

# Fast Full-wave Analysis of Conductor-Loaded Rectangular Cavity Resonators Using Surface Integral Equation and Moment Method

Amir Borji\* and S.Safavi-Naeini\*

\*Dept. of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1 Email: amir@maxwell.uwaterloo.ca

## Abstract

A recently developed technique for rapid calculation of the Green's functions in a rectangular cavity combined with a Surface Integral Equation formulation and Method of Moments (SIE-MoM) is applied for full-wave modeling of arbitrarily oriented conducting objects inside a rectangular enclosure. In a multiply-coupled-resonator structure, each rectangular cavity with apertures of arbitrary shape and orientation on its walls is modeled by a Generalized Admittance Matrix (GAM) obtained through SIE-MoM. Both combine and dual-mode disk resonators with tuning screw are considered for demonstration of the new approach and a remarkable computational efficiency is observed. A commercial EM simulator is used to verify the numerical results with excellent agreement.

## I. INTRODUCTION

Full-wave modeling of various types of conductor-loaded rectangular cavity resonators and filters have been the subject of extensive study in the last decade [1], [2]. Compact size, low cost and easy fabrication combined with high quality factor, high power handling capability, and good spurious performance are the main reasons for increasing applications of these structures in a wide range of filtering and multiplexing purposes in modern communication systems. In particular, two types of these resonators, namely combine and metallic disk resonators, have been widely used in many applications in microwave filter industry. Examples of these types of resonators are shown in Fig. 1 and Fig. 2. Dual-mode resonator made of metallic disk in a rectangular enclosure have also been introduced and investigated which further reduces the overall size and weight of the filter [1].

Efficient electromagnetic analysis of these structures is not an easy task. Commercial EM simulators based on finite element method can be used for any shape or orientation of the conducting objects or apertures inside rectangular enclosures but the accuracy and specially their computational efficiency is questionable particularly for narrow-bandwidth filter designs. Mode-matching technique has been successfully applied for canonical shapes of resonators and coupling irises oriented along the main coordinate axes [1], [2]. This method can become quite tedious for more complex structures or closely spaced objects inside the cavity. Furthermore, irregularly oriented objects and slots or non-canonical shape resonators and coupling apertures can not be treated with mode-matching approach. This includes off-centered tuning screws, excitation probes and tuning screws inserted from the side-walls of the cavity, corner-cut dual-mode disk resonator, cross-shape irises, etc...

In this paper a surface integral equation formulation with moment method (SIE-MoM) is used for full-wave analysis of conductor-loaded rectangular cavities and the coupling apertures between them. A very fast method for calculation of the Green's functions in a rectangular enclosure recently developed by the authors [3] is incorporated into this formulation which leads to a computationally efficient technique for modeling of arbitrary shape metallic objects and coupling structures. In case of aperture coupled resonators, each rectangular cavity with apertures of arbitrary shape and orientation on its walls is modeled as a multi-port network represented by its Generalized Admittance Matrix (GAM). A cascading procedure is then applied to combine the GAM of adjacent resonators in order to obtain the GAM of the entire structure.

## II. FORMULATION

Surface integral equation technique (SIE) with the method of moments (MoM) is a versatile and efficient method for electromagnetic analysis of arbitrary shape dielectric and metallic objects but the slow convergence of the pertinent Green's functions inside a rectangular enclosure makes

it very difficult, if not impossible, for this method to be effectively applied for the analysis of shielded objects. A novel technique for rapid calculation of the potential Green's functions in a rectangular cavity has been recently introduced by the authors [3]. The new approach is based on a fast Chebyshev polynomial approximation of the Green's function and it was observed to be much faster (30 times in most cases) than the best available series acceleration technique known as Ewald sum method. A complete software package for rapid calculation of all the components of electric and magnetic vector and scalar potentials and their derivatives inside a rectangular box has been developed by the authors.

### A. Resonant Frequency

In order to obtain resonant frequencies and their corresponding field patterns inside a closed conductor-loaded cavity, electric field integral equation in mixed potential form (MPIE) is employed which is well-known for its more stable and robust solution due to a less singular kernel. Surface currents on the metallic objects inside the cavity are expanded in terms of well-known RWG triangular basis functions. Galerkin testing procedure leads to a matrix equation with current amplitudes as its unknowns [3]:

$$[Z][J] = 0 \quad (1)$$

The impedance matrix  $[Z]$  is symmetric due to reciprocity and is real-valued because of lossless conditions. Numerical evaluation of  $Z_{mn}$  elements including the treatment of singularities and numerical quadrature for integrals was discussed in [3]. Resonant frequencies correspond to the zeros of the smallest eigenvalue of  $[Z]$  matrix. It was observed that the smallest eigenvalue shows an almost linear behaviour around the resonant frequency and a simple root finding algorithm is used to find the zero. Moreover, the eigenvector of  $[Z]$  associated with the zero eigenvalue corresponds to the resonant mode surface current from which electric and magnetic fields are calculated. Computation times reported in [3] show that the entire process of filling the impedance matrix and finding its smallest eigenvalue at each frequency is extremely fast due to the rapid calculation of the Green's functions.

### B. Coupling Apertures

In a multiply-coupled-cavity structure adjacent resonators are often coupled through different shapes of irises and slots. Therefore, more than one coupling window usually exists for each cavity. A typical resonator with two coupling slots is depicted in Fig. 3. Using equivalence principle, each aperture on the cavity wall is replaced by an equivalent surface magnetic current which is now distributed over the wall of the closed isolated cavity. Continuity of the tangential electric field across the aperture is ensured by opposite signs of the magnetic currents in adjacent cavities. In the next step a triangular mesh is generated for all the apertures as well as the metallic objects inside the resonators and equivalent magnetic and electric currents are then expanded in terms of RWG triangular basis functions. Considering the surface magnetic currents as *known sources* radiating inside the cavity and applying the boundary conditions over the surfaces of the objects within the enclosure a surface integral equation is obtained which is then transformed into a matrix equation using the Galerkin procedure. To obtain a multi-port network representation for each individual cavity, each of the basis functions on aperture boundaries is considered as a *port* and a generalized voltage and current is associated with it which are essentially weighted electric and magnetic fields over that basis function. Finally, a Generalized Admittance Matrix (GAM) representation for each individual resonator is obtained:

$$[Y][V] = [I] \quad (2)$$

in which  $[V]$  and  $[I]$  are the amplitudes of the generalized voltages and currents defined at each port (basis function) and  $[Y]$  has the following form:

$$[Y] = [Y_a] - [Q][Z]^{-1}[Q]^t \quad (3)$$

In Eq. 3  $[Z]$  is the same matrix as in Eq. 1 and  $[Q]$  contains the interactions between the magnetic current basis functions over the aperture boundaries and electric surface currents over the conducting objects.  $[Y_a]$  contains the interactions among the basis functions over the apertures

and is symmetric. A cascading procedure similar to what has been described in [4] can then be applied to successively combine the GAM of adjacent cavities (two at a time) in order to obtain the GAM of the entire structure.

In the case of two isolated coupled resonators, their corresponding admittance matrices are simply added together and the frequency at which the smallest eigenvalue of this matrix becomes zero corresponds to the resonant frequency of the even mode (PMC wall) if the two resonators were identical. Resonant frequency of the odd mode (PEC wall) is obtained from Eq. 1 and the coupling factor can be calculated.

### III. NUMERICAL RESULTS

Two types of resonators shown in Fig. 1 and Fig. 2 are considered for numerical simulation. Ansoft HFSS, a well-known commercial EM simulator, is used for verification of numerical data. All computations were performed on a desktop computer with an AMD AthlonXP processor at 2.2GHz clock frequency.

Resonant frequencies of a circular disk resonator with a tuning screw on its side were calculated and the results are shown in Fig. 4. In all cases 762 RWG basis functions were defined over the circular disk and 114 basis functions were considered for the tuning screw leading to 876 unknowns in total. Total time required for evaluation of the impedance matrix  $[Z]$  and its smallest eigenvalue at each frequency step is only 4.2sec and only a few steps are required to obtain the resonant frequency. Modal electromagnetic fields at  $x = \frac{a}{2}$  and  $y = \frac{a}{2}$  for  $HE_{11}$  mode are shown in Fig. 6.

Coupling factor between two identical combine resonators coupled through a full width iris was also calculated and the results are shown in Fig. 5. The effect of the thickness of the coupling iris is also demonstrated. In all cases 762 basis functions were considered over the surface of the metallic post and the aperture were discretized into 212 triangles leading to 298 basis functions. Therefore,  $[Z]$  matrix in Eq. 3 was 762x762 and  $[Y]$  was 298x298. The total time required for calculation of Eq. 3 and the smallest eigenvalue of the admittance matrix at each frequency step was only 5.7sec which shows the remarkable computational efficiency of the present method.

### IV. CONCLUSION

A novel approach based on SIE-MoM for full-wave analysis and modeling of conductor-loaded rectangular resonators and related coupling structures were presented. A remarkable computational efficiency in terms of speed and memory requirement is observed. Arbitrary shape objects or coupling apertures with irregular orientations can be treated without further manipulation of the computer programs. It is now possible to calculate the entire frequency response of any multiple-coupled cavity filter between the input and output ports including the excitation probes. Dielectric resonator filters can also be treated in a similar fashion. With a drastic reduction in computational time while maintaining high accuracy, it is feasible now to use the present approach for direct optimization of structures made of dielectric or metallic objects inside rectangular cavities.

### REFERENCES

- [1] C.Wang, H.-W. Yao, and K.A.Zaki, "Modeling of Conductor-Loaded Resonators and Filters in Rectangular Enclosures," *IEEE Trans. Microwave Theory Tech.*, vol. 45, no. 12, pp. 2479-2485, Dec. 1997.
- [2] H.W.Yao, K.A.Zaki, A.E.Atia, and R.Hershtig, "Full Wave Modeling of Conducting Posts in Rectangular Waveguides and Its Applications to Slot Coupled Combline Filters," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 12, pp. 2824-2829, Dec. 1995.
- [3] A. Borji and S. Safavi-Naeini, "Rapid Calculation of the Green's Function in a Rectangular Enclosure with Application to Conductor Loaded Cavity Resonators," *submitted to IEEE Trans. Microwave Theory Tech.*, 2004, (revised).
- [4] L.D.Salghetti, A.Freni, and S.Maci, "CAD-Oriented, Generalized Admittance Matrix Approach for Arbitrary Corrugated E-Plane Sectoral Horns," *Proc. IEE, pt. H*, vol. 146, no. 5, pp. 329-334, Oct. 1999.

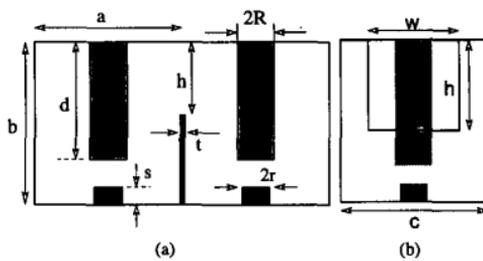


Fig. 1. Combine resonators coupled through a rectangular iris

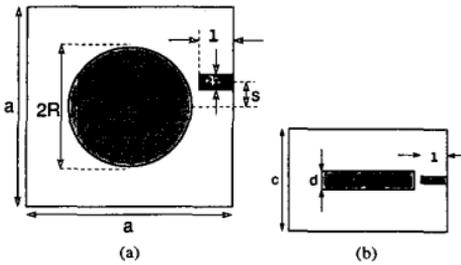


Fig. 2. Metallic disk resonator with tuning screw on the side

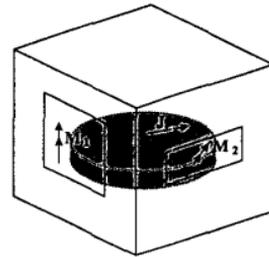


Fig. 3. Typical resonator with different slots on its walls

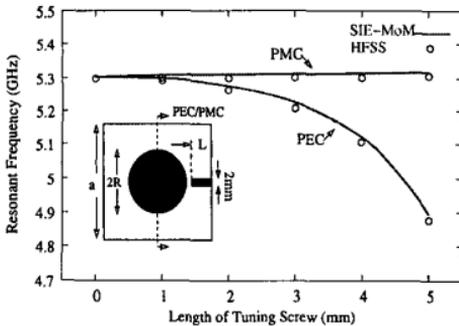


Fig. 4. Resonant frequencies of the structure in Fig. 2 ( $a=30\text{mm}$ ,  $c=26\text{mm}$ ,  $R=9\text{mm}$ ,  $d=4\text{mm}$ ,  $s=0\text{mm}$ ,  $r=1\text{mm}$ )

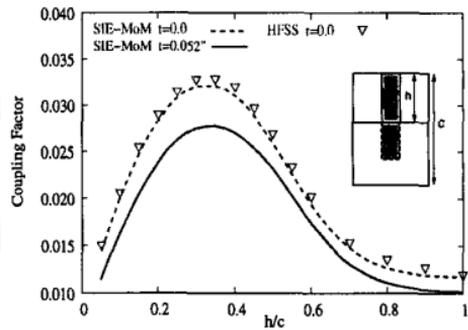


Fig. 5. Coupling factor between two identical combine resonators shown in Fig. 1 ( $a=1''$ ,  $w=c=0.872''$ ,  $b=1.872''$ ,  $R=0.13''$ ,  $d=1.074''$ ,  $s=r=0$ )  $t$  is the thickness of the iris

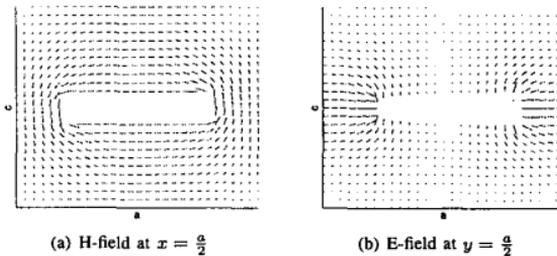


Fig. 6.  $HE_{11}$  electric and magnetic fields at two orthogonal planes for disk resonator shown in Fig. 2 with no tuning screw ( $l = 0$ )