Chapter 5
Metallic and Friction (Hysteretic) Dampers
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• Chapter 5
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1. Introduction

- Displacement-activated supplemental damping systems.
- Metallic dampers:
  - Take advantage of the hysteretic behavior of metals in the post-elastic range to dissipate energy.
- Friction dampers:
  - Dissipate the seismic energy by friction that develops at the interface between two sliding solid bodies.
- Both types of dampers exhibit hysteretic behavior idealized by elastic-perfectly plastic load-displacement relationship.
1. Introduction

Figure 5.1 Idealized Load-Displacement Relationship for Metallic and Friction Dampers
2. Basic Dynamic Structural Response

Equations of motion:

\[ m \ddot{x}(t) + c \dot{x}(t) + kx(t) + F_{hd}(t) = -m \ddot{x}_g(t) \]

Harmonic base motion:

\[ \ddot{x}_g = a_g \sin \omega_g t = -\frac{p_0}{m} \sin \omega_g t \]

Frequency ratio:

\[ \sigma = \frac{\omega_g}{\omega_0} \]

where

\[ \omega_0 = \sqrt{\frac{k + k_d}{m}} \]

Equivalent static displacement:

\[ x_{st} = \frac{p_0}{k} \]

Activation displacement:

\[ x_0 = \frac{F_{lat}}{k_d} \]

Response parameter:

\[ \Lambda_{hd} = \frac{x_{st}}{x_0} = \frac{k_d}{k} \frac{p_0}{F_{lat}} \]
2. Basic Dynamic Structural Response

Unbraced Frame Response
\( \Lambda_{d,e} = \frac{x_{st}}{x_0} = \frac{k_d}{k} \frac{p_0}{F_{lat}} \)

\[ k_d / k = 2.3 \]

Braced Frame Response
\( \Lambda_{d,e} = 0.01 \)
\( \Lambda_{d,e} = 0.05 \)
\( \Lambda_{d,e} = 0.15 \)

2.3
2. Basic Dynamic Structural Response

\[ \Lambda_{hd} = \frac{x_{st}}{x_0} = \frac{k_d}{k} \frac{P_0}{F_{lat}} \]

\[ k_d / k = 2.3 \]

Unbraced Frame Response (\( F_{lat} \) very small)

Braced Frame Response (\( F_{lat} \) very large)

CIE 626 - Structural Control
Chapter 5 – Hysteretic Dampers
2. Basic Dynamic Structural Response

• Addition of hysteretic dampers to a system has three possible effects:
  – Addition of supplemental damping without significantly modifying the dynamic properties of the system ($\Lambda_{hd} > 0.85$ in Fig. 5.3)
  – Addition of supplemental damping along with a modification of the dynamic properties of the system that optimizes the use of the added damper ($\Lambda_{hd} = 0.45$ in Fig. 5.3)
  – The addition of supplemental damping along with a significant effect on the dynamic properties of the system. This modification is the equivalent of adding a brace to the system ($\Lambda_{hd} < 0.15$ in Fig. 5.3)
3. Equivalent Linearization

- May be tempting to replace a nonlinear structure equipped with hysteretic dampers by a linear system with equivalent viscous damping.
- Equivalent viscous damping could be calculated from the amplitude of the nonlinear system at resonance.
- Equivalent linearization of the nonlinear system greatly simplifies the problem.
- Equivalent linearization used in current design and analysis guidelines for structures incorporating passive energy dissipation systems (ASCE 2000, BSSC 2003, ASCE 7-10).
- Equivalent linearization for structures incorporating hysteretic dampers should be used only for preliminary design and for estimating the dynamic response.
- Addition of equivalent viscous damping always causes reduction of the dynamic response of a single-degree-of-freedom system for any seismic input signal.
- Because of the nonlinear nature of actual hysteretic devices, the results obtained with the linear system with equivalent viscous damping can be non-conservative.
3. Equivalent Linearization

- Two-storey frame
- Natural period = 1.35 sec, inherent viscous damping = 2% of critical in each mode
- Structure rehabilitated at each level with an elastic-perfectly plastic hysteretic damper
- Lateral stiffness of damper equal lateral stiffness of frame, i.e. $k_d/k = 1$
- Natural period of rehabilitated structure before dampers are activated = 0.96 sec
- Lateral force corresponding to the onset of yielding (or slipping) of each damper equal to 30% of the weight of the structure at each floor level.
- Original frame and rehabilitated structure subjected to the S00E component of the 1940 El Centro record.
3. Equivalent Linearization

Figure 5.5 Response of Original and Rehabilitated Two-Storey Frame, a) Relative Displacement Time-Histories, b) First Floor Damper Hysteresis
3. Equivalent Linearization

Figure 5.6 Relative Displacement Response Spectra for El Centro Record with Associated Equivalent Periods of Vibration
3. Equivalent Linearization

- For equivalent linearization, pushover analysis of the rehabilitated structure is carried out.
- Lateral load distribution consistent with the first mode shape of the structure in fully braced condition: 38% of the total lateral load at first level and 62% at second level.
- Relationship between base shear and effective lateral displacement taken at 81% of the height of the structure corresponding to center of gravity of applied loading.
3. Equivalent Linearization

\[ k_{\text{eff}} = \frac{816}{0.115} = 7096 \text{ kN/m} \] \hspace{1cm} (5.8)

From Equation (2.9), the effective period of the structure \( T_{\text{eff}} \) can be computed using the effective lateral stiffness \( k_{\text{eff}} \) and the first modal weight of the structure \( W_1 \) equal to 95\% of its total weight:

\[ T_{\text{eff}} = 2\pi \sqrt{\frac{W_1}{g k_{\text{eff}}}} = 1.03s \] \hspace{1cm} (5.9)

The equivalent viscous damping ratio \( \zeta_{eq} \), provided by the hysteretic damper, can be computed by applying Equation (2.8). For a bi-linear system, such as shown in Figure 5.7, this expression can be reduced to:

\[ \zeta_{eq} = \frac{2(1-\alpha)(\mu_\alpha - 1)}{\pi \mu_\alpha (\alpha \mu_\alpha - \alpha + 1)} \] \hspace{1cm} (5.10)

where \( \alpha \) is the ratio of the lateral stiffness once the first level damper is activated to the initial lateral stiffness and \( \mu_\alpha \) is the global displacement ductility at maximum response. From Figure 5.7, these two quantities are computed as \( \alpha = 0.52 \) and \( \mu_\alpha = 1.55 \). (rough)

Substituting back in Equation (5.10), the equivalent viscous damping provided by the friction damper is computed as \( \zeta_{eq} = 8\% \) of critical. The initial equivalent viscous damping of the structure must then be added to obtain the total corrected equivalent viscous damping ratio \( \zeta_{eq} = 10\% \) of critical.

Figure 5.7 Push-Over Analysis of Rehabilitated Structure
3. Equivalent Linearization

- Response of linearized rehabilitated system 25% lower than response of nonlinear rehabilitated structure
- Linearized response even lower than elastic response of initial structure.
4. Study of Analogous Nonlinear Mechanical System

• Scope and Motivation:
  – Study steady-state response of an analogous nonlinear single-degree-of-freedom (SDOF) oscillator subjected to harmonic excitation.
  – When a structure is excited by seismic loading, large portions of its response may be characterized by a quasi-resonant state at its effective fundamental period of vibration.
  – Reveals non-dimensional parameters governing the response of simple structures equipped with hysteretic dampers.
  – Same parameters will be useful in developing a strategy for obtaining the load that activates the damper in order to minimize the seismic response of a structure.
4. Study of Analogous Nonlinear Mechanical System

- Derivation of Closed-Form Frequency Response Function

\[ \ddot{x}_g(t) = a_g \cos(\omega_g t) \]

Figure 5.8 a) Single Storey Hysteretically Damped Structure b) Analogous Nonlinear SDOF Oscillator
4. Study of Analogous Nonlinear Mechanical System

• Derivation of Closed-Form Frequency Response Function

\[ m \frac{d^2x(t)}{dt^2} + k_b \tilde{f}(x, u, t) = -ma_g \cos \omega_g t \]  \hspace{1cm} (5.12)

where \( \tilde{f}(x, u, t) \) is the hysteretic restoring force per unit initial stiffness \( k_b \) shown in Figure 5.9, and given by:

\[ \tilde{f}(x, u, t) = \begin{cases} 
  x & \text{if } 0 \leq x \leq x_0 \\
  x_0 + \alpha(x-x_0) & \text{if } x > x_0 
\end{cases} \]  \hspace{1cm} (5.13)

where \( x \) is the displacement of the mass relative to the moving base, \( x_0 \) is the lateral deflection required to activate the hysteretic damper, and \( \alpha \) is the post-activation stiffness slope ratio, i.e. the ratio of the lateral stiffness of the structure in its unbraced condition after the damper is activated \( k_u \) to the initial lateral stiffness of the structure in its fully braced condition before the damper is activated \( k_b \).
• Derivation of Closed-Form Frequency Response Function

Furthermore, a secondary stiffness parameter $u$ is defined in terms of the stiffness values through:

$$u = 1 - \frac{k_u}{k_b} = 1 - \alpha$$  (5.14)

Finally, the lateral deflection required to activate the hysteretic damper $x_0$ can be expressed as:

$$x_0 = \frac{F_{lat}}{k_d}$$  (5.15)

where $F_{lat}$ is the horizontal lateral load that activates the damper and $k_d$ is the horizontal lateral stiffness provided to the system by the bracing member containing the damper ($k_d = k_b - k_u$).

Note that the inherent damping of the original structure is neglected in order to simplify the analysis and also because it is assumed much smaller than the damping provided by the hysteretic damper.
• Derivation of Closed-Form Frequency Response Function

For the purpose of the derivation, the time variable $t$ is changed to an non-dimensional variable $\tau$ using:

$$\tau = \omega_b t$$ (5.16)

where $\omega_b$ is the natural circular frequency of the fully braced frame (before the damper is activated) given by:

$$\omega_b = \sqrt{\frac{k_b}{m}}$$ (5.17)

Carrying out the change of variable and dividing by $m$, Equation (5.12) becomes:

$$\omega_b^2 \frac{d^2 x(\tau)}{d\tau^2} + \omega_b^2 \tilde{f}(x, u, \tau) = -a_g \cos \sigma \tau$$ (5.18)

where $\sigma$ is the excitation frequency ratio:

$$\sigma = \frac{\omega_g}{\omega_b}$$ (5.19)
4. Study of Analogous Nonlinear Mechanical System

- Derivation of Closed-Form Frequency Response Function

To simplify the notation, the derivative with respect to \( \tau \) is denoted by the overdot symbol in the following development.

Dividing by \( \frac{\omega_b}{2} \), Equation (5.18) becomes:

\[
\ddot{x}(\tau) + f(x, u, \tau) = x_{st}\cos\sigma\tau
\]

(5.20)

where \( x_{st} \) is the static deflection defined by:

\[
x_{st} = \frac{-ma_g}{k_b}
\]

(5.21)

Assuming that a steady-state response of the system exists, by analogy to the linear case, the solution to Equation (5.20) is assumed to take the form:

\[
x(\tau) \approx A(\tau)\cos(\sigma\tau-\varphi(\tau))
\]

(5.22)
• Derivation of Closed-Form Frequency Response Function

Defining \( \theta = \sigma \tau - \varphi(\tau) \) and differentiating Equation (5.22) with respect to \( \tau \) yields:

\[
\dot{x}(\tau) = \dot{A}(\tau) \cos \theta - \sigma A(\tau) \sin \theta + \dot{\varphi}(\tau) A(\tau) \sin \theta
\]  \hspace{1cm} (5.23)

By analogy to Lagrange’s method of variation of a parameter, one may impose the additional restriction:

\[
\dot{A}(\tau) \cos \theta + \dot{\varphi}(\tau) A(\tau) \sin \theta = 0
\]  \hspace{1cm} (5.24)

Equation (5.24) also assures that the expression of \( \dot{x}(\tau) \) has the same form as in the linear case where \( \varphi \) and \( A \) are independent of \( \tau \):

\[
\dot{x}(\tau) = -\sigma A(\tau) \sin \theta
\]  \hspace{1cm} (5.25)

Differentiating again Equation (5.25) with respect to \( \tau \) we obtain:

\[
\ddot{x}(\tau) = -\sigma \dot{A}(\tau) \sin \theta - \sigma^2 A(\tau) \cos \theta + \sigma A(\tau) \dot{\varphi}(\tau) \cos \theta
\]  \hspace{1cm} (5.26)
• Derivation of Closed-Form Frequency Response Function

Substituting Equation (5.26) into Equation (5.20) we get:

\[- \sigma \dot{A}(\tau) \sin \theta + \sigma A(\tau) \dot{\phi}(\tau) \cos \theta - \sigma^2 A(\tau) \cos \theta + \tilde{f}(A \cos \theta, u, \tau) = x_{st} \cos(\theta + \phi(\tau)) \]

(5.27)

Equations (5.24) and (5.27) are used to define a system of two equations for two unknowns \( A(\tau) \) and \( \phi(\tau) \):

First, Equation (5.27) is multiplied by \( \sin \theta \) and Equation (5.24) is multiplied by \( \sigma \cos \theta \), and the two are subtracted:

\[- \sigma \dot{A}(\tau) - \sigma^2 A(\tau) \sin \theta \cos \theta + \tilde{f}(A \cos \theta, u, \tau) \sin \theta
\]

\[= x_{st} \cos(\theta + \phi(\tau)) \sin \theta \]

(5.28)

Then, Equation (5.24) is multiplied by \( \sigma \sin \theta \) and added to Equation (5.27) after the latter is multiplied by \( \cos \theta \):

\[\sigma A(\tau) \dot{\phi}(\tau) - \sigma^2 A(\tau) \cos^2 \theta + \tilde{f}(A \cos \theta, u, \tau) \cos \theta
\]

\[= x_{st} \cos(\theta + \phi(\tau)) \cos \theta \]

(5.29)
• Derivation of Closed-Form Frequency Response Function

Since $A(\tau)$ and $\varphi(\tau)$ are assumed to be slow-varying, they will remain almost constant during one cycle. The time dependence of these two variables can be lifted by taking the averages of Equations (5.28) and (5.29) over one cycle of vibration. In fact, since these two equations are verified for every value of $\tau$, they are also valid for the averages over one cycle. Equations (5.28) and (5.29) averaged over one cycle of $\theta$ yield:

$$-2\sigma \ddot{A} + \frac{1}{\pi} \int_0^{2\pi} f(A \cos \theta, u, \tau \sin \theta d\theta = x_{sf} \sin \varphi$$  \hspace{1cm} (5.30)

and

$$-2\sigma A \dot{\varphi} - \sigma^2 A + \frac{1}{\pi} \int_0^{2\pi} f(A \cos \theta, u, \tau \cos \theta d\theta = x_{sf} \cos \varphi$$  \hspace{1cm} (5.31)

where $A$, $\varphi$, $\dot{A}$ and $\dot{\varphi}$ denote the average values of $A(\tau)$, $\varphi(\tau)$, $\dot{A}(\tau)$ and $\dot{\varphi}(\tau)$ over one cycle.
Derivation of Closed-Form Frequency Response Function

Let

\[ S(A) = \frac{1}{\pi} \int_0^{2\pi} f(A \cos \theta, u, t) \sin \theta d\theta \]  

(5.32)

and

\[ C(A) = \frac{1}{\pi} \int_0^{2\pi} f(A \cos \theta, u, t) \cos \theta d\theta \]  

(5.33)

Equations (5.30) and (5.31) then become:

\[ -2\sigma \hat{A} + S(A) = x_{st} \sin \varphi \]

(5.34)

\[ -\sigma^2 A - 2\sigma A \dot{\varphi} + C(A) = x_{st} \cos \varphi \]

The evaluation of \( S(A) \) and \( C(A) \) is carried out by integrating the force-displacement response (hysteresis) by parts over each linear branch (see Figure 5.9).
• Derivation of Closed-Form Frequency Response Function

It has been shown (Caughey 1960) that:

\[
\bar{S}(A) = \begin{cases} 
0 & \text{for } \bar{A} < 1 \\
\left(-\frac{u}{\pi} \sin^2 \theta^*\right) & \text{for } \bar{A} > 1 
\end{cases}
\] (5.37)

and

\[
\bar{C}(A) = \begin{cases} 
1 & \text{for } \bar{A} < 1 \\
\frac{1}{\pi} \left(u \theta^* + (1 - u) \pi - \frac{u}{2} \sin 2 \theta^*\right) & \text{for } \bar{A} > 1 
\end{cases}
\] (5.38)

where

\[
\bar{C}(A) = \frac{C(A)}{A} \\
\bar{S}(A) = \frac{S(A)}{A} \\
\theta^* = \cos^{-1} \left(1 - \frac{2(x_0/x_{st})}{(A/x_{st})}\right) \\
\bar{A} = \frac{A}{x_0}
\]
• Derivation of Closed-Form Frequency Response Function

When the steady-state response is reached, the average values of the derivatives of $A(\tau)$ and $\phi(\tau)$ over one full cycle $\dot{A}$ and $\dot{\phi}$ are equal to zero. Recalling Equation (5.34), squaring and adding the two equations eliminates the terms in $\dot{\phi}$ and yields:

$$\sigma^2 = \frac{C(A)}{A} \pm \sqrt{\left(\frac{x_{st}}{A}\right)^2 - S^2(A)}$$

(5.40)

Substituting Equations (5.37) and (5.38) into Equation (5.40) leads to the following transcendental steady-state amplitude solution:

$$\sigma^2 = \begin{cases} 1 \pm \frac{x_{st}}{A} & \text{for } \bar{A} \leq 1 \\ \frac{1}{\pi} \left[u \theta^* + (1 - u) \pi - \frac{u}{2} \sin 2\theta^* \right] \pm \sqrt{\left(\frac{x_{st}}{A}\right)^2 - \left(\frac{u \sin^2 \theta^*}{\pi}\right)^2} & \text{for } \bar{A} > 1 \end{cases}$$

(5.41)
• Closed-Form Frequency Response Analysis

For any particular values of $u$ and $x_0/x_{st}$, $\sigma^2$ can be solved for specific values of $x_{st}/A$ using Equation (5.41). The maximum steady-state amplitude, corresponding to a resonance state, occurs where $\sigma$ has a double root, that is at the point where:

$$\left(\frac{x_{st}}{A}\right)^2 = \left(\frac{u \sin^2 \theta^*}{\pi}\right)^2$$  \hspace{1cm} (5.42)

Substituting Equation (5.39) into Equation (5.42) yields an explicit expression for the steady-state amplitude of the response at resonance:

$$\frac{A}{x_{st}} = \frac{4u}{\pi} \frac{(x_0/x_{st})}{\frac{4u}{\pi} - (x_{st}/x_0)}$$  \hspace{1cm} (5.43)

Since the steady-state amplitude $A$ is by definition positive, bounded response at resonance occurs provided that:

$$\frac{x_{st}}{x_0} < \frac{4u}{\pi}$$  \hspace{1cm} (5.44)
• Closed-Form Frequency Response Analysis

Substituting Equations (5.14), (5.15) and (5.21) into (5.44) yields the following condition on the activation load of the hysteretic damper for bounded response at resonance:

\[
\frac{F_{\text{lat}}}{W} \geq \frac{\pi}{4} \frac{a_g}{g}
\]  

(5.45)

where \( W \) is the seismic weight of the system and \( g \) the acceleration of gravity. From Equation (5.40), the resonant frequency ratio \( \sigma_r^2 \) is given by:

\[
\sigma_r^2 = \bar{C}(A)
\]  

(5.46)

\[
x_0 = \frac{F_{\text{lat}}}{k_d}
\]  

(5.14)

\[
x_{st} = \frac{-ma_g}{k_b}
\]  

(5.15)

\[
u = 1 - \frac{k_u}{k_b}
\]  

(5.21)
Closed-Form Frequency Response Analysis

The value of the activation load of the hysteretic damper minimizing the amplitude of motion at resonance can be obtained from (see Equation (5.43)):

$$\frac{\partial(A/x_{st})}{\partial(x_0/x_{st})} = 0$$

which leads to:

$$\frac{x_{st}}{x_0} = \frac{2u}{\pi}$$

Substituting Equations (5.14), (5.15), and (5.21) into Equation (5.48) provides a closed form expression for the lateral load required to activate the damper $F_{lat}^*$ that minimizes the resonant amplitude:

$$\frac{F_{lat}^*}{W} = \frac{\pi a_g}{2g}$$
• Closed-Form Frequency Response Analysis

The minimum resonant amplitude \( A^* \) can be obtained by substituting Equation (5.48) into Equation (5.43):

\[
\frac{A^*}{x_{st}} = \frac{2x_0}{x_{st}} = \frac{\pi}{u} = \frac{\pi k_b}{k_b - k_u}
\]

Equation (5.50) also reveals that the hysteretic damper will always be activated at resonance regardless of the value of the activation load used, since \( A^* > x_0 \). The frequency ratio \( \sigma^*_r \) at which this optimum resonance occurs can be obtained by substituting Equations (5.48) and (5.50) into Equation (5.46):

\[
\sigma^*_r = \sqrt{\frac{1}{2} + \frac{\omega_u}{2\omega_b}}
\]

where:

\[
\omega_u = \sqrt{\frac{k_u}{m}} \quad \text{and} \quad \omega_b = \sqrt{\frac{k_b}{m}}
\]
• Closed-Form Frequency Response Analysis

The condition for which the amplitude of the response is a minimum for a particular excitation frequency $\omega_g$ (other than resonance) cannot be found analytically since Equation (5.41) is transcendental and $A/x_{st}$ cannot be solved for every frequency ratio $\sigma$. However, an analysis of the stability of the steady-state solution of a bilinear hysteretic system (Caughey 1960) shows that this system is always stable and that the families of frequency response curves that can be generated from Equation (5.41) are all single valued. Hence a jump phenomenon, which is characteristic of certain nonlinear systems (Nayfeh and Mook 1979), is not expected to occur in this case. Therefore, Equation (5.41) can be symbolically inverted, such as:

$$\frac{A}{x_{st}} = G\left(\frac{\omega_0}{\omega_b}, \frac{x_0}{x_{st}}, u\right)$$

(5.53)

where $G$ is a single valued function.
4. Study of Analogous Nonlinear Mechanical System

• Closed-Form Frequency Response Analysis

The value of the activation load of the hysteretic damper that minimizes the amplitude of the response at any forcing frequency can be obtained by:

$$\frac{\partial G}{\partial (x_0/x_{st})} = 0$$

(5.54)

which yields a condition such as:

$$\frac{x_0}{x_{st}} = H\left(\frac{\omega_g}{\omega_b}, u\right)$$

(5.55)

where $H$ is also a single valued function.
• Closed-Form Frequency Response Analysis

Substituting Equations (5.14), (5.15) and (5.21) into Equation (5.55) yields a symbolic expression for the optimum value of the activation load of the hysteretic damper $F_{lat}^{opt}$:

$$
\frac{F_{lat}^{opt}}{W} = \frac{a_g}{g} \left(1 - \frac{k_u}{k_b}\right) H\left(\frac{\omega_g}{\omega_b}, \left(1 - \frac{k_u}{k_b}\right)\right)
$$

(5.56)

or

$$
\frac{F_{lat}^{opt}}{W} = \frac{a_g}{g} Q\left(\frac{T_b}{T_g}, \frac{T_b}{T_u}\right)
$$

(5.57)

where $Q$ is another single valued function and:

$T_b = 2\pi/\omega_b$ is the natural period of the fully braced frame;
$T_u = 2\pi/\omega_u$ is the natural period of the unbraced frame;
$T_g$ is the period of the ground motion;
$a_g$ is the peak ground acceleration;
g is the acceleration of gravity, and
$W$ is the seismic weight of the structure.

$$
\begin{align*}
x_0 &= \frac{F_{lat}}{k_d} \\
x_{st} &= \frac{-ma_g}{k_b} \\
u &= 1 - \frac{k_u}{k_b}
\end{align*}
$$

(5.14) (5.15) (5.21)
• **Closed-Form Frequency Response Analysis**
  
  – **Significance of Equation (5.57):**

  \[
  \frac{F_{lat}^{opt}}{W} = \frac{a_g}{g} Q\left(\frac{T_b}{T_g}, \frac{T_b}{T_u}\right)
  \]

  • Reveals non-dimensional parameters governing the optimum activation load of hysteretic damper for a single storey hysteretically damped structure excited by a harmonic ground motion.
  
  • Can be expected that same non-dimensional parameters will also play an important role in optimum response of a structure excited by a general earthquake ground motion.
  
  • Optimum activation load of a hysteretic damper depends on the frequency and amplitude of ground motion and is not strictly a structural property.
  
  • Anticipated ground motion must be carefully characterized in the design of structures equipped with hysteretic dampers.
  
  • Optimum activation load of hysteretic damper is linearly proportional to the peak ground acceleration.
4. Study of Analogous Nonlinear Mechanical System

- Numerical Verification of Closed-Form Frequency Response Function
  - Closed form solutions obtained in previous section verified by computing numerically steady-state response of a single storey hysteretically damped structure

Table 5-1: Physical Properties of Single Storey Hysteretically Damped Structure

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seismic weight</td>
<td>$W = 445$ kN</td>
</tr>
<tr>
<td>Lateral stiffness of fully braced frame</td>
<td>$k_b = 1521$ kN/m</td>
</tr>
<tr>
<td>Lateral stiffness of unbraced frame</td>
<td>$k_u = 454$ kN/m</td>
</tr>
<tr>
<td>Peak ground acceleration</td>
<td>$a_g = 0.05$ g</td>
</tr>
</tbody>
</table>

1. Based on these properties, the values of $T_b = 1.08$ s, $T_u = 1.99$ s, $u = 0.70$ and $x_{st} = 14.1$ mm can be computed. Table 5-2 presents the different values of the activation load of
• Numerical Verification of Closed-Form Frequency Response Function

Table 5-2: Lateral activation loads of the hysteretic damper in the numerical verification

<table>
<thead>
<tr>
<th>$F_{lat}$ (kN)</th>
<th>$x_{st}/x_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.5 *</td>
<td>0.89</td>
</tr>
<tr>
<td>25.7</td>
<td>0.61</td>
</tr>
<tr>
<td>35.0 **</td>
<td>0.45</td>
</tr>
<tr>
<td>42.9</td>
<td>0.36</td>
</tr>
<tr>
<td>51.4</td>
<td>0.30</td>
</tr>
<tr>
<td>60.0</td>
<td>0.26</td>
</tr>
</tbody>
</table>

* Minimum activation load for bounded response at resonance (Equation (5.45))
** Activation load minimizing amplitude at resonance (Equation (5.49))
4. Study of Analogous Nonlinear Mechanical System

- Numerical Verification of Closed-Form Frequency Response Function
  - Hysteretically damped system excited by a series of harmonic ground acceleration time-histories having constant amplitude and different forcing frequencies.
  - Integration time-step of 0.01 s
- Numerical Verification of Closed-Form Frequency Response Function

Figure 5.10 Frequency Response Functions for Hysteretically Damped Structures
• Numerical Verification of Closed-Form Frequency Response Function

Figure 5.11 Frequency Response Functions for Three Values of Hysteretic Damper Activation Load
• Numerical Verification of Closed-Form Frequency Response Function

![Graph]

**Figure 5.12** Optimum Lateral Activation Load of Hysteretic Damper for Harmonic Ground Motion
4. Study of Analogous Nonlinear Mechanical System

• Summary

- A lower bound value of the activation load of the hysteretic damper has been established such that bounded amplitudes occur at resonance (Equation (5.45))
- The value of the activation load of the hysteretic damper minimizing the amplitude at resonance has been determined along with the frequency at which this optimum resonance occurs (Equations (5.49) and (5.51))
- The system is always stable and a jump phenomenon does not occur
- The non-dimensional parameters governing the value of the optimum activation load of the hysteretic damper minimizing the steady-state amplitude for any forcing frequency were determined (Equation (5.57)).
- Similar parameters will also influence the optimum activation load distribution of the array of hysteretic dampers distributed in multi-storey structures under general earthquake loading. These parameters clearly indicate that the optimum activation load of an hysteretic damper is a function of the amplitude and frequency of the ground motion and cannot strictly be defined as a structural property.
5. Metallic Dampers

- Hysteretic Behaviour of Yielding Steel Elements

Figure 5.13 Cyclic Stress-Strain Hysteresis of Steel Elements
5. Metallic Dampers

- Geometrical Considerations
  - Metallic dampers can be used as part of chevron bracing systems
  - Yielding devices dissipate energy through relative horizontal displacement between apex of chevron and above floor level
  - If metallic plates used, act as fixed-fixed beams
  - To maximize energy dissipation, desirable that plastic moment at any section be reached simultaneously
  - Geometry of the device must be optimized

![Figure 5.14 Use of Yielding Devices as Part of Chevron Bracing Systems](image-url)
5. Metallic Dampers

- Geometrical Considerations

Consider first a damper made of a single steel plate with a constant width \( b_0 \) and a variable depth \( d(x) \) as illustrated in Figure 5.15a. When the plastic moment is reached at the ends of the device the bending moment at any section of the device \( M(x) \) is given by:

\[
M(x) = \frac{2M_{po}}{h} \times = \left( \frac{2}{h} \right) \left( \frac{b_0 d_0^2}{4} \right) F_y x, \text{ for } -\frac{h}{2} \leq x \leq \frac{h}{2}
\]  

(5.59)

where \( M_{po} \) is the plastic moment at the ends of the device, \( h \) is the height of the plate, \( d_0 \) is the depth at the ends of the plate, \( x \) is the distance taken from the mid-height of the plate and \( F_y \) is the yield strength of the steel.

The plastic moment at any section of the plate \( M_p(x) \) is

\[
M_p(x) = \frac{b_0 d(x)^2}{4} F_y
\]  

(5.60)
5. Metallic Dampers

To insure simultaneous plastic yielding at all sections, when the plastic moment is reached at the ends of the plate, the moment acting at every section must be equal to the plastic moment of that section. This is achieved by equating Equation (5.59) to Equation (5.60), while neglecting the effect of the axial and shear forces acting on the element. Therefore the variation of the depth of the metallic damper to maximize energy dissipation is given by:

\[ d(x) = d_0 \sqrt{\frac{2x}{h}} \]  

(5.61)

This optimum geometry is illustrated in Figure 5.15a. Obviously, from a practical point of view, the depth of the plate can not be zero at mid-height since the plate must also carry the corresponding shear:

\[ V = \frac{2M_{p0}}{h} \]  

(5.62)
5. Metallic Dampers

Now let's consider a metallic damper made of a single steel plate with a constant depth \(d_0\) and a variable width \(b(x)\). The plastic moment at any section of the plate \(M_p(x)\) is given by:

\[
M_p(x) = \frac{b(x) d^2}{4} F_y
\]  

(5.63)

Again, to insure plastic yielding simultaneously at all sections, Equation (5.59) is equated to Equation (5.63); the optimum variation of the depth of the plate is then given by:

\[
b(x) = \frac{2x}{h} b_0
\]  

(5.64)

As shown in Figure 5.15b, this optimum geometry results in a linear variation of the width along the height of the plate. Again, from a practical point of view, the width of the device can not be zero at mid-height, as shown in the photograph on the left side of Figure 4.1.
5. Metallic Dampers

• Geometrical Considerations

In practice, the second geometry is preferred with $d_0$ much smaller than $b_0$ to minimize the possibilities of local buckling and to minimize the thickness of the steel required for uniform yielding along the height of the plate.
5. Metallic Dampers

- Experimental Studies
  - Added Damping - Added Stiffness Systems (ADAS)
    - ADAS device originally manufactured by Bechtel Corporation is an evolution of earlier Xplate devices used as damping supports for piping systems (Stiemer et al. 1981).
    - Geometry of the ADAS incorporates several interconnected yielding plates in series.

![Figure 4.1 Example of Metallic Damping System: ADAS Damper](image)
5. Metallic Dampers

- Experimental Studies
  - Added Damping - Added Stiffness Systems (ADAS)
    - Component Testing (Bergman and Goel 1987, Whittaker et al. 1991)
      - ADAS elements capable of sustaining 100 loading cycles at three times the measured yield displacement without signs of degradation.
      - ADAS elements can be safely designed for 10 times the

![Hysteresis Loop Graph](image)

Figure 5.16 Hysteresis Loops of ADAS Elements in a Plane Frame with Gravity Loads (after Whittaker et al. 1991)
5. Metallic Dampers

• Experimental Studies
  – Added Damping - Added Stiffness Systems (ADAS)
    • System Testing (Whittaker et al. 1991)

Figure 5.17 Geometry of ADAS Test Structure (from Whittaker et al. 1991, reproduced with the permission of the Earthquake Engineering Research Institute)

<table>
<thead>
<tr>
<th></th>
<th>1st Mode</th>
<th>2nd Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMRSF</td>
<td>Period</td>
<td>Damping ratio (%)</td>
</tr>
<tr>
<td></td>
<td>0.74</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>2.10</td>
<td>1.30</td>
</tr>
<tr>
<td>ADAS</td>
<td>Period</td>
<td>Damping ratio (%)</td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>3.40</td>
<td>1.70</td>
</tr>
</tbody>
</table>
5. Metallic Dampers

- Experimental Studies
  - Added Damping - Added Stiffness Systems (ADAS)
    - System Testing (Whittaker et al. 1991)
5. Metallic Dampers

- Experimental Studies
  - Added Damping - Added Stiffness Systems (ADAS)
    - System Testing (Whittaker et al. 1991)

![Graph showing response comparison for ADAS Test Structure, 1985 Llollelo, Chile (N10E) Earthquake (after Whittaker et al. 1991, reproduced with the permission of the Earthquake Engineering Research Institute).]
5. Metallic Dampers

- **Experimental Studies**
  - Added Damping - Added Stiffness Systems (ADAS)
    - System Testing (Whittaker et al. 1991)

![Hysteresis Loops of ADAS Elements, ADAS-3 Frame, 1940 El Centro S00E](image)

1 inch = 25.4 mm  
1 kip = 4.45 kN

*Figure 5.19 Hysteresis Loops of ADAS Elements, ADAS-3 Frame, 1940 El Centro S00E (from Whittaker et al. 1991)*
5. Metallic Dampers

• Experimental Studies
  – Added Damping - Added Stiffness Systems (ADAS)
    • System Testing (Whittaker et al. 1991)

![Graph showing energy time-histories](image)

Figure 5.20 Energy Time-Histories, ADAS-3 Frame, 1940 El Centro S00E (from Whittaker et al. 1991)

1 inch = 25.4 mm
1 kip = 4.45 kN

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Chapter 5 – Hysteretic Dampers
5. Metallic Dampers

• Experimental Studies
  – Triangular Added Damping Added Stiffness (TADAS) Systems
    • Developed by Tsai et al. (1993).
    • Variation of ADAS system using triangular metallic plate dampers.
    • Triangular plates rigidly welded to a top plate but simply connected to a slotted base.
    • Main advantages of TADAS:
      – Not affected by gravity loads because of slotted holes in base plate.
      – No rotational restraint required at the top of the brace connection assemblage.
    • Disadvantages of TADAS:
      – Construction more complicated.
      – Careful welding required.
5. Metallic Dampers

- **Experimental Studies**
  - **Cast Steel Yielding Fuse For Concentrically Braced Frames**
    - Developed by Gray et al. (2010) at the University of Toronto, Canada.
    - Ductile cast steel connector in a concentrically braced frame.
    - Seismic energy dissipated through inelastic flexural yielding of specially designed yielding elements similar to TADAS elements in the cast connector.
5. Metallic Dampers

- Experimental Studies
  - Cast Steel Yielding Fuse For Concentrically Braced Frames
    - Main advantages:
      - No welding or bolting in the connector.
      - Stable hysteretic response with tension stiffening at large displacements.
    - Main disadvantage:
      - Cost?
    - Tested at full-scale at the University of Toronto.
The *Cast ConneX® Scorpion™ Yielding Brace System™* (YBS) is a special class of concentrically braced frame. Just as in Special Concentrically Braced Frames (SCBF; AISC) / Moderately Ductile Concentrically Braced Frames (Type-MD CBF; CSA), the centerlines of YBS members that meet at a joint intersect at a point to form a complete vertical truss system that resists lateral forces. YBS have more ductility and energy absorption than SCBF / Type-MD CBF because overall brace buckling, and its associated strength degradation, is eliminated.

A YBS is composed of columns, beams and bracing elements, all of which are subjected primarily to axial forces. Braces of YBS are composed of a *Cast ConneX® Scorpion Yielding Connectors™* connected to the end of a conventional W-Shape or HSS member. Scorpion Yielding Connectors have specially designed fingers which are intended to yield under seismic loading to dissipate energy while all other elements of the braced frame remain essentially elastic.

Engineers employing this system select Scorpion Yielding Connectors based on the desired activation load for the brace and then select a conventional W-Shape or HSS brace element based on capacity design requirements and on the desired axial stiffness of the brace assembly. In so doing, the yield force and elastic stiffness of each brace comprising a YBS can be independently tuned with this unique bracing system.

An additional benefit of the system is its unique post-yield response. At large deformations, *Cast ConneX® Scorpion Yielding Braces™* exhibit post-yield strengthening and stiffening due to second-order geometric effects. This stiffening behavior allows for a better distribution of yielding in braces over a building’s height at large drift levels. In the event that deformations begin to collect in a single story, the system’s post-yield stiffening and strengthening will cause braces in adjacent stories to be activated, thereby reducing the likelihood of the formation of a “soft-story”. Neither of these advantages are available in systems which exhibit little or no post-yield stiffness.

The combination of high axial stiffness and low activation force also makes the YBS ideal for retrofitting existing seismically deficient
5. Metallic Dampers

- Experimental Studies
  - Lead Extrusion Devices (LED)
    - Developed in the mid-1970s in New Zealand (Robinson and Greenbank 1976).
    - Metallic dampers that take advantage of extrusion of lead through orifices.
    - Two different types of LED devices: constricted tube and bulged shaft.
5. Metallic Dampers

- Experimental Studies
  - Lead Extrusion Devices (LED)

- Component Testing

Figure 5.23 Typical Hysteretic Behaviour of LED Dampers at 1 cm/min:
  a) Constricted Tube: 15 kN, b) Constricted Tube: 150 kN c) Bulged Shaft: 30 kN, d) Bulged Shaft: 170 kN (from Robinson and Greenback 1976, copyright John Wiley & Sons Ltd., reproduced with permission)
5. Metallic Dampers

• Experimental Studies
  – Lead Extrusion Devices (LED)

  • Desirable characteristics of LED dampers:
    – Hysteretic behaviour is stable and repeatable and is unaffected by number of load cycles.
    – Environmental factors have no significant influence on the behaviour.
    – Fatigue is not a major concern since lead is hot worked at room temperature.
    – Strain rate has only a minor effect on the hysteretic response.
    – Tests

  ![Figure 5.24 Insignificant Aging Effect on LED Dampers (after Robinson and Cousins 1987)](image-url)
5. Metallic Dampers

- Experimental Studies
  - Buckling Restrained Braces (BRB) / Unbonded Braces
    - Originally manufactured by Nippon Steel Corporation in Japan.
    - Steel core plate encased in a steel tube filled with concrete.
    - Steel core carries the axial load while the outer tube, via the concrete, provides lateral support to the core and prevents global buckling.
    - Thin layer of lubricating material at the concrete interface.
    - Cyclic qualification tests included in AISC Seismic Requirements.

Figure 5.27 Components of an Unbonded Brace
5. Metallic Dampers

- Experimental Studies
  - Buckling Restrained Braces (BRB) / Unbonded Braces

Figure 5.28 Hysteretic Response of Unbonded Brace (from Black et al. 2004, reproduced with permission from the American Society of Civil Engineers)
4. Metallic Dampers

- **Experimental Studies**
  - Buckling Restrained Braces (BRB) in Propped Rocking Wall System (Nicknam and Filiatrault, 2012)
4. Metallic Dampers

- Experimental Studies
  - Buckling Restrained Braces (BRB) in Propped Rocking Wall System (Nicknam and Filiatrault, 2012)
5. Metallic Dampers

- Structural Implementations

**APPENDIX A: Implementation of Metallic Dampers in Structures. [N]: New Construction, [R]: Retrofit**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Location</th>
<th>Year</th>
<th>Damper Type</th>
<th>Number of Dampers</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rangitiki Bridge [R-N]</td>
<td>New Zealand</td>
<td>1980</td>
<td>Torional Beam</td>
<td>10</td>
<td>Skinner et al. 1980</td>
</tr>
<tr>
<td>Chimney [R]</td>
<td>Christchurch, New Zealand</td>
<td>---</td>
<td>Tapered Plate</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Motorway Overbridge [R-N]</td>
<td>Dunedin, New Zealand</td>
<td>---</td>
<td>Tapered Plate</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Cromwell Bridge [R-N]</td>
<td>New Zealand</td>
<td>---</td>
<td>Fensal Beam</td>
<td>6</td>
<td>---</td>
</tr>
<tr>
<td>Shpping Highways [R-N]</td>
<td>Wellington, New Zealand</td>
<td>---</td>
<td>Lead Extrusion</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5/9 Story Buildings (connected) [R-N]</td>
<td>Japan</td>
<td>1987</td>
<td>Bell Damper</td>
<td>3</td>
<td>Kobori et al. 1988</td>
</tr>
<tr>
<td>Steel Suspension Building [R-N]</td>
<td>Naples Italy</td>
<td>1990</td>
<td>Tapered Plate</td>
<td>---</td>
<td>Ciampe 1993</td>
</tr>
<tr>
<td>Irazaga #30-40 Building [R]</td>
<td>Mexico City, Mexico</td>
<td>1990</td>
<td>ADA5</td>
<td>250</td>
<td>Martinez-Romero 1993</td>
</tr>
<tr>
<td>Cardiology Hospital [R]</td>
<td>Mexico City, Mexico</td>
<td>1990</td>
<td>ADA5</td>
<td>90</td>
<td>Martinez-Romero 1993</td>
</tr>
<tr>
<td>Reforma #475 Building [R]</td>
<td>Mexico City, Mexico</td>
<td>1992</td>
<td>ADA5</td>
<td>400</td>
<td>Martinez-Romero 1993</td>
</tr>
</tbody>
</table>
5. Metallic Dampers

- Structural Implementations
  Wells Fargo Bank, San Francisco, CA
  - Two-story non-ductile concrete frame.
  - Constructed in 1967.
  - Damaged during the 1989 Loma Prieta Earthquake.
  - Voluntary upgrade with chevron braces and ADAS devices.
  - Conventional retrofit rejected because of foundation work.
  - Seven ADAD devices, each with a yield force of 150 kips.
5. Metallic Dampers

• Structural Implementations

<table>
<thead>
<tr>
<th>Structure</th>
<th>Location</th>
<th>Year</th>
<th>Damper Type</th>
<th>Number of Dampers</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Osaka Int. Conf. Centre [N]</td>
<td>Osaka, Japan</td>
<td>2000</td>
<td>Unbonded</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>UC-San Francisco IBBQB Center [N]</td>
<td>San Francisco, California</td>
<td>2000</td>
<td>Unbonded</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>UC-Davis, Enr Sciences Bldg. [N]</td>
<td>Berkeley, California</td>
<td>Year</td>
<td>Unbonded</td>
<td>---</td>
<td>Clark et al. 2003</td>
</tr>
<tr>
<td>San Bernardino Library [N]</td>
<td>San Bernardino, California</td>
<td>2003</td>
<td>Unbonded</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Taipei County Building [N]</td>
<td>Taipei, Taiwan</td>
<td>2003</td>
<td>Unbonded</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Matsushita YRP Research Lab. [N]</td>
<td>Yokosuka City, Japan</td>
<td>2003</td>
<td>Unbonded</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
5. Metallic Dampers

- Structural Implementations

  Unbonded Braces in New 4-story Central Dining Facility
  Stanley Hall
  UC Berkeley, 2003

Photo: Courtesy of M. Constantinou
6. Friction Dampers

- Dissipate seismic energy by friction that develops between two solid bodies sliding relative to one another.
- Variety of devices developed.
- Macroscopic modeling of friction dampers is similar to the modeling of metallic devices, slip load is considered as an equivalent yield force.
6. Friction Dampers

• Basic Principles of Solid Coulomb Friction
  – Based on three assumptions validated experimentally:
    • Total frictional force is independent of the apparent surface area of contact.
    • Frictional force proportional to normal force acting across the sliding interface.
    • For low relative velocities, frictional force is independent of sliding velocity.

Therefore during slipping, the relation between the frictional force $F_f$ acting tangentially within the interfacial plane in the direction opposing the motion and the normal force $N$ can be expressed as:

$$F_f = \mu N$$  \hspace{1cm} (5.65)

where $\mu$ is the coefficient of friction.
6. Friction Dampers

- Basic Principles of Solid Coulomb Friction
  - Although frictional forces are simple to measure or calculate based on Equation (5.65), the frictional phenomena in sliding interfaces are multiple and complex.
  - Factors influencing friction force:
    - True contact area:
      - Shape and contour of faying surfaces, way asperities on faying surfaces deform and adhere under normal pressure, role of surface films, and how energy is lost when the surfaces are deformed during sliding.
    - Normal pressure:
      - $\mu$ and $N$ not independent.
    - Sliding velocity:
      - $\mu$ varies with sliding velocity.
6. Friction Dampers

- Basic Principles of Solid Coulomb Friction
  - Two kinds of frictional forces:
    - Static frictional force
    - Sliding frictional force

Figure 5.29 Typical Frictional Force-Sliding Displacement Relation (after Kim et al. 2004)
6. Friction Dampers

• Basic Principles of Solid Coulomb Friction
  – Three components contribute to the work done at interface between two sliding surfaces:
    • i) adhesion component along the interface,
    • ii) ploughing component in the bulk zone, and
    • iii) presence of third bodies such as contaminants or wearing debris (Bowden et al. 1973).
  – These components were first suggested to explain friction between metallic surfaces and further extended to polymers.
6. Friction Dampers

- Basic Principles of Solid Coulomb Friction
  - i) Adhesion Component:

  ![Image of Asperities and Junctions in Sliding Interface](image)

  (a) Before applied normal pressure
  (b) After applied normal pressure

  Figure 5.30 Asperities and Junctions in Sliding Interface

The frictional force due to adhesion $F_{fa}$ can be rewritten in terms of the real contact area and the shear strength of the frictional material as (Bowden et al. 1973, Constantinou et al. 1999):

$$F_{fa} = sA_r$$

(5.66)

where $s$ is the shear strength of the junctions and $A_r$ is the true contact area.
6. Friction Dampers

- Basic Principles of Solid Coulomb Friction
  - ii) Ploughing Component:
    - During sliding, one body must lift over the roughness of the other.
    - Both positive and negative slopes between frictional bodies coexist.

![Figure 5.31 Slopes at Contact Points](image)

- Asperities located on the harder frictional material dig into the surface of the softer one while junctions are formed in other asperities.
- Sharp edges on the surface of the harder frictional material make scores or grooves on the surface of the softer one.
- Debris from scores or grooves accumulate at the ploughing edge.
6. Friction Dampers

• Basic Principles of Solid Coulomb Friction
  – iii) Third Bodies Component:
    • Third bodies, such as contaminants or wearing debris, and cold welding affect frictional properties.
    • Difficult to predict the contribution of third bodies on the friction properties because it is dependent on the shape and strength of third bodies as well as the roughness of faying surfaces.
    • If third bodies can remain between asperities in high roughness faying surfaces: reduce the degree of roughness of the faying surface and increase the true contact area.
    • If contaminants are round in shape and made of strong material: facilitate sliding “rolling friction” phenomenon.
    • Sharp contaminants: cause plowing and increase frictional force.
    • In general, it is observed that contaminants increase the frictional force.
6. Friction Dampers

• Stick-Slip Motion
  – Stick-Slip (or jerky) motion at a sliding interface.
  – Arises whenever static coefficient of friction is markedly greater than the kinetic coefficient of friction.
  – Upon reversal of motion, the frictional interface undergoes a momentary stop (movement changes direction).
  – Upon initiation of motion in the reverse direction, the static frictional force, which is usually larger than the sliding friction, is mobilized.
  – Subsequently, irregular stick-slip motion takes place when the frictional force drops with increasing displacement then increases due to the increase in sliding velocity.
6. Friction Dampers

- **Wear Phenomenon**
  - Removal of material from a solid surface as a result of a mechanical action.
  - Characterized by the amount of material removed during sliding.
  - Classified in four categories:
    - i) adhesive wear
    - ii) abrasive wear
    - iii) corrosive wear
    - iv) surface fatigue wear
6. Friction Dampers

- Wear Phenomenon
  - Adhesive wear
    - During sliding, fragments are disassembled from their sliding surface and adhere to the other surface.
    - Fragments under continued sliding action, may come off the surface on which they adhere and be transferred back to the original surface or be removed completely from the sliding interface.
    - Particles which adhere to the other surface due to adhesive wear can change the characteristics of friction of the sliding interface since the surfaces tend to become rougher.
    - Universal in nearly all mechanical sliding systems and can not be eliminated but only reduced.
    - Proportional to the normal pressure and to the travel length and inversely proportional to the hardness of the surface.
6. Friction Dampers

• Wear Phenomenon
  – Abrasive wear
    • Occurs when a rough harder surface, or a softer surface containing hard particles slides on a softer surface.
    • Hard particles plow the soft surface.
    • Abrasive wear is closely related to the plowing phenomena.
  – Corrosive wear and surface fatigue wear
    • Occur in a corrosive environment and in sliding interfaces subjected to repeated sliding.
    • Sliding action wears a corroded or fatigued surface film away.
    • Can be eliminated by good preparation of faying surfaces such as clean smooth surfaces without hard particles or the use of surfaces with higher corrosion resistance.
6. Friction Dampers

- **Effect of Normal Load on Sliding Interfaces**
  - Most friction experiments carried out in special testing machines using actuators for applying normal load (e.g. Bondonet and Filiatrault 1997, Constantinou et al. 1999, Wolff 1999).
  - Normal load is assumed uniformly distributed on the sliding interface and is constant during the tests.
  - Assumption usually not true for sliding surfaces where normal force applied through bolt preload.
  - Variation of the bolt preload causes a change of slip resistance and unstable hysteresis loops.
  - Amount of dissipated energy by friction can be unpredictable.
6. Friction Dampers

- Effect of Normal Load on Sliding Interfaces
  - Bolt Behaviour under Preload
    - Bolt preload usually introduced by torquing (turn-of-nut method).
    - Behaviour of bolts subjected to the preload governed by threaded part.
    - Load versus elongation characteristics of a bolt more significant than stress versus strain relationship of fastener metal itself (Kulak et al. 2001, 2002).

Figure 5.33 Bolt Preload of 22-mm Torqued A325 and A490 Bolts

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6. Friction Dampers

• Effect of Normal Load on Sliding Interfaces
  – Bolt Behaviour under Preload
    • Long-term effects of bolt preload difficult to predict in seismic applications.
    • Relaxation of bolt preload is most important long-term effect.
      – Depends on stress level, grip length, and number of plies.
      – Concentrates on the threaded part.
      – Relaxation increases as grip length is decreased.
      – Increasing number of plies increases bolt relaxation.
6. Friction Dampers

• Effect of Normal Load on Sliding Interfaces
  – Effect of Bolt Preload on Sliding Interfaces
    • Main components influencing elongation of a bolt and bolt preload:
      – wear, temperature rise and accumulation of debris or contaminants.
    • Wear:
      – Decreases the elongated bolt length and bolt preload.
    • Temperature rise:
      – Causes expansion of connected steel plates causing increase of bolt length and bolt preload if the bolt is in the elastic range.
    • Debris or contaminants:
      – Causes increase of bolt length and bolt preload if the bolt is in the elastic range.
    • Poisson’s effect and initial non-uniformity in thickness of connected plates can also represent sources of variation of bolt preload (Tremblay and Stiemer 1993).
6. Friction Dampers

- Effect of Normal Load on Sliding Interfaces
- Effect of Bolt Preload on Sliding Interfaces

![Diagram of friction damper](image)

Figure 5.34 Model of the assembly of bolted connection (after Tremblay and Steiner 1993)

In Figure 5.34(b), $\Delta_{bi}$ is the initial elongation of the bolt when it is tightened and the bolt preload at this point is $P_u$. When only the wearing effect governs the elongation of the bolt, the elongation becomes $\Delta_{bw}$ which is smaller than the initial elongation of the bolt $\Delta_{bi}$ and the corresponding bolt preload is $P_w$ which is smaller than the initial bolt preload. However, temperature rise does not influence the bolt preload even if the elongation of the bolt increases to $\Delta_{bt}$. If there is an accumulation of debris instead of wear with temperature rise, the elongation of the bolt further increases to $\Delta_{btd}$. But the bolt preload is still $P_u$. On the other hand, if the wear exceeds the accumulation of debris and the wearing effect is larger than the temperature rise effect, the elongation of the bolt decreases to $\Delta_{bwt}$ and the bolt preload also decreases to $P_w$. It is therefore inferred that the variation of bolt preload can be minimized by reducing the effect of wear.
6. Friction Dampers

- Effect of Normal Load on Sliding Interfaces
  - Effect of Normal Pressure with Bolt Pre-load
    - Distribution of normal pressure applied by prestressed bolts not uniform.
    - Wearing concentrates on contact areas with highest normal pressure.
    - Variable distribution of normal pressure calculated by a modified semi-infinite wedge model derived from theory of elasticity (Ugural et al. 1995).

![Diagram of friction dampers with labeled key parameter](image)
6. Friction Dampers

- Effects Influencing Frictional Behaviour
  - Coefficient of friction depends on:
    - Apparent pressure
    - Sliding velocity
    - Temperature
    - Load dwell

Figure 5.36 Relation Between Sliding Velocity, Pressure and Frictional Coefficient (adapted from Constantinou et al. 1999)

Figure 5.37 Influence of Temperature to Coefficient of Friction (adapted from Constantinou et al. 1999)
6. Friction Dampers

- Studies on the Variation of Coefficient of Friction for Teflon-Steel Interfaces (Bondonet and Filiatrault 1997)
6. Friction Dampers

- Studies on the Variation of Coefficient of Friction for Teflon-Steel Interfaces (Bondonet and Filiatrault 1997)

![Graphs showing frictional response for different frequencies](image)

Figure 5.38 Frictional Response of Unfilled Teflon-Steel Interface Under a Confining Pressure of 30 MPa (from Bondonet and Filiatrault 1997, reproduced with permission from the American Society of Civil Engineers)
6. Friction Dampers

• Studies on the Variation of Coefficient of Friction for Teflon-Steel Interfaces (Bondonet and Filiatrault 1997)

Figure 5.39 Variations of Initial and Steady State Coefficients of Friction with Absolute Maximum Velocity (after Bondonet and Filiatrault 1997)
6. Friction Dampers

- Studies on the Variation of Coefficient of Friction for Teflon-Steel Interfaces (Bondonet and Filiatrault 1997)

![Graph showing the influence of confining pressure on initial and steady-state coefficients of friction.](image-url)
6. Friction Dampers

- Studies on the Variation of Coefficient of Friction for Metal-Metal Interfaces
  - Pall et al. (1980)

Figure 5.42 Hysteretic Behaviour of Slip Bolted Joints (from Pall et al. 1980, reproduced with the permission of the Prestressed Concrete Institute)
6. Friction Dampers

- Studies on the Variation of Coefficient of Friction for Metal-Metal Interfaces
  - Tremblay and Stiemer (1993)

Figure 5.43 Hysteretic behaviour of Friction-Type Bolted Brace Connections (from Tremblay and Stiemer 1993, reproduced with the permission of the authors)
6. Friction Dampers

- Studies on the Variation of Coefficient of Friction for Metal-Metal Interfaces
  - Grigorian et al. (1993)

![Steel on Steel and Steel on Brass Hysteresis Diagrams](image)

*Figure 5.44 Cyclic Response of Slotted Bolted Connections (from Grigorian et al. 1993, reproduced with the permission of the Earthquake Engineering Research Institute)*
6. Friction Dampers

- Existing Friction Damping Systems
  - Slotted-Bolted Connections
    - Simplest form of friction dampers.
    - Slotted-bolted connections at the ends of conventional bracing members.
    - To maintain constant slip load, disc spring washers can be used.

Figure 5.45 Slotted-Bolted Connection for Steel Framed Building (from Tremblay and Stiemer 1993, reproduced with the permission of the authors)
6. Friction Dampers

- Existing Friction Damping Systems
  - Slotted-Bolted Connections

Figure 5.46 Full Scale Testing of Slotted-Bolted Friction Damped Braced Frame (from Tremblay and Stieimer 1993, reproduced with the permission of the authors)

Figure 5.47 Quasi-Static Response of Slotted-Bolted Friction Braced Frame (from Tremblay and Stieimer 1993, reproduced with the permission of the authors)
6. Friction Dampers

- Existing Friction Damping Systems
  - Sumitomo Friction Device (Sumitomo Metal Industries Ltd., Japan)
    - More sophisticated friction device.
    - Incorporates a pre-compressed internal spring that induces a force that is converted through the action of inner and outer wedges into a normal force on copper alloy friction pads containing graphite plug inserts for lubrication.

![Diagram of Sumitomo Friction Device](image)

Figure 5.48 Sumitomo Friction Device (after Aiken and Kelly 1993)
6. Friction Dampers

- **Existing Friction Damping Systems**
  - Pall Friction Device (Pall Dynamics Ltd., Canada)
    - Most implemented friction damping system.
    - Designed to be mounted in a moment-resisting framed structure.
    - Mechanism containing slotted slip joints introduced at intersection of frame cross-braces.

![General Arrangement of Pall Friction Device](image)
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)

\[ P_g = 2P_l - P_{cr} \]

\[ P_g = \text{Global Slip Load} \]

\[ P_l = \text{Local Slip Load} \]

Free-Body Diagram of Friction Damper when Slippage Occurs
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)

![Figure 5.51 Hysteretic Behaviour of Simple Moment-Resisting Frame Equipped with Pall Friction Damping System](image-url)
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)

Figure 5.51 Hysteretic Behaviour of Simple Moment-Resisting Frame Equipped with Pall Friction Damping System
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)

![Diagram showing hysteretic behaviour of a simple moment-resisting frame equipped with a Pall friction damping system.](image)
6. Friction Dampers

• Existing Friction Damping Systems
  – Pall Friction Device (Pall Dynamics Ltd., Canada)

Figure 5.52 Hysteretic Response of Pall Friction Device Incorporating Asbestos Brake Lining Pads (from Filiatrault and Cherry 1987, reproduced with the permission of the Earthquake Engineering Research Institute)
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)

Table 5-4: Dynamics Characteristics of Friction Damped Test Structure (after Filiatrault and Cherry 1987)

<table>
<thead>
<tr>
<th></th>
<th>Fundamental Period (s)</th>
<th>Damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRF</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28</td>
</tr>
<tr>
<td>BMRF</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.27</td>
</tr>
<tr>
<td>FDBF</td>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.60</td>
</tr>
</tbody>
</table>
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)
6. Friction Dampers

- Existing Friction Damping Systems
  - Pall Friction Device (Pall Dynamics Ltd., Canada)

Figure 5.53 Measured Top Accelerations of MRF, BMRF, and FDBF under Taft Earthquake Scaled to PGA = 0.9g (from Filiatrault and Cherry 1987, reproduced with the permission of the Earthquake Engineering Research Institute)
6. Friction Dampers

- Structural Implementations

APPENDIX B: Implementation of Friction Dampers in Structures. [N]: New Construction, [R]: Retrofit

<table>
<thead>
<tr>
<th>Structure</th>
<th>Location</th>
<th>Year</th>
<th>Damper Type</th>
<th>Number of Dampers</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>McConnell Library</td>
<td>Montreal, Canada</td>
<td>1991</td>
<td>Fall</td>
<td>143</td>
<td>Fall et al. 1987</td>
</tr>
<tr>
<td>Concordeia Union [N]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Some Office Building [N]</td>
<td>Omiyama City, Japan</td>
<td>1988</td>
<td>Somitomo</td>
<td>5</td>
<td>Aiken and Kelly 1990</td>
</tr>
<tr>
<td>Ecole Polyvalente Sorel [R]</td>
<td>Sorel, Canada</td>
<td>1990</td>
<td>Fall</td>
<td>64</td>
<td>Fall and Fall 1996</td>
</tr>
<tr>
<td>Canadian Space Agency [N]</td>
<td>St-Hubert, Canada</td>
<td>1992</td>
<td>Panel dampers</td>
<td>55</td>
<td>Vezina et al. 1992</td>
</tr>
<tr>
<td>CCRIT Building [R]</td>
<td>Laval, Canada</td>
<td>1992</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Montreal, Casino [R]</td>
<td>Montreal, Canada</td>
<td>1993</td>
<td>Fall</td>
<td>32</td>
<td>Pasquin et al. 1994</td>
</tr>
<tr>
<td>Canadian Dept. of Nat. Defence [N]</td>
<td>Ottawa, Canada</td>
<td>1992</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>L. H. Mooney Bldg Stanford [R]</td>
<td>Palo Alto, California</td>
<td>1994</td>
<td>Slotted Bolted Connections</td>
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<tr>
<td>Bldg. 610 Stanford [R]</td>
<td>Palo Alto, California</td>
<td>1994</td>
<td>Slotted Bolted Connections</td>
<td>80</td>
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<tr>
<td>Maison 1, McGill [R]</td>
<td>Montreal, Canada</td>
<td>1995</td>
<td>Fall</td>
<td>65</td>
<td>Savard et al. 1995</td>
</tr>
<tr>
<td>Hamilton Courthouse [R]</td>
<td>Hamilton, Canada</td>
<td>1995</td>
<td>Fall</td>
<td>74</td>
<td>Wagner et al. 1995</td>
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<tr>
<td>Ecole Technologie Superieure [R]</td>
<td>Montreal, Canada</td>
<td>1995</td>
<td>Fall</td>
<td>74</td>
<td>Godin et al. 1995</td>
</tr>
<tr>
<td>Federal Building [R]</td>
<td>Sherbrooke, Canada</td>
<td>1995</td>
<td>Fall</td>
<td>30</td>
<td>---</td>
</tr>
<tr>
<td>Desjardins Life Insurance Bldg [R]</td>
<td>Levis, Canada</td>
<td>1995</td>
<td>Fall</td>
<td>30</td>
<td>---</td>
</tr>
</tbody>
</table>
6. Friction Dampers

• Structural Implementations

Concordia University Library, Montreal, Canada, 1991

This ten-storey McConnell Building is a masterpiece in innovative structural design. High energy dissipating Pall friction-dampers are installed at the junction of steel cross-bracings in rigid concrete frames. The use of steel bracings eliminated the need of expensive concrete shearwalls and the introduction of supplemental damping provided by friction-dampers eliminated the need of dependence on ductility of structural members.

Unlike concrete shearwalls, the bracings were generally not continuous one over the other and thus provided greater flexibility in space planning. The architects have boldly exposed several bracings as these add to the aesthetic appearance.

Pall friction-dampers provided an economical design solution to safeguard the building and its valuable contents against earthquakes.

The building was designed in 1987 and its construction was completed in 1991.
6. Friction Dampers

- Structural Implementations

<table>
<thead>
<tr>
<th>Structure</th>
<th>Location</th>
<th>Year</th>
<th>Damper Type</th>
<th>Number of Dampers</th>
<th>Reference</th>
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</thead>
<tbody>
<tr>
<td>Oversea Water Tank</td>
<td>Bremerton, WA</td>
<td>1995</td>
<td>Fall</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>St. Luc Hospital [R]</td>
<td>Montreal, Canada</td>
<td>1995</td>
<td>Fall</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>Residence</td>
<td>Montreal, Canada</td>
<td>1996</td>
<td>Fall</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>Maisonnette</td>
<td>Montreal, Canada</td>
<td>1996</td>
<td>Fall</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Harry Stevens Building [R]</td>
<td>Vancouver, Canada</td>
<td>1996</td>
<td>Fall</td>
<td>84</td>
<td>Elliott et al. 1990</td>
</tr>
<tr>
<td>Justice Headquarters [R]</td>
<td>Ottawa, Canada</td>
<td>1996</td>
<td>Fall</td>
<td>46</td>
<td>Hale et al. 1995</td>
</tr>
<tr>
<td>UC Davis Water Tank</td>
<td>Davis, California</td>
<td>1996</td>
<td>Fall</td>
<td></td>
<td></td>
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<tr>
<td>BCBC Selkirk</td>
<td>Victoria, Canada</td>
<td>1997</td>
<td>Fall</td>
<td>74</td>
<td></td>
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<tr>
<td>Maison de Beaucours [R]</td>
<td>Quebec City, Canada</td>
<td>1997</td>
<td>Fall</td>
<td>42</td>
<td></td>
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<tr>
<td>Maison Sherway Williams [R]</td>
<td>Montreal, Canada</td>
<td>1997</td>
<td>Fall</td>
<td>64</td>
<td></td>
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<tr>
<td>251 South Lake Ave. Building [R]</td>
<td>Pasadena, California</td>
<td>1998</td>
<td>Fall</td>
<td>---</td>
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<tr>
<td>Quebec Provincial Police Mdo. [R]</td>
<td>Montreal, Canada</td>
<td>1999</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
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<tr>
<td>Freeport Water Tower [R]</td>
<td>Sacramento, CA</td>
<td>1999</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
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<tr>
<td>Boeing Airplane Factory [R]</td>
<td>Everett, WA</td>
<td>2001</td>
<td>Fall</td>
<td>---</td>
<td>Vail and Hubbard 2002</td>
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<tr>
<td>Moscone West Court Center [R]</td>
<td>San Francisco, CA</td>
<td>2001</td>
<td>Fall</td>
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<td>---</td>
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<tr>
<td>1000 Lennox Squ. Arts Building [R]</td>
<td>Seattle, WA</td>
<td>2001</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
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<tr>
<td>Boeing Cafeteria and Auditorium [R]</td>
<td>Seattle, WA</td>
<td>2001</td>
<td>Fall</td>
<td>56</td>
<td>---</td>
</tr>
<tr>
<td>ACC Sharp Mem. Hospital [R]</td>
<td>San Diego, CA</td>
<td>2001</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
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<tr>
<td>La Gardena Tower [R]</td>
<td>Guwahati, India</td>
<td>2001</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
6. Friction Dampers

- Structural Implementations

Freeport Water Tower, Sacramento, CA, 1999

The Freeport water tower, a distinctive landmark visible from Interstate-5, was built in 1956. The steel reservoir stands about 120 feet high. The supporting structure consists of 27 steel columns with two levels of 60 feet long tension cross bracing.

Of the several seismic retrofit options, the scheme with Pall friction-dampers in tension cross bracing was chosen. When tension in one of the braces forces the damper to slip, the damper’s mechanism shortens the other brace, thus preventing buckling.

Due to high damping provided by the Pall friction-dampers, the strengthening of columns and foundations was not necessary. Seismic retrofit was completed in 1999.
6. Friction Dampers

- Structural Implementations

Boeing Plant, Seattle, Washington, 2001

The mammoth Boeing plant, which could contain Disneyland under one roof, is the world’s largest building in volume. It was built in phases from 1968-1991, for the assembly of wide-bodied 747 jetliners – world’s largest commercial airplane. The steel frame building is 120 feet high with clear spans of 350 feet and covers more than 98 acres.

In 1998, the Boeing engineers considered several seismic upgrade schemes for this structure. They chose Pall friction-dampers because they are foolproof in construction and offer reliable and maintenance free performance at low cost. The performance of friction-dampers is independent of velocity, therefore the forces on the connections remain constant for any future earthquake. Economy in the design of connections and easy installation of dampers provided significant savings in construction cost and time.

Several types of Pall friction-dampers suitably modified to adapt to site conditions, were incorporated in different types of existing bracings.
6. Friction Dampers

- Structural Implementations

<table>
<thead>
<tr>
<th>Structure</th>
<th>Location</th>
<th>Year</th>
<th>Damper Type</th>
<th>Number of Dampers</th>
<th>Reference</th>
</tr>
</thead>
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<tr>
<td>Eaton's Building</td>
<td>Montreal, Canada</td>
<td>2002</td>
<td>Fall</td>
<td>161</td>
<td>Pasquin et al. 2002</td>
</tr>
<tr>
<td>MUCTC Building</td>
<td>Montreal, Canada</td>
<td>2002</td>
<td>Fall</td>
<td>88</td>
<td>Vezina et al. 2002</td>
</tr>
<tr>
<td>Water Towers</td>
<td>Renton, Washington</td>
<td>2003</td>
<td>Fall</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

http://www.palldynamics.com/
7. Design of Structures with Hysteretic Dampers

- Various proposed design procedures based on various concepts:
  - Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)
  - Transfer functions concepts: (Dowdell and Cherry 1996)
  - Equivalent force reduction factor, R: Fu and Cherry (1999, 2000)
  - Uniform drift distribution: Levy et al. (2000, 2001)

- Optimum planar positioning of friction dampers in asymmetric structures: De LaLlera et al. (2005)
  - Friction dampers can control effectively lateral-torsional coupling by positioning dampers so that the so-called empirical center of balance (ECB) of the structure is at equal distance form all edges of the building.
  - Simple equation determining optimum location of dampers derived.
  - Approach can be extended to multi-storey structures.
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

Earlier numerical studies (Filiatrault and Cherry 1988) have indicated the feasibility of using an optimum distribution of shear forces required to activate all hysteretic dampers in a structure that is proportional to the interstorey drift arising from a first mode vibration of the structure. For a building structure equipped with a pair of tension-compression bracing members at every level, the activation shear at a given storey $i$, $V_{ai}$ is related to the activation load of each damper $F_{ai}$ by:

$$V_{ai} = 2F_{ai} \cos \gamma_i$$  \hspace{1cm} (5.79)

where $\gamma_i$ is the angle of inclination from the horizontal of the braces in storey $i$.  

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7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

It has also been shown (Filiatrault and Cherry 1988), however, that little benefit is achieved from the use of this optimum distribution as compared with the use of the simpler uniform activation shear distribution $V_{ai} = V_a$ for $i = 1, 2, ..., N_f$ where $N_f$ is the total number of floor levels. Based on this assumption, an optimum activation shear distribution was determined based on a numerical parametric study that takes into account the frequency content of the ground motion and the dynamic properties of the structure with and without the added bracing system (Filiatrault and Cherry 1990). For a given ground motion, the optimum activation shear distribution is determined by minimizing a Relative Performance Index $RPI$ derived from energy concepts:

$$ RPI = \frac{1}{2} \left[ \frac{SEA}{SEA_0} + \frac{U_{max}}{U_{max0}} \right] $$  \hspace{1cm} (5.80)

where $SEA$ is the strain energy area, i.e., the area under the strain energy time-history for all structural members of an hysteretically damped structure, $SEA_0$ is the strain energy area for a zero activation load, $U_{max}$ is the maximum strain energy stored in all structural members of an hysteretically damped structure and $U_{max0}$ is the maximum strain energy for a zero slip load.
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)
- Strain Energy Area (SEA) for elastic structure
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

Values of the $RPI$ are such that:

- $RPI = 1$, the response corresponds to the behaviour of an unbraced structure (activation load = 0);
- $RPI < 1$, the response of the hysteretically damped structure is "smaller" than the response of the unbraced structure;
- $RPI > 1$, the response of the hysteretically damped structure is "larger" than the response of the unbraced structure.
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)
- Example for an industrial building

![Diagram of structural components with dimensions and specifications]

Diagonal Braces: 1858 mm²
Friction Devices: 0.96mx0.34m
All Columns: W14x68
Floor Weight: 910 kN (1st and 2nd Floor)
416 kN (3rd Floor)

**FIG. 6. Slip Load Optimization from FDBFAP: Newmark-Blume-Kapur Artificial Earthquake (0.5 g)**

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7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

The procedure then requires the estimation of the design peak ground acceleration $a_g$ and the predominant period of the design ground motion $T_g$ for the site.

Recalling Equation (5.56), an equation can be written for the total shear force $V_0$ required to activate all hysteretic dampers in a structure (Filiatrault and Cherry 1990):

$$\frac{V_0}{W} = \frac{a_g}{g} Q\left(\frac{T_b}{T_g}, \frac{T_b}{T_u}, N_f\right)$$  \hspace{1cm} (5.82)

where $Q$ is an unknown single valued function and $N_f$ is the number of floors.
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NS$</td>
<td>1, 3, 5, 10</td>
</tr>
<tr>
<td>$T_{0}/T_u$</td>
<td>0.1 sec/$T_u$, 0.7 sec/$T_u$, 1.4 sec/$T_u$, 2.0 sec/$T_u$</td>
</tr>
<tr>
<td>$T_b/T_u$</td>
<td>0.20, 0.40, 0.60, 0.80 for $NS = 1$</td>
</tr>
<tr>
<td></td>
<td>0.20, 0.50, 0.80 for $NS = 3, 5, 10$</td>
</tr>
<tr>
<td>$a_g/g$</td>
<td>0.005, 0.05, 0.10, 0.15, 0.20, 0.30, 0.40 for $NS = 1$</td>
</tr>
<tr>
<td></td>
<td>0.05, 0.10, 0.20, 0.40 for $NS = 3, 5, 10$</td>
</tr>
</tbody>
</table>
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

**TABLE 1. Versions of Basic Structural Configuration**

<table>
<thead>
<tr>
<th>NS number</th>
<th>Version number</th>
<th>W (kN)</th>
<th>$I_1$ (mm$^4$)</th>
<th>$T_u$ (sec)</th>
<th>$T_b/T_u$ = 0.20</th>
<th>$T_b/T_u$ = 0.40</th>
<th>$T_b/T_u$ = 0.50</th>
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CIE 626 - Structural Control
Chapter 5 – Hysteretic Dampers
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)
  
  - Stochastic ground motion representation
    
    - Kanai-Tajima power spectral density function
      \[ S_o(T) = \frac{1 + 4h_g^2\left(\frac{T_g}{T}\right)^2}{\left[1 - \left(\frac{T_g}{T}\right)\right]^2 + 4h_g^2\left(\frac{T_g}{T}\right)} \]
      \[ S_A = |H(T)|^2S_A \]
      \[ s_o = 30 \exp (-3.254a_g^{0.35}) \]
      \[ h_g = 0.30 \]
      
      where \(|H(T)|^2\) represents the transfer function of the soil layers and \(S_A\) the power spectral density function at the bedrock level.

- Earthquake duration (sec):
- 10 synthetic records per values of \(T_g\) and \(a_g\).
- 4880 nonlinear time-history dynamic analyses.
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

A comprehensive parametric numerical study was conducted by Filiatrault and Cherry (1990) in order to provide an estimate of the function $Q$ that minimizes the $RPI$ value given by Equation (5.80):

$$
Q = \begin{cases} 
\frac{T_g}{T_u} \left[ (-1.24N_f - 0.31) \frac{T_b}{T_u} + 1.04N_f + 0.43 \right] & \text{for } 0 \leq \frac{T_g}{T_u} \leq 1 \\
\frac{T_b}{T_u} \left[ (0.01N_f + 0.02) \frac{T_g}{T_u} - 1.25N_f - 0.32 \right] + \frac{T_g}{T_u} (0.002 - 0.002N_f) + 1.04N_f + 0.42 & \text{for } \frac{T_g}{T_u} > 1
\end{cases}
$$

(5.83)
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

\[
O_1 = (-1.24N_f - 0.31) \frac{T_b}{T_u} + 1.04N_f + 0.43
\]

\[
O_2 = (-1.07N_f - 0.10) \frac{T_b}{T_u} + 1.01N_f + 0.45
\]
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

Figure 5.55 Optimum Hysteretic Damping Design Spectra for Structures Ranging from 1 to 10 Stories
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

![Graph showing Average Values of RPI at Optimum Slip Load]

\[ N_f = 1 \]
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

These authors recommended the selection of diagonal cross braces such that:

$$\frac{T_b}{T_u} < 0.40$$  \hspace{1cm} (5.81)

where $T_u$ is the fundamental period of the unbraced structure and $T_b$ is the fundamental period of the braced structure. It was found that the most desirable response (minimum $RPI$) of hysteretically damped structures occurs for small values of $T_b/T_u$, which corresponds to large diagonal cross-braces. Therefore the diagonal cross-braces should be chosen with the largest possible cross-sectional area within the limits of cost and availability of material.
7. Design of Structures with Hysteretic Dampers

- Optimum Hysteretic Design Spectra (Filiatrault and Cherry 1988)

Note that several assumptions were made in developing the optimum hysteretic design spectra:

- The study was conducted within the following range of parameters:
  \[0.2 \leq \frac{T_b}{T_u} \leq 0.8, \quad 0.2 \leq \frac{T_b}{T_g} \leq 0.8, \quad 0.05 \leq \frac{a_g}{g} \leq 0.4 \quad \text{and} \quad N_f \leq 10\]

- The response of the main structural elements was assumed to remain elastic.

- The ground motions that were used to develop the hysteretic design spectra did not contain the damageable severe acceleration pulses that are expected in the near-field of major earthquake events.

- The choice of the two parameters (SEA and \(U_{max}\)) may not be appropriate to minimize structural and nonstructural damage in actual buildings. Other parameters such as peak interstorey drifts or floor absolute accelerations are now commonly used to evaluate the seismic performance of a structure.
7. Design of Structures with Hysteretic Dampers

- Design Example (Hysteretic Design Spectra)

![Diagram of Steel Structure used in Hysteretic Damper Design Example]

Figure 5.56 Steel Structure used in Hysteretic Damper Design Example

(HSS 300x300x13mm)
7. Design of Structures with Hysteretic Dampers

- Design Example (Hysteretic Design Spectra)

For the structure considered, $T_h = 1.304 \text{ s}$ and $T_y = 0.648 \text{ s}$. Therefore: $T_h/T_y = 0.497$ in this case, which is close to the recommended value of 0.40 (see Equation (5.81)).

The design procedure outlined above then requires the estimation of the design peak ground acceleration $a_g$ and the predominant ground period $T_g$ for the site. A value of $a_g = 0.40 \text{ g}$, corresponding to the $Z$ factor for a zone 4 in the UBC code (ICBO 1994), was retained. Also, a value of $T_g = 0.4 \text{ s}$ was assumed as it corresponds to the predominant period of simulated ground motions for the Los Angeles region (Graves and Saikia 1995).

From these parameters, an optimum hysteretic design spectrum can be constructed to estimate the total activation shear $V_0$ of the structure. For this six-storey structure, applying Equation (5.83) yields $Q = 0.863$ or:

$$\frac{V_0}{W} = 0.863 \frac{a_g}{g} = (0.863)(0.4) = 0.345$$

(5.85)

which leads to the following value of the optimum activation shear:

$$V_0 = 0.345 W = (0.345)(28950) = 9994 \text{ kN}$$

(5.86)
7. Design of Structures with Hysteretic Dampers

- Design Example (Hysteretic Design Spectra)

The total activation shear is distributed uniformly among the bracing at each floor of the two exterior frames:

\[ V_{si} = \left( \frac{1}{2} \right) \left( \frac{9994}{6} \right) = 830 \text{ kN} \quad (5.87) \]

Using Equation (5.79), the optimum activation load for each of the two dampers located in the first storey \( F_{a1} \) is given by:

\[ F_{a1} = \frac{830}{2 \cos(56.3^\circ)} = 750 \text{ kN} \quad (5.88) \]

The optimum activation load for each damper in all other stories \( F_{ai} \) is given by:

\[ F_{ai} = \frac{830}{2 \cos(46.2^\circ)} = 600 \text{ kN} \quad \text{for } i = 2 \text{ to } 6 \quad (5.89) \]
7. Design of Structures with Hysteretic Dampers

- Design Example (Hysteretic Design Spectra)

The structure considered was subjected to a suite of 5 historical (LA) ground motion recordings from earthquakes with local magnitudes ranging from 6 to 7.3, which were scaled to match the 10% probability of exceedence in 50 years uniform hazard spectrum for Los Angeles (SAC Joint Venture 1997). The unscaled S00E El Centro record from the 1940 Imperial Valley Earthquake and the S69E Taft Lincoln Tunnel record from the 1952 Kern County Earthquake were also included in the analyses. These two ground motions have been used extensively in past studies, and are considered herein as a reference for comparison purposes.
7. Design of Structures with Hysteretic Dampers

- Design Example (Hysteretic Design Spectra)

\[ \frac{V_o}{W} = 0.345 \]

![Figure 5.57 Peak Inter-Storey Drifts for Example Steel Structure (after Filiatrault et al. 2001)](image_url)
7. Design of Structures with Hysteretic Dampers

• Significance of Optimum Hysteretic Design Spectra
  – Seismic response of Friction Damped Braced Frame (FDBF) not sensitive to slip load variations of ±20% from optimal slip load

![Graph showing the effect of variation in slip load on displacement response](image-url)

*Figure 5.58 Effect of Variation in Slip Load on Displacement Response*
• Seismic Design Flow Chart

$V \geq 0.75V_{\text{code}}$ (ASCE 7-10)
see Chapter 7 of class notes
7. Design of Structures with Hysteretic Dampers

- Performance-Based Design of Hysteretically Damped Structures
  - Current design procedures do not allow selection among number of possible design options.
  - Often iterative process to achieve target performance objective.
  - Mansour and Christopoulos (2005) developed performance-based design procedure for structures equipped with hysteretic dampers.
    - Inelastic displacements and corresponding acceleration response spectra for hysteretically damped SDOFs derived for range of damper activation loads and of values of $T_b/T_u$ for a suite of twenty historical records.
• Performance-Based Design of Hysteretically Damped Structures

SL: Activation Load of Hysteretic Damper
SL\textsubscript{e}: Elastic Damper load

\[ T_u = 2.0 \text{ sec} \]

Northridge Record

Figure 5.59 Three-Dimensional Displacement and Acceleration Response Spectra for Hysteretically Damped SDOFs, \( T_u = 2.0 \text{ s} \)
- Performance-Based Design of Hysteretically Damped Structures

$T_u = 2.0 \text{ sec}$

Northridge Record

Figure 5.60 Displacement and Acceleration Response Spectra for Hysteretically Damped SDOFs with Various Damper Activation Loads, $T_u = 2.0 \text{ s}$
• Performance-Based Design of Hysteretically Damped Structures

\[ \Delta_{act} k_b + (\Delta_{opt} - \Delta_{act}) k_u = m a_{opt} \]

• Solve for \( \Delta_{act} \)

• \( SL_{opt} = \Delta_{act} k_b \)
Questions/Discussions

http://www.palldynamics.com/