Resonant Length of Longitudinal Slots and Validity of Circuit Representation: Theory and Experiment

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Abstract—**Pertinent theory for the design of longitudinal slot arrays is reviewed and its dependence on the dominant mode scattering off a single slot is pinpointed. The critical need to know resonant length versus slot offset is emphasized and the desirability of determining this information theoretically rather than experimentally is argued. Then method of moments solutions are used to calculate resonant length versus slot offset for given waveguide dimensions and frequency. These theoretical results are compared to new, carefully obtained experimental data. Agreement is found to be so good, it is concluded that one can dispense with the costly gathering of experimental input data when designing longitudinal slot arrays fed by standard rectangular waveguide. A critical look is taken at the validity of representing the longitudinal slot as a shunt element on an equivalent transmission line. This assumption is found to be more and more questionable as the *b* dimension is reduced. For quarter-height guide, an alternate design procedure is suggested as being more accurate.**

INTRODUCTION

IN 1978 a theory was presented [1], [2] that would permit the design of linear and planar arrays of waveguide-fed slots, including the effects of external mutual coupling. That theory was generalized and improved [3] in 1983. Briefly stated, the theory culminates in three design equations which contain the lengths and offsets of the slots as unknowns, with the desired aperture distribution, the desired input match, and the self-admittance of a single slot versus its offset and length as knowns.

Up to the present time, this design procedure has been used by first amassing experimental data on the self-admittance of a single slot, fitting functions of slot length and offset to that data, and then using these fitted functions in the three design equations. This experimental input data-gathering is costly, time-consuming, and inherently less accurate than one would wish when designing high-performance arrays. Thus it would be desirable to replace this *experimentally* obtained input data by *theoretically* obtained input data, if the latter could be generated with sufficient accuracy and at less cost.

That quest has been going on for a considerable number of years. In 1948 Stevenson [4] published a classic paper in which he established the internal Green's functions for a rectangular waveguide and used those functions to analyze scattering off a longitudinal slot excited by an incident TE_{10} mode. He assumed that the waveguide was air-filled, that the slot length was $\lambda_0/2$, that this was the *resonant* length, that the electric field in the slot was totally transverse and possessed an equiphase half-cosinusoid amplitude distribution, that the waveguide walls were vanishingly thin, and that the broad wall containing the slot was imbedded in an infinite ground plane. Under these circumstances Stevenson was able to show that dominant TE_{10} mode scattering off the slot was symmetrical, as a result of which the slot could be represented by a shunt element on an equivalent transmission line. Using a power balance, Stevenson derived a formula for the normalized resonant conductance of a single longitudinal slot as a function of its offset x from the centerline, viz.,

$$g(x)=\frac{G_{\text{res}(x)}}{G_0}=\left[2.09\,\frac{(a/b)}{(\beta_{10}/k_0)}\cos^2\left(\frac{\beta_{10}}{k_0}\frac{\pi}{2}\right)\right]\sin^2\frac{\pi x}{a}. \tag{1}$$

In (1), a and b are the transverse waveguide dimensions, β_{10} is the propagation constant of the TE_{10} mode, and $k_0 = 2\pi/\lambda_0$.

Stevenson then attempted to generalize his result to embrace a nonresonant slot but was unable to find a companion expression for slot susceptance.

In 1951 Stegen [5] carefully performed a series of experiments on longitudinal slots cut in a broad wall of standard X-band waveguide ($a = 0.900$ in, $b = 0.400$ in, $t = 0.050$ in). He used round-ended slots, 0.063 in wide, of various offsets and lengths, and measured relative backscattering B_{10}/A_{10} at a frequency of 9.375 GHz. Stegen wanted to achieve a standard against which theory could be judged, and it is a tribute to his painstaking work that his data served as the standard for 30 years.

Stegen found that he could present the data in a universal form by normalizing slot admittance to resonant conductance, in effect writing

$$\frac{Y(x,\,l)}{G_0}=[h_1(y)+jh_2(y)]g(x). \tag{2}$$

In (2), $Y(x, l)$ is the admittance of the slot, seen as a shunt element on an equivalent lossless transmission line of characteristic conductance G_0; x and $2l$ are the offset and length of the slot; $g(x) = G_{\text{res}}(x)/G_0$ is the normalized conductance of a resonant-length slot whose offset is x; $y = l/l_{\text{res}}$ is actual slot length in ratio to the length of a resonant slot possessing the same offset. Stegen's fitted curves and data points for $h_1(y)$, $h_2(y)$, and $g(x)$ are reproduced as [2, figs. 8.35 and 8.36]. His fitted curve for $g(x)$ is in the form $g(x) = K\sin^2(\pi x/a)$ and is seen to be in good agreement with the experimental data, thus validating the form of (1). However, the value of his constant K, chosen for a best fit, was not quite in agreement

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with Stevenson's derived constant, contained within the square brackets in equation (1).

Additionally, Stegen found that resonant length, contrary to Stevenson's assumption, was offset-dependent. His fitted curve and data points relating $k_0 l_{res}$ to slot offset are shown as [2, fig. 8.37]. As we shall see, agreement between theory and experiment for $v(x) = k_0 l_{res}(x)$ is critical if one wishes to substitute theoretical input data for experimental data of the type accumulated by Stegen.

Working in parallel with Stegen, Oliner [6] developed the first theory for longitudinal slots which would permit calculation of the susceptance as well as the conductance. He did this by formulating a stationary expression for reactive power and found good agreement with Stegen when he calculated $h_1(y)$, $h_2(y)$, and $g(x)$. However, Oliner had two small obstacles to overcome before he could compare his calculated result for $v(x)$ with Stegen's measured values. First, Oliner's original theory assumed "zero" wall thickness, and Stegen's wall thickness of $t = 0.050$ in was not negligible. Oliner overcame this difficulty by assuming that the rectangular hole which linked the lower and upper slot apertures was a section of waveguide of length t, in which only the dominant TE_{10} mode need be considered as the coupling agent between internal and external fields. With this assumption, Oliner was able to derive a "wall thickness correction," which for Stegen's case amounted to 2 percent. Second, Oliner's theory assumed square-ended slots whereas Stegen's slots were round-ended, having been cut by a vertical milling machine. Oliner made the assumption that round-ended and square-ended slots were equivalent if they had the same area. This led to an additional 2 percent correction. Oliner concluded that his theoretical square-ended slot in a "zero" thickness wall should be 4 percent shorter than Stegen's actual slot. With this adjustment, theoretical and experimental curves for $v(x)$ were in fair-to-good agreement. Unfortunately, in terms of today's design requirements, the agreement was insufficient. More will be said about this point later in the present paper.

The next significant theoretical attack on this problem was undertaken by Khac [7], who took full advantage of the capabilities of the modern computer. Khac assumed a square-ended longitudinal slot in a "zero" thickness wall, assumed that the electric field in the slot aperture was totally transverse, used Stevenson's internal Green's function and the external half-space Green's function to formulate integral expressions (involving the unknown slot aperture electric field) for the internal and external scattered fields, and matched the total longitudinal H-field across the slot aperture. This gave him an integral equation in $E_x(P')$, the unknown aperture field, with the longitudinal component of the incident TE_{10} mode playing the role of driving function. Khac then used the method of moments, employing pulse functions and point matching, to solve for $E_x(P')$ and used that knowledge to find the dominant mode backscattering, and thus $Y(x, l)/G_0$.

Khac's agreement with Stegen on $g(x)$ was excellent but on $h_1(y)$ and $h_2(y)$ only qualitative. Before he could compare $v(x)$ results and improve on his $h_1(y)$ and $h_2(y)$ calculations, he had to overcome the same two obstacles as had confronted Oliner. Khac resorted to coupled integral equations to account for wall thickness and this resulted in good agreement with experiment on $h_1(y)$ and $h_2(y)$. If Khac were then to use the same 2 percent correction for round ends as Oliner, his agreement with Stegen on $v(x)$ could only be characterized as fair. Khac had an additional difficulty in that his use of pulse functions violated the boundary condition that $E_x(P')$ be zero at the two ends of the slot. His ad hoc assumption to add one cell to the length of his theoretical slot brought his $v(x)$ curve within 2 percent of Stegen's, but is difficult to justify on a rigorous basis. As we shall see, a 2 percent error in $v(x)$ is not good enough.

To obtain the theoretical results reported later in this paper, we have used Khac's integral equation and the method of moments, but have chosen basis functions which model $E_x(P')$ more realistically. Also, to overcome some of the difficulties which confronted both Oliner and Khac in comparing their theoretical results to Stegen's experimental data, we have carefully fabricated and tested a family of seven waveguide sections, each containing a longitudinal slot with a different offset. Standard X-band dimensions were used ($a = 0.900$ in, $b = 0.400$ in) but the wall thickness was only $t = 0.005$ in, and the slots were *square*-ended. This brings the theoretical model and the actual test fixtures into closer agreement and permits a more precise comparison. Further, with the modern trend toward thin waveguide walls and slot fabrication via stamping or etching, square-ended slots are just as easy to fabricate as round-ended slots, so this convenient assumption in the theory is no longer academic.

The Design Procedure

The procedure that we use to design linear and planar arrays of longitudinal slots fed by rectanguar waveguide is fully described in [3]. It rests on three design equations:

$$\frac{Y^a_{mn}/G_0}{Y^a_{pq}/G_0} = \frac{f_{mn} V^s_{mn} V_q}{f_{pq} V^s_{pq} V_n} \tag{3}$$

$$Y^a_{mn}/G_0 = \frac{2f^2_{mn}}{\dfrac{(2/K)L^2_{mn}}{h_1(y_{mn}) + jh_2(y_{mn})} + \mathrm{MC}_{mn}} \tag{4}$$

$$\sum_{m=1}^{M(n)} \frac{Y^a_{mn}}{G_0} = C_n \tag{5}$$

in which

$$f_{mn} = L_{mn} \sin^2 \frac{\pi x_{mn}}{a} \tag{6}$$

$$L_{mn} = \frac{[(\pi/2)/y_{mn} v(x_{mn})] \cos [(\beta_{10}/k) y_{mn} v(x_{mn})]}{[(\pi/2)/y_{mn} v(x_{mn})]^2 - (\beta_{10}/k)^2} \tag{7}$$

$$\mathrm{MC}_{mn} = j(\beta_{10}/k)(kb)(a/\lambda)^3 \sum_r \sum_s{}' \frac{V^s_{rs}}{V^s_{mn}} g_{mnrs}. \tag{8}$$

Many of the symbols which appear in (3)-(8) have been defined in the Introduction. Equation (3) gives the ratio of the active admittances of the mnth slot and the pqth slot in terms of the desired slot voltage ratio V^s_{mn}/V^s_{pq}, the ratio V_q/V_n of the mode voltages in the nth and qth branch lines, and the functions f_{mn} and f_{pq}, these latter depending on the offsets and lengths of the two slots, as can be seen from (6) and (7). Equation (4) contains the mutual coupling term MC_{mn} which, from (8), is seen to involve the summation on all other slots of the functions g_{mnrs}, weighted by the slot voltage ratio. g_{mnrs} is a complex number arising from the calculation of a double integral which depends on the lengths on the mnth and rsth slots and their relative positions. Equation (5) gives the desired admittance level C_n in the nth branch line and assumes a standing wave feed. (A more general form for (5) can be used when a traveling wave feed is employed.)

Briefly stated, the design technique proceeds as follows: One assumes initial offsets and lengths for all the slots and computes every g_{mnrs}. Using the desired aperture distribution V^s_{rs}/V^s_{mn}, one then calculates every MC_{mn}. Next, for the mnth slot, one searches for a couplet (x_{mn}, y_{mn}) that will make the denominator of (4) pure real, that is, one seeks to make

$$\mathcal{I}m\left[\frac{(2/K)L^2_{mn}(x_{mn}, y_{mn})}{h_1(y_{mn})+jh_2(y_{mn})}+MC_{mn}\right]=0. \tag{9}$$

The realization emerges that there is a *continuum* of couplets that will satisfy (9). Similarly, there is a continuum of couplets (x_{pq}, y_{pq}) that will satisfy (9) for the pqth slot. But, for a particular couplet (x_{mn}, y_{mn}), there is only one (x_{pq}, y_{pq}) that will also satisfy (3). Which particular couplet (x_{mn}, y_{mn}) to choose depends on (5). Thus (3), (4), and (5) jointly determine all the slot offsets and lengths. Of course, one must iterate, since this improved information about offsets and lengths permits improved calculation of MC_{mn}. Iterating can be terminated when all newly found couplets are negligibly different from the previous set or when, for a given set of couplets, the computed slot voltage distribution gives a satisfactory pattern.

The purpose for reviewing the design procedure is so we can focus on the crucial step embodied in satisfying (9), the first term of which can be recognized as an altered form of

$$\frac{2f^2_{mn}}{Y(x_{mn}, l_{mn})/G_0}.$$

We thereby appreciate that (9) is being satisfied by detuning the mnth slot (changing its length away from self-resonance) so as to cancel out the imaginary component of MC_{mn}. But a study of Stegen's $h_1(y_{mn})$ and $h_2(y_{mn})$ curves (and his curves are typical) shows that the useful dynamic range of y_{mn} is about ± 2.5 percent around unity. (This is normally sufficient to achieve a satisfactory design.) However, if our knowledge of resonant length were in error by only 1 percent, that would imply an error in $h_1(y_{mn})$ + $jh_2(y_{mn})$ far more serious. Indeed, for many slots that error would swamp the desired compensation for mutual coupling. At best, such an error in our knowledge of $v(x)$ would cause the array to perform satisfactorily at other than the design frequency. More likely, it would also cause a degraded sidelobe level and a degraded input admittance over the entire frequency band. It is for this reason that input experimental data on $Y(x, l)/G_0$ must be obtained with great care. For the same reason, if one is planning to substitute a theoretical calculation of $Y(x, l)/G_0$ for experimental data, a high reliability for those calculations must be established. We now address that problem.

An Integral Equation for the Slot Aperture E-Field

The boundary conditions

$$H_x^{\text{inc}}(P)+H_x^{\text{int}}(P)=H_x^{\text{ext}}(P) \tag{10}$$

$$H_z^{\text{inc}}(P)+H_z^{\text{int}}(P)=H_z^{\text{ext}}(P) \tag{11}$$

can be used to develop integral equations for the unknown slot aperture field $\mathbf{E}(P')$ if the wall is vanishingly thin. The slot geometry is shown in Fig. 1; P and P' are points in the slot aperture, $\mathbf{H}^{\text{inc}}$ is the incident TE_{10} mode, $\mathbf{H}^{\text{int}}$ is the internal magnetic field scattered by the slot, and $\mathbf{H}^{\text{ext}}$ is the external field scattered by the slot. Taken together, (10) and (11) represent the statement that the total tangential **H**-field is being matched across the slot aperture. This is true even when the guide is dielectric-filled.

Stevenson's Green's functions and the external half-space Green's function can be used to formulate $\mathbf{H}^{\text{ext}}_{\text{tang}} - \mathbf{H}^{\text{int}}_{\text{tang}}$ as a pair of integral equations, with the unknown tangential slot aperture electric field components $E_x(P')$, $E_z(P')$ occuring in the integrands. A method of moments type of solution can yield approximations to $E_x(P')$ and $E_z(P')$. We have followed this procedure, dividing the slot aperture into a two dimensional array of rectangular cells, using pulse functions and point matching, and have found for standard X-band waveguide and narrow slots ($w \leq 0.063$ in) that the maximum value of $E_z(P')$ in the slot aperture is less than one three thousandth of the maximum value of $E_x(P')$. We therefore decided to ignore the longitudinal component of E-field in the slot, to discard (10), and to cast (11) in the form

$$\begin{aligned}H_z^{\text{inc}}(P)&=jA_{10}\cos\left[\frac{\pi}{a}(x_o+\xi)\right]e^{-j\beta_{10}\zeta}\\&=H_z^{\text{ext}}(P)-H_z^{\text{int}}(P)=\int_{\text{slot}}E_x(P')G(P, P')\,dS'\end{aligned} \tag{12}$$

where A_{10} is the amplitude of the incident TE_{10} mode and

$$\begin{aligned}G(P, P')=&\left(\frac{\partial^2}{\partial\zeta'^2}+k_0^2\right)\left(\frac{e^{-jk_0R}}{2\pi j\omega\mu_0R}\right)+\frac{2}{j\omega\mu_0ab}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\epsilon^2_{mn}}{\gamma_{mn}}\\&\cdot\left\{\cos\left[\frac{m\pi}{a}(x_0+\xi)\right]\cos\left[\frac{m\pi}{a}(x_0+\xi')\right]\right.\\&\left.\cdot\left(\frac{\partial^2}{\partial\zeta'^2}+k^2\right)e^{-\gamma_{mn}|\zeta-\zeta'|}\right\}.\end{aligned} \tag{13}$$

In (13), $\epsilon^2_{00} = 1/4$, $\epsilon^2_{m0} = \epsilon^2_{0n} = 1/2$, $\epsilon^2_{mn} = 1$ otherwise. Also, $k_0^2 = \omega^2\mu_0\epsilon_0$, $k^2 = \omega^2\mu_0\epsilon$ with ϵ the permittivity of the dielectric filling the guide, and

$$\gamma_{mn}=\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2-k^2}\quad R=\sqrt{(\xi-\xi')^2+(\zeta-\zeta')^2}$$

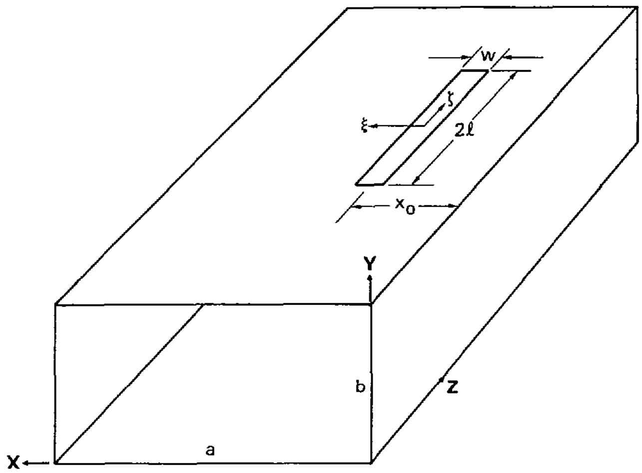

Fig. 1. Geometry of waveguide-fed longitudinal slot.

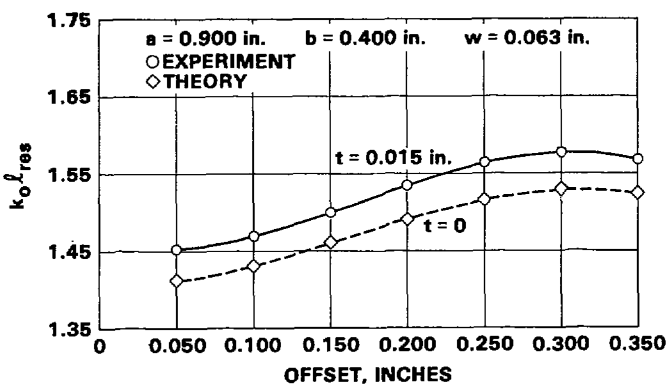

Fig. 2. Khac-type theory versus experiment. Resonant length versus offset for longitudinal slots. Square-ended slots in theory and experiment. No cell added. Wall thickness not taken into account.

where the local coordinates defined in Fig. 1 are used to represent $P(\xi, \zeta)$ and $P'(\xi', \zeta')$.

By dividing the slot aperture into N rectangular cells of size w by $(2l/N)$, by assuming $E_x(P')$ to be constant in a cell, and by satisfying (12) at the central point of each cell, we were able to reproduce Khac's results. The approximate solution obtained for $E_x(P')$ was used in the dominant mode backscattering term of Stevenson's Green's function to determine B_{10}, the amplitude of the backscatter. B_{10}/A_{10} was then used to find $Y(x, l)/G_0$, with resonance occurring when B_{10}/A_{10} was pure real negative. We found, as had Khac, that the computed results were typically in excellent agreement with experiment for $h_1(y)$, $h_2(y)$, and $g(x)$, but satisfactory agreement was not achieved for $v(x)$. For example, Fig. 2 shows the comparison between theory and experiment for longitudinal slots 0.063 in wide, cut in a 0.015 in broad wall of otherwise standard X-band waveguide ($a = 0.900$ in, $b = 0.400$ in). The actual slots were square-ended, so the principal difference between theory and experiment was that the theory assumed a "zero" wall thickness. We see that the two curves track each other quite well but that the theoretical curve lies 3 percent below the experimental curve. Even if we were to attribute 1 percent to wall thickness,[1] the remaining 2 percent gap is not acceptable when it comes to using this data in the design of arrays.

Attention next turned to improving on Khac's model. First, it was appreciated that, with a "zero" wall thickness, $E_x(P')$ should become infinite at $\xi = \pm w/2$ [8]. If we continue to make Khac's assumption that $E_x(\xi', \zeta')$ is separable, a better choice than pulse functions is to assume that

$$E_x(P') = \frac{w/2}{\sqrt{(w/2)^2 - (\xi')^2}} F(\zeta'). \tag{14}$$

This causes (12), after an integration by parts, to adopt the form

$$\begin{aligned} jA_{10} \cos\left[\frac{\pi}{a}(x_0+\xi)\right] e^{-j\beta_{10}\zeta} &= \frac{k_0^2}{2\pi j\omega\mu_0} \int_{-l}^{l}\int_{-w/2}^{w/2} F(\zeta') \frac{w/2}{\sqrt{(w/2)^2-(\xi')^2}} \\ &\cdot \frac{e^{-jk_0R}}{R}\, d\xi'\, d\zeta' \\ &- \frac{1}{2\pi j\omega\mu_0} \int_{-l}^{l}\int_{-w/2}^{w/2} \frac{dF(\zeta')}{d\zeta'} \frac{w/2}{\sqrt{(w/2)^2-(\xi')^2}} \\ &\cdot (1+jk_0R)(\zeta-\zeta') \frac{e^{-jk_0R}}{R^3}\, d\xi'\, d\zeta' \\ &+ \frac{\pi w}{j\omega\mu_0 ab} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\epsilon_{mn}^2}{\gamma_{mn}} J_0\left(\frac{m\pi w}{2a}\right) \\ &\cdot \cos\frac{m\pi x_0}{a} \cos\left[\frac{m\pi}{a}(x_0+\xi)\right] \\ &\cdot \int_{-l}^{l} F(\zeta') \cdot \left(\frac{\partial}{\partial \zeta'^2} + k^2\right) e^{-\gamma_{mn}|\zeta-\zeta'|}\, d\zeta'. \end{aligned} \tag{15}$$

In (15), J_0 is the Bessel function of zero order of the first kind. This equation is the departure point for various models which we employed. The most successful models all satisfied the boundary condition $E_x(P') = 0$ at $\zeta' \pm l$, that is, $F(l) = F(-l) = 0$. Good results were obtained with a piecewise sinusoidal formulation for $F(\zeta')$, viz.,

$$F(\zeta') = \sum_{n=1}^{N} K_n F_n(\zeta') \tag{16}$$

in which

$$F_n(\zeta') = \begin{cases} \dfrac{\sin\,[k_0(\zeta' - \zeta_{n-1})]}{\sin\,[k_0(\zeta_n - \zeta_{n-1})]}, & \zeta_{n-1} \le \zeta' \le \zeta_n \\[2ex] \dfrac{\sin\,[k_0(\zeta_{n+1} - \zeta')]}{\sin\,[k_0(\zeta_{n+1} - \zeta_n)]}, & \zeta_n \le \zeta' \le \zeta_{n+1} \end{cases}$$

with

$$\zeta_n = n\frac{2l}{N+1}, \qquad n = 0,\ 1,\ \cdots,\ N+1.$$

[1] Such an attribution would be crude because it turns out that Oliner's wall thickness correction is sensitive to slot length.

TABLE I
RESONANT LENGTH OF SQUARE-ENDED LONGITUDINAL SLOTS VERSUS OFFSET

	RESONANT LENGTH, INCHES		
OFFSET, INCHES	PIECEWISE SINUSOIDAL	GLOBAL GALERKIN	PERCENT DEVIATION
.050	.5935	.5925	.17
.100	.6022	.6017	.09
.150	.6159	.6157	.03
.200	.6328	.6326	.04
.250	.6494	.6486	.13
.300	.6597	.6587	.15
.350	.6595	.6583	.18

Theoretical calculations using (15). a = 0.900 in, b = 0.400 in, w = 0.0625 in, t = 0 in, ν = 9 GHz.

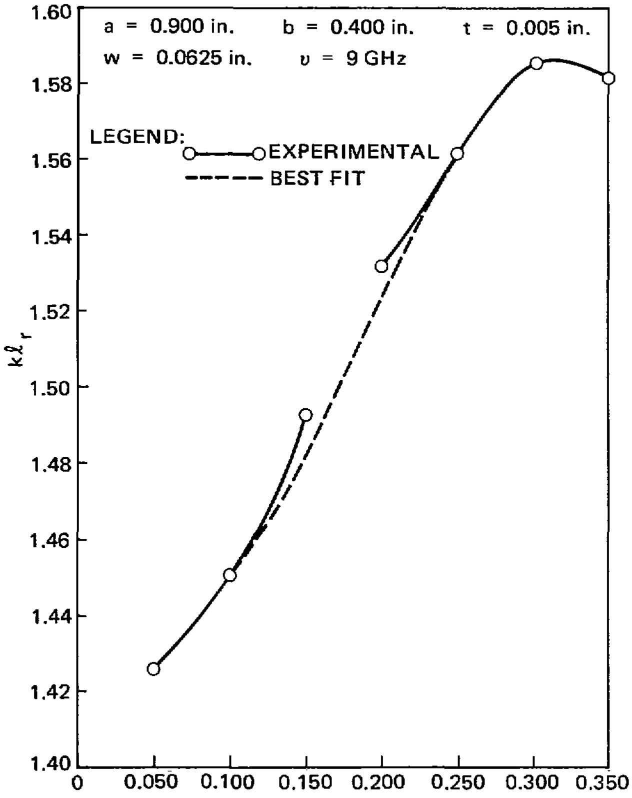

Fig. 3. Experimental values of resonant length versus slot offset.

Good results were also obtained using a global Galerkin formulation, viz,

$$F(\zeta')=\sum_{m=1}^{N} K_m \sin\left[\frac{m\pi}{2l}(\zeta'+l)\right]. \tag{17}$$

As with Khac's model, use of (16) or (17) gave satisfactory agreement with experiment for $h_1(y)$, $h_2(y)$, and $g(x)$. But the crucial question remained "how good is the agreement for $v(x)$?" In anticipation of the desire to test a set of fixtures fabricated to address this question, computations were undertaken, using either (16) or (17) in conjunction with (15), for the case a = 0.900 in, b = 0.400 in, w = 0.0625 in, t = 0 in, and ν = 9 GHz. In using (16), we chose $\zeta_{n+1} - \zeta_n = 0.02$ in; in using (17), we chose $M = 20$.

Table I indicates the deviations between the two calculations for various offsets; the maximum deviation is 0.18 percent; the average deviation is only 0.11 percent. Computations using the piecewise sinusoidal model were less costly.

Test Fixtures

We desired to test the theoretical results for resonant length displayed in Table I, by constructing a family of fixtures, each containing one slot (but with different offsets), and with the slot imbedded in as thin a wall as possible. The technique which proved most satisfactory involved the machining of aluminum mandrels of rectangular cross section, with a = 0.900 in and b = 0.400 in. Each mandrel contained a raised rectangular boss whose position and dimensions (0.0625 in by $2l$) corresponded to the size and placement of the desired slot. Copper was electroplated onto the aluminum mandrel to a depth of 0.005″ after which a central area 1 in by 2 in, and including the boss, was masked. Electroplating was then continued until all unmasked walls were 0.05 in. The result, after the aluminum mandrel was etched away, was a piece of standard X-band waveguide except for a rectangular region around a square-ended slot, that region being only 0.005 in thick.

Seven such fixtures were constructed, each 7.6590 in long ($4\lambda_g$ at 9 GHz), and were fitted with flanges at both ends. The slot offsets relative to the waveguide centerline were to be 0.050 in, 0.100 in, $\cdots$, 0.350 in and the desired lengths $2l$ were read from the piecewise sinusoidal data of Table I.

Two of the fixtures proved defective (those for 0.150 in and 0.200 in offset). Both had rippled walls in the thin wall region and the 0.150 in fixture additionally had a bump in one broad wall near a flange. Added to this visual evidence was the experimental observation that each had a measured length about one percent higher than one would predict using a best-fit curve, as can be seen from Fig. 3.

Thus our sample was reduced from seven fixtures to five. The actual slot offsets and lengths for the remaining five fixtures are entered in the first and second columns of Table II.

The actual lengths of all five fixtures were within 0.001 in of specification and each slot was equidistant from the two ends of its fixture within 0.001 in. Four equispaced measurements of the **a** dimension of each fixture were made and the maximum deviation from 0.900 in was 0.0005 in.

After a fixture was flush-mounted into a 6.5 in by 8 in ground plane, dominant mode forward and backward scatter were measured on an automatic network analyzer over the frequency range 8 to 10 GHz. Column 3 of Table II lists the resonant frequencies for the five slots.

TABLE II
RESONANT LENGTH VERSUS OFFSET FOR SQUARE-ENDED LONGITUDINAL SLOTS; COMPARISON OF THEORY AND EXPERIMENT

ACTUAL SLOT OFFSET, INCHES	ACTUAL SLOT LENGTH, INCHES	MEASURED RESONANT FREQUENCY, MHz	SLOT LENGTH ADJUSTED TO 9 GHz	THEORETICAL COMPUTATION OF RESONANT LENGTH	PERCENT DEVIATION
.0507	.5960	8987	.5951	.5946	-.08
.1000	.6031	9031	.6053	.6035	-.30
.2020	.6486	9034	.6516	.6499	-.26
.3005	.6599	9028	.6625	.6586	-.59
.3498	.6597	8999	.6596	.6597	+0.02

$a = 0.900$ in, $b = 0.400$ in, $w = 0.0625$ in, $t = 0.005$ in, $\nu = 9$ GHz (nominal).

TABLE III
RESONANT LENGTH OF SQUARE-ENDED LONGITUDINAL SLOTS VERSUS OFFSET; THEORETICAL CALCULATIONS

	FULL-HEIGHT GUIDE, b = .400"			QUARTER-HEIGHT GUIDE, b = .100"		
OFFSET, INCHES	$2\ell_{res}$, INCHES	B_{10}/A_{10}	C_{10}/A_{10}	$2\ell_{res}$, INCHES	B_{10}/A_{10}	C_{10}/A_{10}
.050	.5935	-.0229	.0230 $\angle -179.97°$	.6575	-.0832	.0834 $\angle -176.70°$
.100	.6022	-.0832	.0833 $\angle -177.75°$	.7127	-.2546	.2565 $\angle -174.09°$
.150	.6159	-.1615	.1618 $\angle -177.30°$	.8303	-.3960	.4270 $\angle -164.49°$
.200	.6328	-.2400	.2407 $\angle -176.60°$	.9782	-.3877	.6064 $\angle -142.64°$
.250	.6494	-.3077	.3092 $\angle -175.70°$	1.0335	-.2590	.7582 $\angle -130.68°$
.300	.6597	-.3602	.3630 $\angle -174.40°$	1.0291	-.2031	.8295 $\angle -127.87°$
.350	.6595	-.3988	.4018 $\angle -174.52°$	1.0184	-.2072	.8818 $\angle -128.36°$

$w = 0.0625$ in, $t = 0$ in, $\nu = 9$ GHz.

Next, we used computed data for $l_{res}(x)$ at 9 GHz and 10 GHz, plus linear interpolation, to deduce what the actual length of each slot should have been in order for the slot to be resonant at 9 GHz. These values are entered in the fourth column of Table II. Finally, we performed an Oliner-type wall thickness correction [6] on the piecewise sinusoidal curve of Fig. 3, thereby obtaining the entries shown in the fifth column of Table II. The percent deviation between the values shown in columns 4 and 5 is listed in column 6 and can fairly be interpreted as the discrepancy between theory and experiment. We see that the maximum deviation is 0.59 percent; the average deviation is only 0.24 percent. Despite the excessive care we took in measuring back scatter for these five fixtures, we do not believe that our experimental error had an average value as low as one quarter of one percent. Therefore we reach the important conclusion that our theoretical calculations of resonant length versus offset are as trustworthy as the most careful experimental measurements we are able to make. When one pauses to appreciate that the theoretical approach works equally well for a dielectric-filled guide, whereas the experimental approach becomes more difficult, this conclusion becomes even more compelling.

THE SLOT VIEWED AS A SHUNT ELEMENT

It has been common practice to assume that a longitudinal slot is equivalent to a shunt element on a transmission line. This assumption is based on the premise that the dominant mode forward and backward scattering off the slot are equal and in phase, which in turn requires that the electric field in the slot aperture be symmetrical. Our numerical solutions based on (15) can test this assumption. We list in Table III the forward scatter and backward scatter for various slot offsets

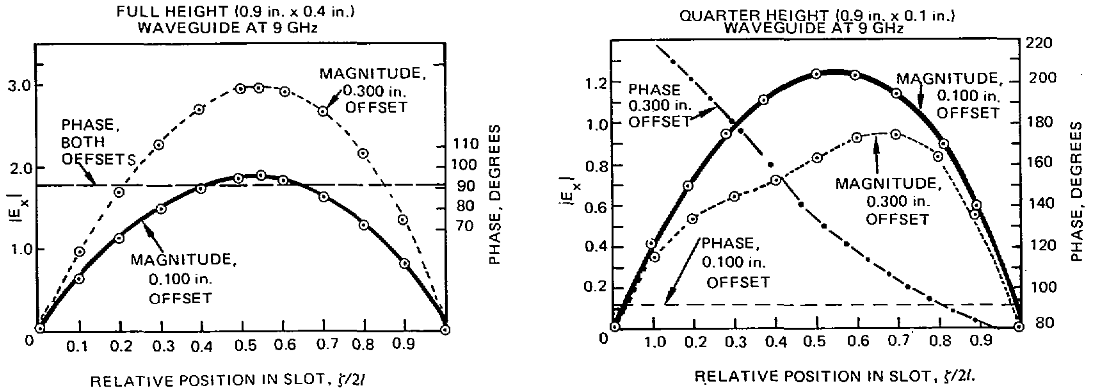

Fig. 4. Electric field in slot aperture versus slot offset for resonant-length slots. Full-height and quarter-height waveguide.

TABLE IV
FORWARD AND BACKWARD SCATTERED TE_{10} WAVES VERSUS SLOT OFFSET AND LENGTH FOR FULL-HEIGHT AND QUARTER-HEIGHT WAVEGUIDE

OFFSET	FULL HEIGHT (0.9" x 0.4") WAVEGUIDE @ 9 GHz				
0.050	LENGTH	0.560	0.580	0.600	0.620
	B/A	−0.015−j.011	−0.021−j.006	−0.023+j.003	−0.019+j.009
	C/A	−0.016−j.011	−0.021−j.007	−0.023+j.002	−0.019+j.008
0.200	LENGTH	0.600	0.620	0.640	0.660
	B/A	−0.2−j.086	−0.234−j.036	−0.24+j.02	−0.22+j.067
	C/A	−0.2−j.097	−0.234−j.05	−0.24+j.005	−0.22+j.05
0.350	LENGTH	0.620	0.640	0.660	0.680
	B/A	−0.35−j.13	−0.39−j.066	−0.4+j.002	−0.39+j.064
	C/A	−0.35−j.15	−0.39−j.098	−0.4−j.035	−0.39+j.021

OFFSET	QUARTER HEIGHT (0.9" x 0.1") WAVEGUIDE @ 9 GHz				
0.050	LENGTH	0.620	0.640	0.660	0.680
	B/A	−0.06−j.036	−0.078−j.020	−0.083+j.003	−0.077+j.023
	C/A	−0.06−j.039	−0.078−j.024	−0.083−j.002	−0.078+j.017
0.200	LENGTH	0.940	0.960	0.980	1.000
	B/A	−0.39−j.044	−0.39−j.021	−0.39+j.002	−0.38+j.026
	C/A	−0.46−j.35	−0.47−j.36	−0.48−j.37	−0.50−j.38
0.350	LENGTH	0.980	1.000	1.020	1.040
	B/A	−0.24−j.037	−0.23−j.017	−0.2+j.0015	−0.18+j.019
	C/A	0.60−j.66	−0.52−j.68	−0.55−j.69	−0.57−j.71

when slot length causes the back scatter to be out of phase with the incident TE_{10} mode. This is done for full-height and quarter-height X-band guide (a = 0.900 in, w = 0.0625 in, t = 0, ν = 9 GHz). We see that B_{10} and C_{10} become more disparate as the slot offset increases and/or as the b dimension decreases.

Concomitant with this is increasing asymmetry in the slot aperture field, as Fig. 4 attests. We see that for full-height guide, the asymmetry is slight enough to justify the equivalent shunt element assumption. However, for quarter-height guide, this is no longer the case. At the modest offset of 0.100 in, the phase of the aperture field is quite flat, but the amplitude is visibly skewed. At the large offset of 0.300 in, both amplitude and phase are strongly asymmetric.

These asymmetries have severe consequences not only at resonance, but also for slots of nonresonant length. Table IV shows B_{10} and C_{10} versus slot offset and length for full-height and quarter-height quide. A study of the entries in this table reveals that, for slot offsets of 0.200 in or more in quarter-height guide, C_{10}/A_{10} does not even go through a resonance, throwing into question the concept of resonant length. Even at 0.050 in offset, C_{10}/A_{10} goes through a resonance at a slot length 1 percent greater than does B_{10}/A_{10}.

The field asymmetries noted in Fig. 4 also have serious implications when one contemplates using an Oliner-type correction for wall thickness, since his theory relies on the assumption that the aperture field is essentially an equiphase half-cosinusoid.

One can conclude from all this that the shunt element assumption is less appropriate at larger offsets and can be more seriously questioned as the **b** dimension is reduced. For quarter-height guide the assumption has doubtful validity.

When it is inappropriate to use the shunt element assumption the design procedure described in [3] can be replaced by one which does away with equivalent circuits and deals directly with scattered waves.

Conclusion

It has been demonstrated that a method of moments type solution can give results for the resonant length of a waveguide-fed longitudinal slot versus its offset that are comparable in accuracy to a carefully performed experiment. This creates the opportunity to dispense with the more costly experimental gathering of input data in the design of slot arrays.

It has also been shown that the standard model in which a longitudinal slot is represented by a shunt element on an equivalent transmission line is less justified at larger slot offsets and for smaller **b** dimensions.

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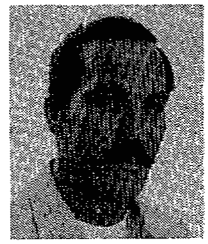

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