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ABSTRACT

The emerging field of compressed sensing deals with the techniques of combining the two blocks of sampling and compression into a single unit without compromising the performance. Clearly, this is not feasible for any general signal; however, if we restrict the signal to be sparse, it becomes possible.

There are two main challenges in compressed sensing, namely the sampling process and the reconstruction methods. In this thesis, we will focus only on the deterministic sampling process as opposed to the random sampling. The sampling methods discussed in the literature are mainly linear, i.e., a matrix is used as the sampling operator. Here, we first consider linear sampling methods and introduce some deterministic designs. The constructed matrices are derived from OOC, BCH and non-binary BCH codes. The cyclic property of BCH codes enables us to implement fast reconstruction methods by using the FFT algorithm. The channel coding matrices are based on the finite Galois field algebra, which restricts the number of rows in such matrices to some subsets of the integer numbers. We also introduce means to combine these matrices to obtain sampling matrices with arbitrary number of rows.

Non-linear sampling methods are discussed in this thesis for the first time. When the sparsity domain is unknown at the time of sampling, no linear sampling method can guarantee perfect recovery; however, we show that non-linear methods can be used to recover ١-sparse signals. Furthermore, if the sparsity domain is known, non-linear methods can reduce both the number of required samples and the reconstruction complexity. The drawback of these methods is their sensitivity to additive noise.

Sparsity and compressibility are fundamental concepts in the field of compressed sensing. Although it is straightforward to define these concepts for finite dimensional vectors, the generalization to the infinite dimension and continuous domain is completely different. On the other hand, in order to be able to apply compressed sensing results to the real world problems, we need to consider continuous signals. Here we show that sparsity and compressibility concepts can be generalized to infinite deterministic and random sequences. Although the generalization from discrete to continuous signals is the main goal in many research works, the well-known generalization deals with substituting the vectors with matrices. For the latter case, instead of the zero/non-zero status of the elements, sparsity is usually defined through the rank of the matrix. In the last part of this thesis, we show how low-rank matrices can be retrieved from their point-wise distorted versions.

KEYWORDS

1. Compressed Sensing.
2. Sparsity.
3. Linear Projection.
4. Nonlinear Sampling.
5. i.i.d. Sequence.
6. Compressibility.
7. Low-rank Matrix.

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