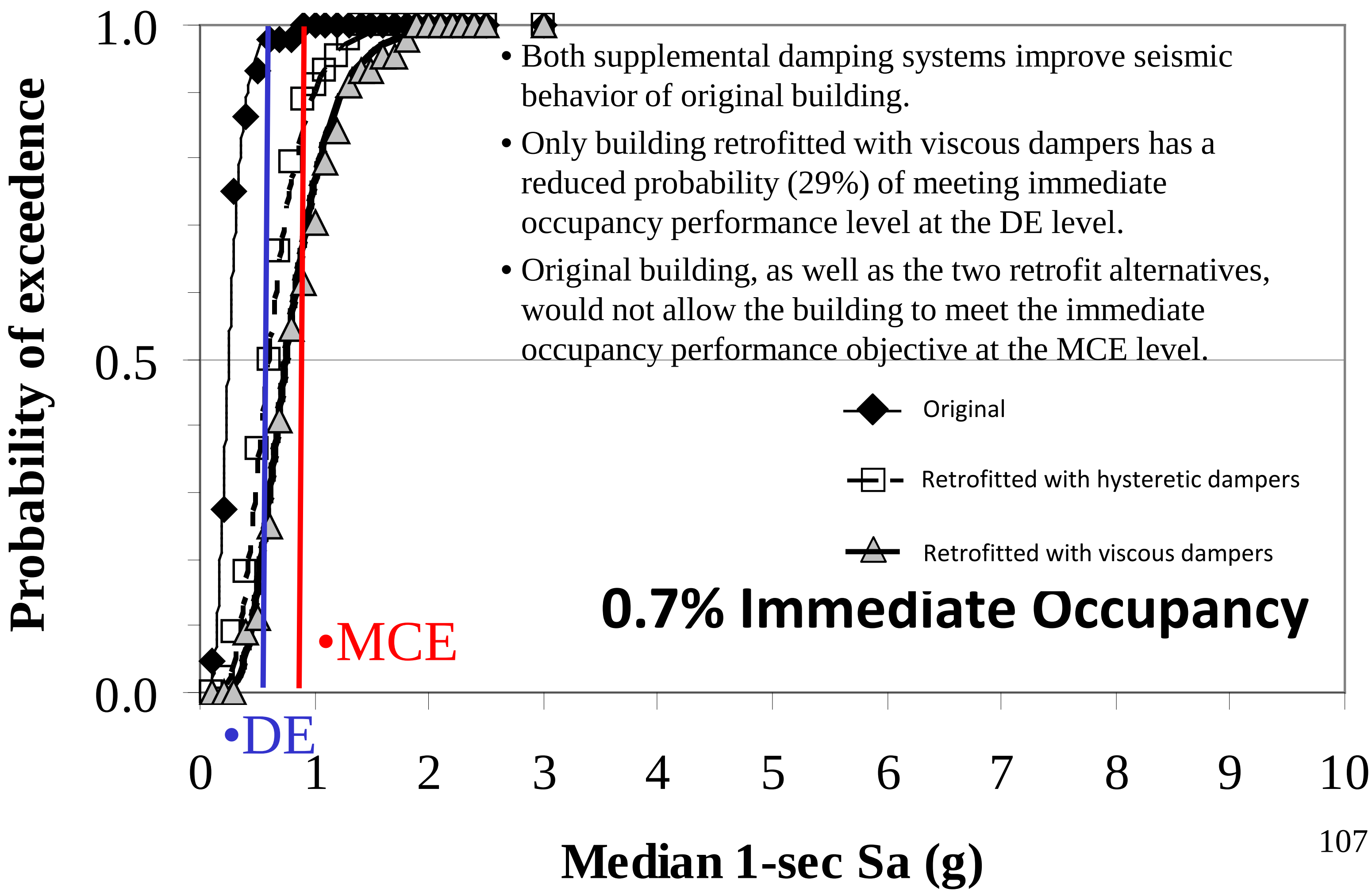
- Earthquake Ground Motions
  - FEMA P695 Far-Field Ground Motion Set
  - 22 pairs of scaled historical ground motion records (44 records total)



•Note: Median 1-sec S_a = 0.60 g for DBE level and 0.90 g for MCE level

11. Performance-Based Design Example

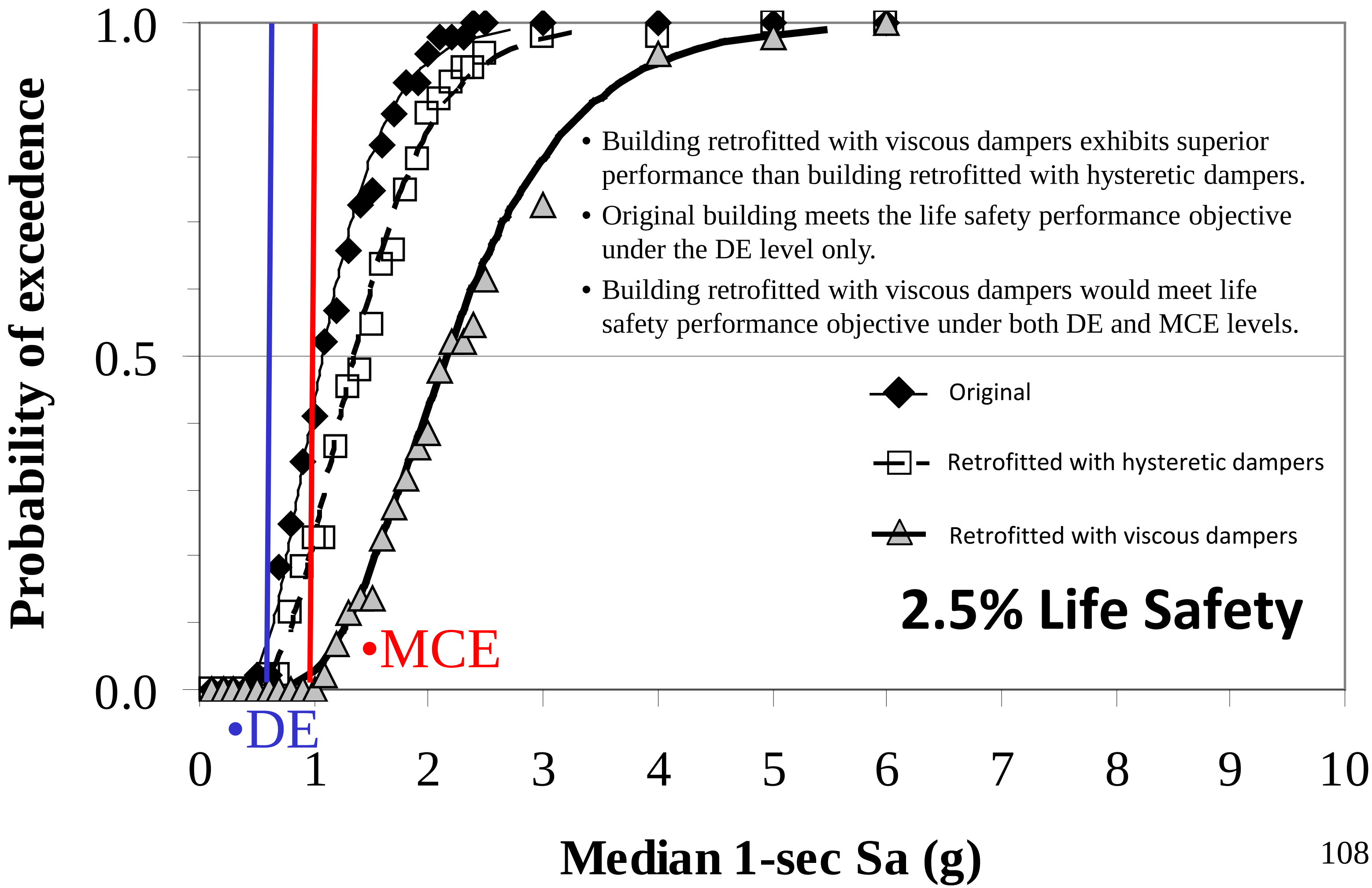
- Structural Fragilities



- Both supplemental damping systems improve seismic behavior of original building.
- Only building retrofitted with viscous dampers has a reduced probability (29%) of meeting immediate occupancy performance level at the DE level.
- Original building, as well as the two retrofit alternatives, would not allow the building to meet the immediate occupancy performance objective at the MCE level.

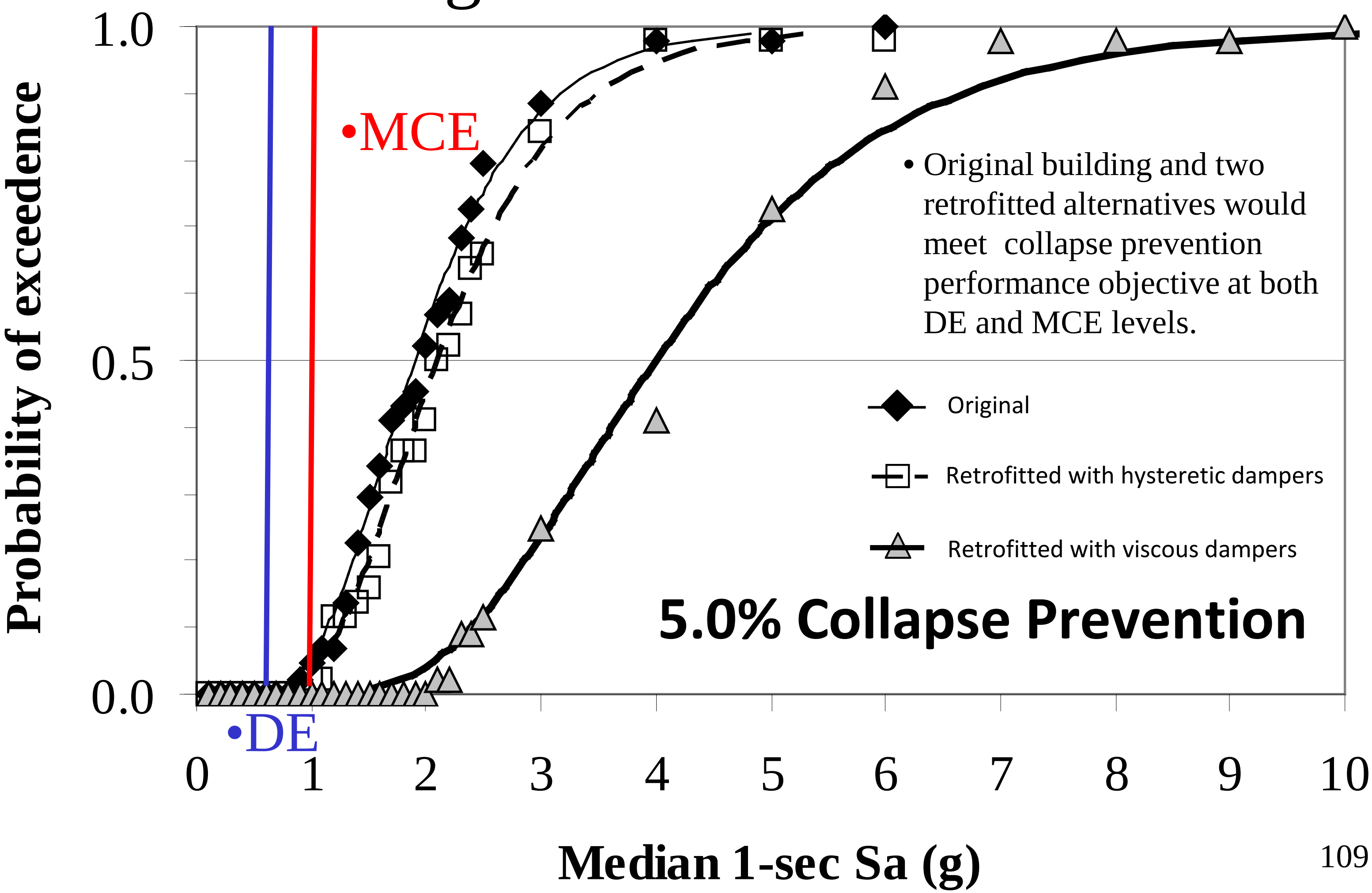
11. Performance-Based Design Example

- Structural Fragilities



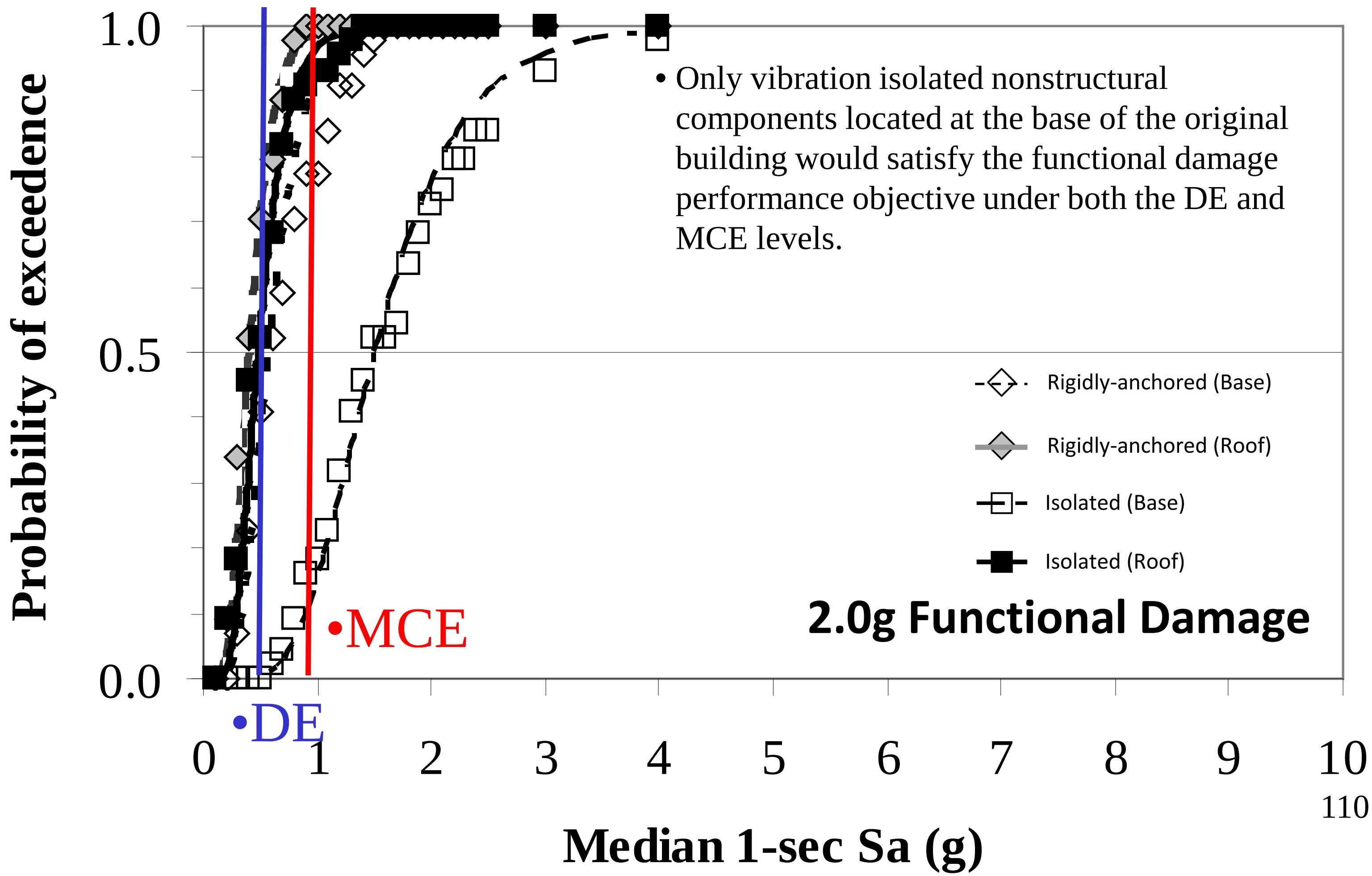
11. Performance-Based Design Example

- Structural Fragilities



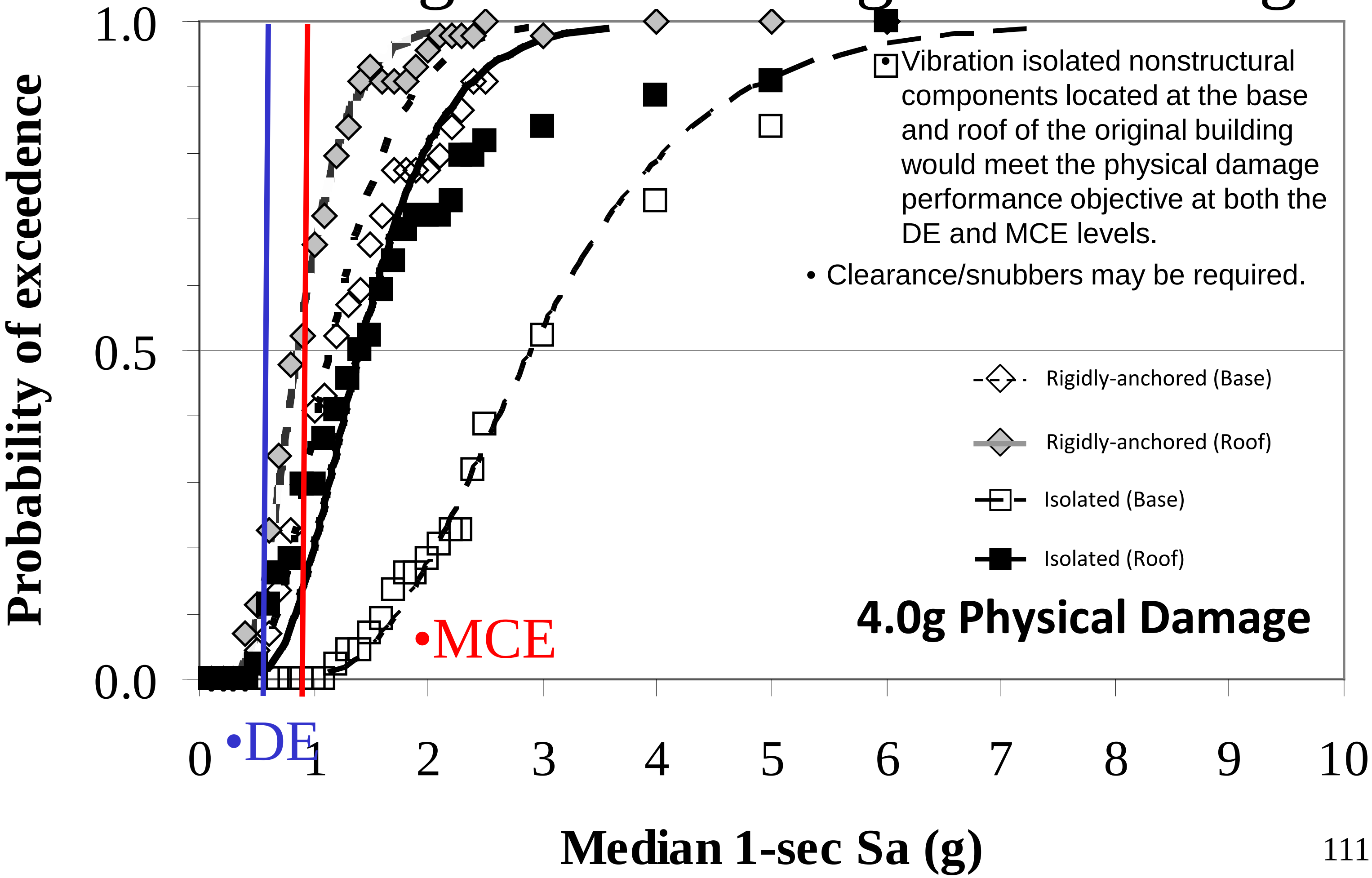
11. Performance-Based Design Example

- Nonstructural Fragilities for Original Building



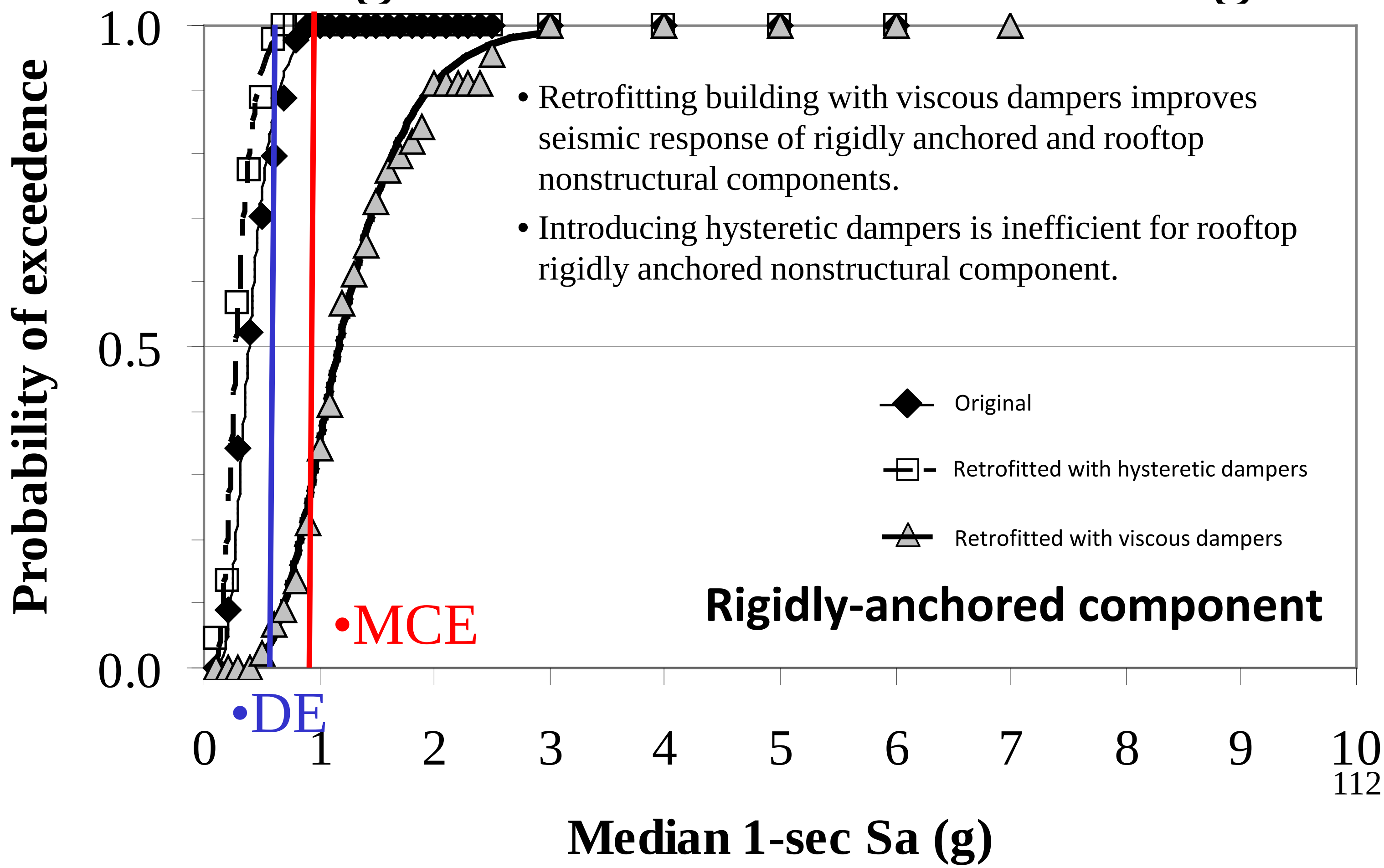
11. Performance-Based Design Example

- Nonstructural Fragilities for Original Building



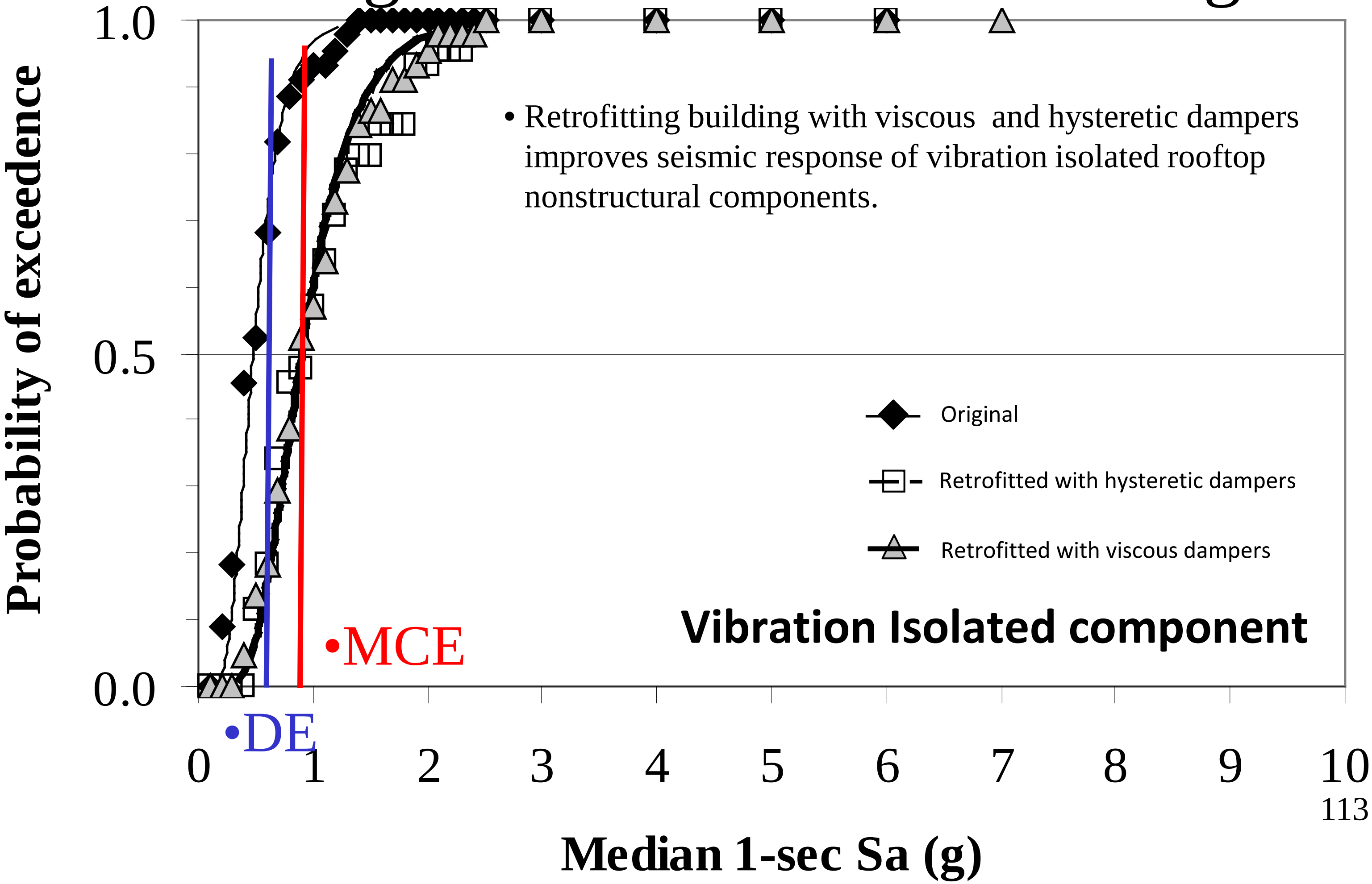
11. Performance-Based Design Example

- Nonstructural fragilities for functional damage at the roof level of original and retrofitted buildings.



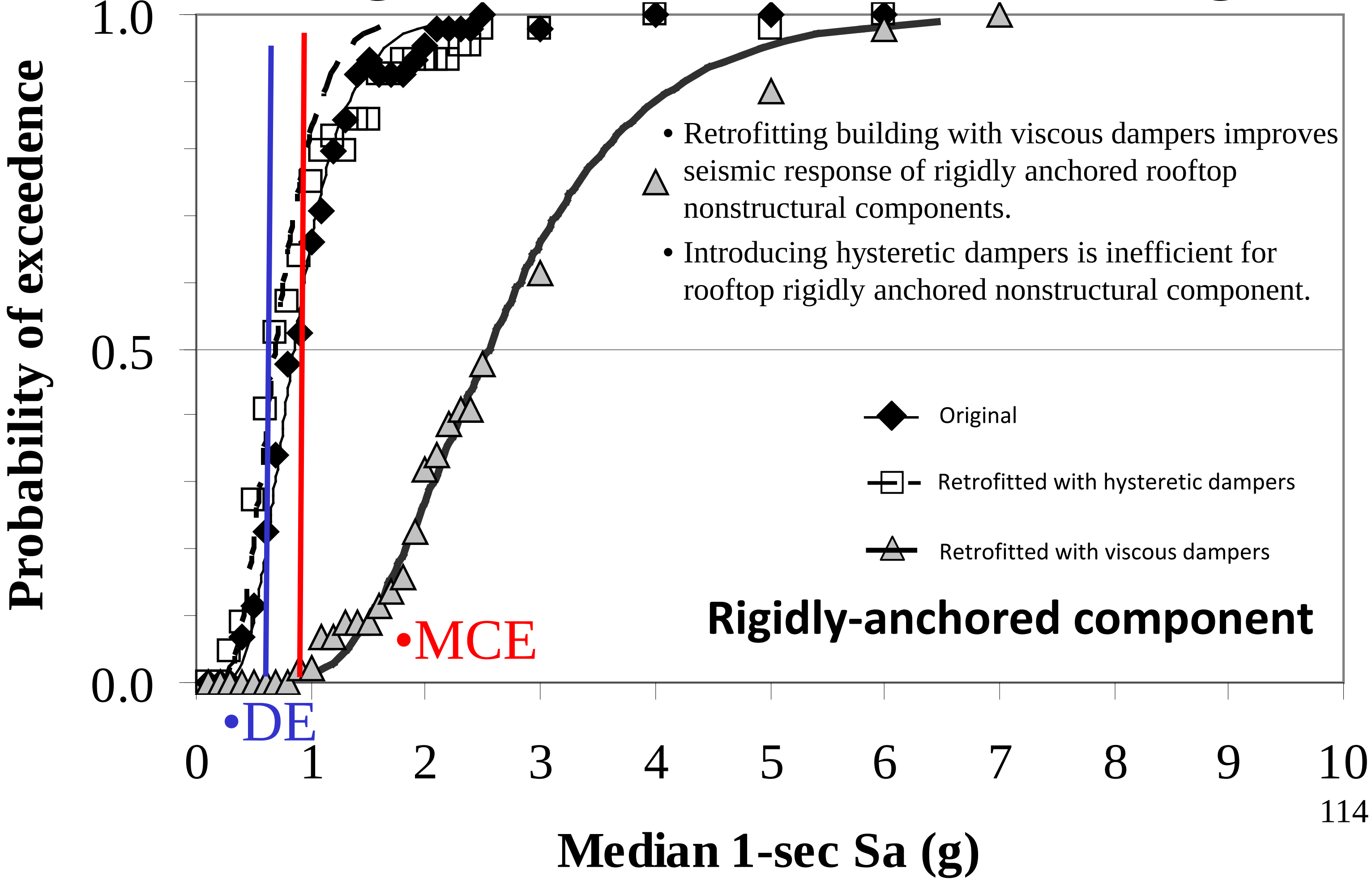
11. Performance-Based Design Example

- Nonstructural fragilities for functional damage at the roof level of original and retrofitted buildings.



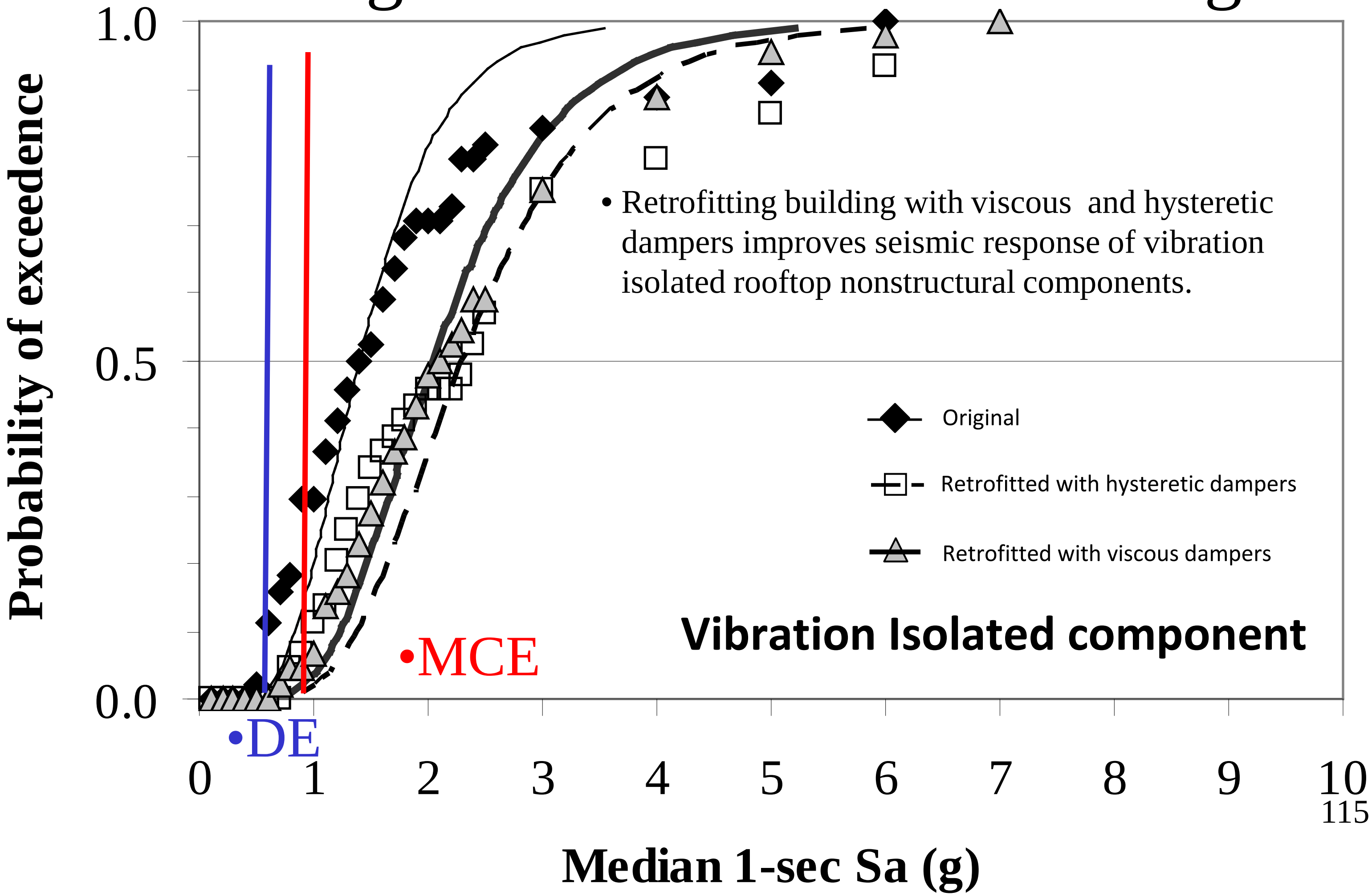
11. Performance-Based Design Example

- Nonstructural fragilities for physical damage at the roof level of original and retrofitted buildings.



11. Performance-Based Design Example

- Nonstructural fragilities for physical damage at the roof level of original and retrofitted buildings.



11. Performance-Based Design Example

- Final Comment
  - The simple direct analysis procedure illustrated in this example can be used to support the decision process of selecting supplemental damping systems for the performance-based seismic design or retrofit of a building taking into account the seismic performance of both structural and nonstructural components.

Questions/Discussions