Sensor Selection for Sparse Source Detection in Planar Arrays

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The sensor selection is a technique to reduce the cost, the energy consumption, and the complexity of a system by discarding redundant or less useful sensors. In this technique, one attempts to select a subset of sensors within a larger set so as to optimize a performance criterion. In this work, we consider the detection of sources in the 3-D space when the source space could be sparsely represented. The application of compressive sensing (CS) methods in this setting has been extensively studied. Our aim in this work is to carry out the task of sensor selection in a planar array without sacrificing the detection performance. Our approach is to adopt the equivalent CS model and reduce the mutual coherence of the associated sensing matrix. This minimization task is non-convex, for which we propose a convex relaxation. Numerical simulations demonstrate that the gap between our relaxed solution and its optimal counterpart is negligible. The proposed sensor selection method is also shown to outperform the uniform approach in terms of source detection.

Introduction: On one hand, the research in detection and localization of the sources using sensor arrays has gained substantial attention in a wide range of applications including radar, sonar, and communication systems [1,2]. On the other hand, the theory of compressive sensing (CS) provides an elegant way of exploiting the spatial sparsity of the sources in improving the performance of classical methods [3, 4]. As a result, nowadays we observe multiple applications of CS theory in sparse source detection and localization.

The key ingredients for the success of CS in solving a source detection problem are: (a) the scene should be sparse, indicating that only few sources exist in the direction of arrival (DOA) grid space, and (b) the associated sensing matrix should be well-behaved for the sparse recovery. The former is valid in most setups. For the latter, a number of performance ensuring properties are introduced. The most well-known is the restricted isometry property (RIP) [5] that guarantees stable recovery even in the noisy scenarios. Unfortunately, it is NP-hard to verify that a given sensing matrix satisfies RIP of a desired order. The mutual coherence has been commonly used as a practical alternative to RIP for ensuring the success of sparse recovery methods. The mutual coherence of the matrix $\mathbf{\Phi}$ is defined as

$$\mu(\mathbf{\Phi}) = \max_{i \neq j} \frac{|\boldsymbol{\phi}_i^H \boldsymbol{\phi}_j|}{\|\boldsymbol{\phi}_i\| \|\boldsymbol{\phi}_j\|}. \tag{1}$$

where $\boldsymbol{\phi}_i$'s are the columns of $\mathbf{\Phi}$.

In source detection, besides the physics of the problem, the sensing matrix is determined by the number and the location of the sensors. When the number of sensors equal or exceed the size of the DOA grid, simple least-square techniques can recover the sources. However, this approach is wasteful of the resources. In particular, when the sources are sparse, a small fraction of the sensors is sufficient for the successful detection of all the sources in the DOA space, of course with the use of sparse recovery methods. However, the selected sensors shall be such that the associated sensing matrix behaves well.

In general, in the approach of sensor selection, we choose a set of sensors out of a pool of sensors in order to reduce the cost, the weight, and the energy consumption of the system. Moreover, sensor selection helps us to reduce the amount of data to be processed and stored. Depending on the application, the selection rule can be designed based on optimizing different performance measures [6, 7]. For the special case of CS-based source detection, the selection rule affects the overall sensing matrix. Therefore, it is desirable to perform the selection task based on optimizing a property such as the mutual coherence of the sensing matrix, thereby improving the sparse recovery performance. This problem looks combinatorial, and to the best of our knowledge is not addressed in the literature.

In this letter, considering a CS-based source detection system, a sensor selection framework is presented in which the coherence of the sensing matrix is minimized. The selection problem corresponds to optimizing a cost over a $0/1$-valued vector (called selection vector) with a predetermined number of 1s (number of sensors). Then, we relax the $0/1$ constraint and derive a convex problem, the solution of which can be obtained efficiently using off-the-shelve solvers. A quantization algorithm is applied at the end to convert the final selection vector into binary format. The results of an exhaustive search in low-dimensional settings indicate negligible increase in the objective value using the relaxed scheme. In addition, numerical simulations confirm superior performance of the proposed method compared to the uniform selection scheme.

System Model: We consider a 3-D source detection scenario using an $M \times N$ standard rectangular array (SRA) with inter-sensor spacing $d = \lambda/2$, the positions of which in the x-y plane can be represented as $(x_n, y_n) = (nd, md)$, $n = 0, \ldots, N-1$, and $m = 0, \ldots, M-1$.

We refer this $M \times N$ SRA as the complete array in the sequel. Let us assume there exists a source in the far-field of the array with the DOA parameters $u = \sin(\theta)\cos(\phi)$ and $v = \sin(\theta)\sin(\phi)$, where θ and ϕ are the elevation and the azimuth angles, respectively. The steering vector for the complete array, which is denoted by $\mathbf{a}(u,v) \in \mathbb{C}^{MN}$, is given by

$$\mathbf{a}(u,v) = \left\{ \mathrm{e}^{\mathrm{j}\pi(mu+nv)} \right\}_{\substack{m=0,\ldots,M-1 \\ n=0,\ldots,N-1}}. \tag{2}$$

Considering the narrowband assumption, the baseband received signals in the complete array can be written in matrix form as

$$\mathbf{r} = \mathbf{a}(u,v)x + \mathbf{n} \tag{3}$$

where x denotes the transmitted signal and $\mathbf{n}$ is the additive noise term modeled by a circularly symmetric complex Gaussian random vector. It should be mentioned that we consider a single snapshot source detection scenario.

Similarly, when there exists K sources in the surveillance area with DOA parameters (u_k, v_k) and the corresponding transmitted signals x_k, $k = 1, \ldots, K$, the received vector becomes

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{a}(u_k, v_k)x_k + \mathbf{n}. \tag{4}$$

By considering a uniform grid from possible span of DOA parameters (i.e. u and v) as $(\alpha_1, \beta_1), \ldots, (\alpha_G, \beta_G)$ such that $\alpha_i^2 + \beta_i^2 \leq 1$ and $G \gg K$, and assuming that each of the existing sources in the surveillance area lie approximately on the grid points, the received vector $\mathbf{r}$ can be recast using a sparse representation as

$$\mathbf{r} = \mathbf{A}\mathbf{x} + \mathbf{n} \tag{5}$$

where $\mathbf{A} = [\mathbf{a}(\alpha_1, \beta_1), \ldots, \mathbf{a}(\alpha_G, \beta_G)]$ is the sensing matrix. In addition, $\mathbf{x}$ is a sparse vector whose l-th element will be equal to x_k if the k-th source is at the l-th DOA bin (α_l, β_l) and will be zero otherwise.

Equation (5) provides a matrix expression for the received vector in the complete array. Now, we aim to consider the case of a partial array obtained via selecting N_s elements from the complete array. To this end, we define the selection vector $\mathbf{s} = [s_1, s_2, \ldots, s_{MN}]^T$ where s_i is a binary variable being one if the associated array element is selected and otherwise is zero. We shall have

$$\sum_{i=1}^{MN} s_i = N_s. \tag{6}$$

Then, we form a selection matrix $\mathbf{S} \in \mathbb{C}^{N_s \times MN}$ through removing the all-zero rows of diag$(\mathbf{s})$ (there exist $MN - N_s$ of such rows). It is easy to verify that

$$\mathbf{S}^H \mathbf{S} = \mathrm{diag}(\mathbf{s}). \tag{7}$$

In this case, the sparse representation of the received signal in (5) can be written as

$$\mathbf{r} = \tilde{\mathbf{A}}\mathbf{x} + \mathbf{n} \tag{8}$$

where $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}_1, \ldots, \tilde{\mathbf{a}}_G] = \mathbf{S}\mathbf{A}$ and $\tilde{\mathbf{a}}_l = \mathbf{S}\mathbf{a}(\alpha_l, \beta_l)$, $l = 1, \ldots, G$. To recover the sparse vector $\mathbf{x}$ in (8), we employ NESTA, a well-known recovery method in the case of complex vectors and matrices proposed in [8]. To detect possible sources in the scene, we compare the amplitude of the elements of the recovered vector with a predefined threshold determined by a constant false alarm strategy.

Sensor Selection: In this section, we aim to select a given number of sensors from a complete array with the goal of coherence minimization (so as to improve the CS-based detection performance).

The above-mentioned problem can be expressed in general form as

$$\begin{aligned} \min_{\mathbf{s}} \quad & f(\tilde{\mathbf{A}}) \\ \text{s.t.} \quad & \sum_{i=1}^{MN} s_i = N_s \\ & s_i \in \{0,1\},\ i = 1, \ldots, MN \end{aligned} \tag{9}$$

where the objective function is the coherence of the sensing matrix (for the partial array), which is given by

$$f(\tilde{\mathbf{A}}) = \max_{l \neq l'} \frac{\left|\tilde{\mathbf{a}}_{l'}^H \tilde{\mathbf{a}}_l\right|}{\|\tilde{\mathbf{a}}_{l'}\| \|\tilde{\mathbf{a}}_l\|} \tag{10}$$

where $\tilde{\mathbf{a}}_l$ is the l-th column of the sensing matrix $\tilde{\mathbf{A}}$ defined below (8). Note that the tunable parameter in the above objective function is the sensor selection vector. To simplify this function in terms of the selection vector $\mathbf{s}$, it is enough to derive such expression for $\tilde{\mathbf{a}}_{l'}^H \tilde{\mathbf{a}}_l$. Let us denote the notation $\mathbf{a}(\alpha_l, \beta_l)$ in brief by $\mathbf{a}_l$. Then, we have

$$\tilde{\mathbf{a}}_{l'}^H \tilde{\mathbf{a}}_l = (\mathbf{S}\mathbf{a}_{l'})^H (\mathbf{S}\mathbf{a}_{l'}) = \mathbf{a}_{l'}^H \mathrm{diag}(\mathbf{s}) \mathbf{a}_l \tag{11}$$

in which we employed the relation presented in (7). Taking $\mathrm{Tr}\{*\}$ on the right-side of (11) and using the property of Tr operator gives $\tilde{\mathbf{a}}_{l'}^H \tilde{\mathbf{a}}_l = \mathrm{Tr}\{\mathbf{A}_{ll'} \mathrm{diag}(\mathbf{s})\}$, where $\mathbf{A}_{ll'} = \mathbf{a}_l \mathbf{a}_{l'}^H$. Furthermore, in the case of $l = l'$, all diagonal entries of $\mathbf{A}_{ll}$ are equal to 1, which yields

$$\mathrm{Tr}\{\mathbf{A}_{ll} \mathrm{diag}(\mathbf{s})\} = \sum_{i=1}^{MN} s_i = N_s. \tag{12}$$

Hence, for $\|\tilde{\mathbf{a}}_l\|$ we can write

$$\|\tilde{\mathbf{a}}_l\|^2 = \tilde{\mathbf{a}}_l^H \tilde{\mathbf{a}}_l = N_s, \quad \forall l \in \{1, \ldots, G\}. \tag{13}$$

Since the term $\|\tilde{\mathbf{a}}_l\|$ is constant for all l, the denominator of the objective function in (10) can be ignored. Thus, the objective function can be stated as

$$f(\tilde{\mathbf{A}}) = \max_{l \neq l'} |\operatorname{Tr}\{\mathbf{A}_{ll'} \operatorname{diag}(\mathbf{s})\}|. \tag{14}$$

The Relaxed Problem: By considering (14) as the objective function, it is still difficult to solve the resulting problem since the variables (i.e., the selection vector entries) are binary-valued, making the search problem NP-hard. To somehow relax the problem, we allow the selection vector to be continuous-valued in $[0, 1]$. Then, if we be able to solve the resulting problem for the continuous selection vector, there remain a quantization process to obtain the final binary selection vector, which will be discussed in the next section. The relaxed problem can be written as

$$\begin{aligned} \min_{\mathbf{s}} \quad & f(\tilde{\mathbf{A}}) \\ \text{s.t.} \quad & \sum_{i=1}^{MN} s_i = N_s, \qquad \mathbf{0} \le \mathbf{s} \le \mathbf{1} \end{aligned} \tag{15}$$

where $f(\tilde{\mathbf{A}})$ is defined in (14).

The associated problem can be reformulated by incorporating a dummy variable t as

$$\begin{aligned} \min_{\mathbf{s},t} \quad & t \\ \text{s.t.} \quad & |\operatorname{Tr}\{\mathbf{A}_{ll'} \operatorname{diag}(\mathbf{s})\}|^2 \le t, \; l, l' \in \{1, \ldots, G\}, l \neq l' \\ & \sum_{i=1}^{MN} s_i = N_s, \qquad \mathbf{0} \le \mathbf{s} \le \mathbf{1} \end{aligned} \tag{16}$$

The above problem satisfies conditions of a convex problem and so can be solved efficiently using such toolboxes as CVX [9].

Quantiaztion: The solution of problem (16) (i.e., the selection vector) is continuous-valued and must somehow be quantized to obtain the final selection. To quantize a selection vector one may replace N_s largest values in $\mathbf{s}$ with one and the other entries with zero. However, a more suitable way to quantize such vector would be available using the so-called randomized rounding. In this method, the obtained continuous values are regarded as the probabilities of selecting the corresponding sensors. Then, several realizations of binary-valued selection vectors are generated from these probabilities, among which the final selection vector is adopted such that satisfies the problem constraint (6) and has the smallest objective value. For more details on randomized rounding see [7].

Numerical Results: In this section, we aim to assess the performance of the proposed sensor selection method. First, we evaluate optimality level of the proposed method (i.e., solving the relaxed problem and rounding) by comparing its achieved objective value with the optimal value obtained through an exhaustive search over all possible selections. To this end and to make the exhaustive search possible, we consider a small 5×5 SRA. Such an array offers a resolution proportional to $1/M$ in both u and v domains. Accordingly, we constitute a uniform two-dimensional grid over $u \in [-1, 1]$ and $v \in [-1, 1]$ in the visible region $u^2 + v^2 \le 1$ with the spacing $1/M$. For different values of N_s, we solve the relaxed problem (16) to obtain the continuous-valued selection vector. Then, we conduct randomized rounding with 1000 independent feasible samples (i.e. ones who satisfying (6)) to obtain the final binary-valued selection vectors and report the achieved coherence values for the resulting partial arrays. We also include the results of the uniform sensor selection, i.e., applying the randomized rounding process to a uniform selection vector (a vector with equal elements satisfying (6)). This indeed translates to realizing 1000 independent selections using uniform probabilities and choosing the one with the least coherence value. The obtained objective value of the proposed suboptimal method is compared with its optimal counterpart and the uniform method in Fig. 1 for different values of N_s. As shown in this figure, the proposed method while consistently achieves lower values than those of the uniform method, exhibits close behavior to that of the optimal selections (It has achieved exactly the same value as optimal in most cases).

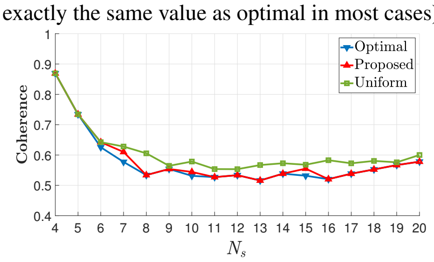

Fig. 1: Comparison of the achieved coherence values of the proposed method with the optimal values and uniform sensor selection for a 5×5 SRA.

To evaluate the detection performance of the proposed method, we consider a larger and more realistic SRA with $M = N = 10$. As before, the spacing of the grid (both in u and v domains) is set to $1/M$. In Fig. 2, the achieved coherence values of our proposed method and the uniform method are depicted which again demonstrates that our method significantly outperforms the uniform sensor selection. Moreover, using a Monte Carlo simulation with 10000 independent runs, we obtain the curves of the probability of detection P_d versus SNR for a fixed probability of false alarm $P_{fa} = 10^{-5}$. In particular, in each run, K sources are realized randomly on the assumed grid. Source signals are modeled as $e^{j\phi_t}$ where ϕ_t is uniformly distributed over $[0, 2\pi]$. Using NESTA, we attempt to recover the scene from the noisy measurements. A source is detected at a specific grid point if its corresponding value in the recovered vector exceeds a prescribed threshold. The threshold value itself is determined according to the value of P_{fa} which is set to 10^{-5} and computed over all runs. Note that P_{fa} equals the total number of source declarations in the grid points in which there are no sources, divided by the total number of such grid points. In addition, P_d is computed by dividing the number of exact detections by the total number of sources (here $K \times 10000$). The above procedure is conducted for different values of the SNR to obtain the curves. In Fig. 3, the results are shown for the proposed method and the uniform approach for two different settings using $N_s = 26, K = 3$ and $N_s = 30, K = 4$, which demonstrates the superiority of our method in CS-based detection.

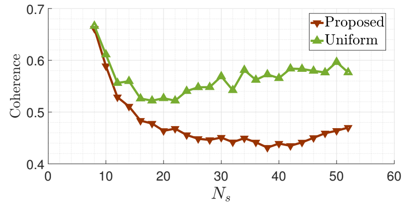

Fig. 2: The coherence values vs N_s for sensor selection in a 10×10 SRA.

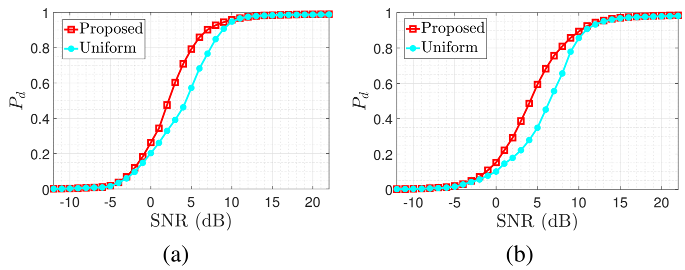

Fig. 3: Evaluation of the CS-based detection performance through P_d vs SNR curves with $P_{fa} = 10^{-5}$: (a) $N_s = 26, K = 3$ and (b) $N_s = 30, K = 4$.

Conclusion: In this letter, a sensor selection scheme in a CS-based source detection scenario was proposed. The sensor selection approach considered in this letter is based on minimizing the coherence of the sensing matrix so as to improve the CS-based source detection performance of the resulting partial array. Due to being combinatorial in nature, solving the sensor selection problem is intractable when the size of array is large. Thus, we relaxed the aforementioned problems utilizing the continuous-valued selection vectors and obtained the sub-optimal selection using convex programming followed by the randomized rounding technique for quantization. The proposed sensor selection method was shown to provide desirable performance in multiple source detection.

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