General Physics I

chapter 6

Sharif University of Technology
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M. Reza Rahimi Tabar

Chapter 6
Force and Motion-II

6-1 FRICTION
What Is Physics?

6-2 THE DRAG FORCE AND TERMINAL SPEED

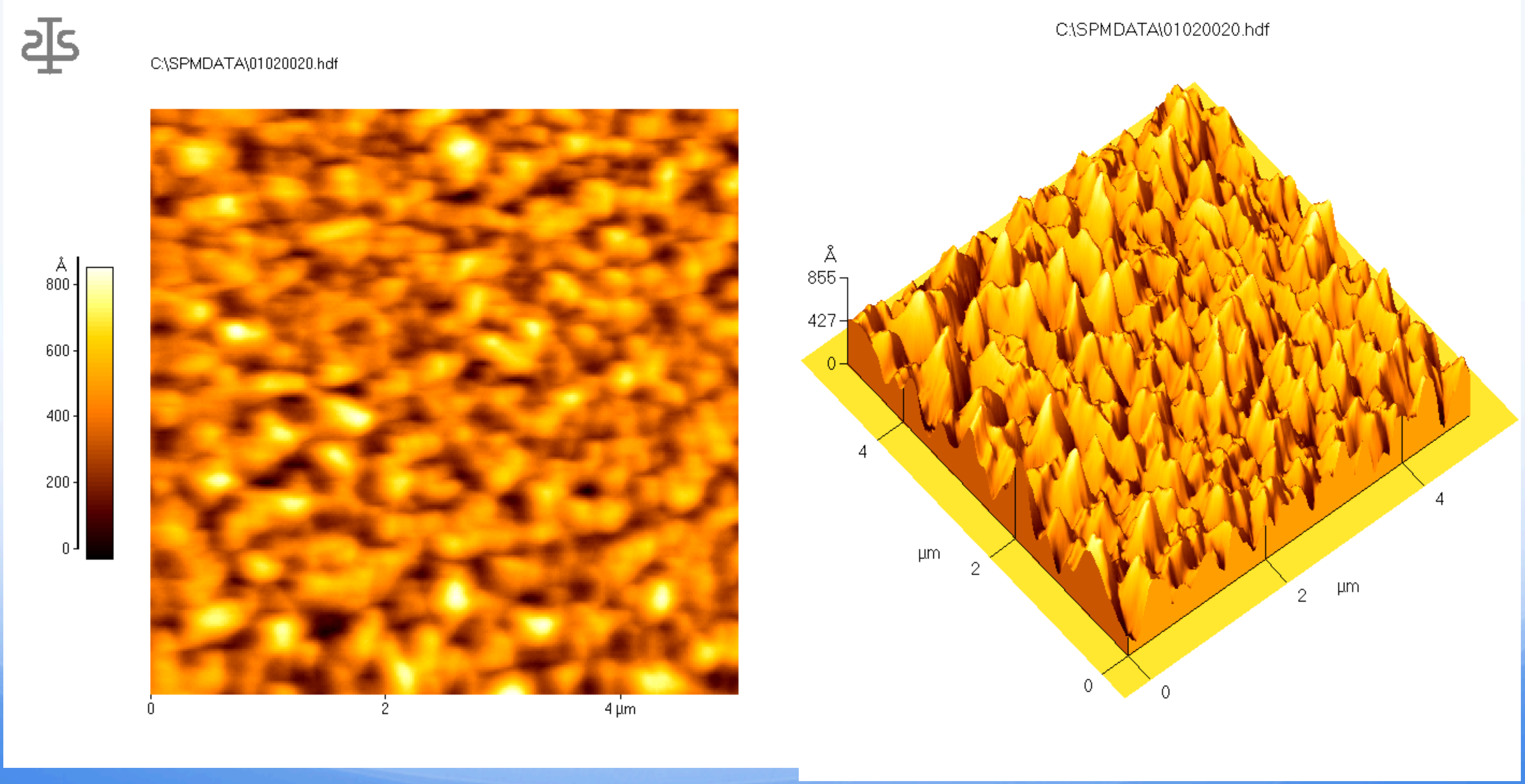
6-3 UNIFORM CIRCULAR MOTION

Karl-Josef Hildenbrand/dpa/Landov LLC

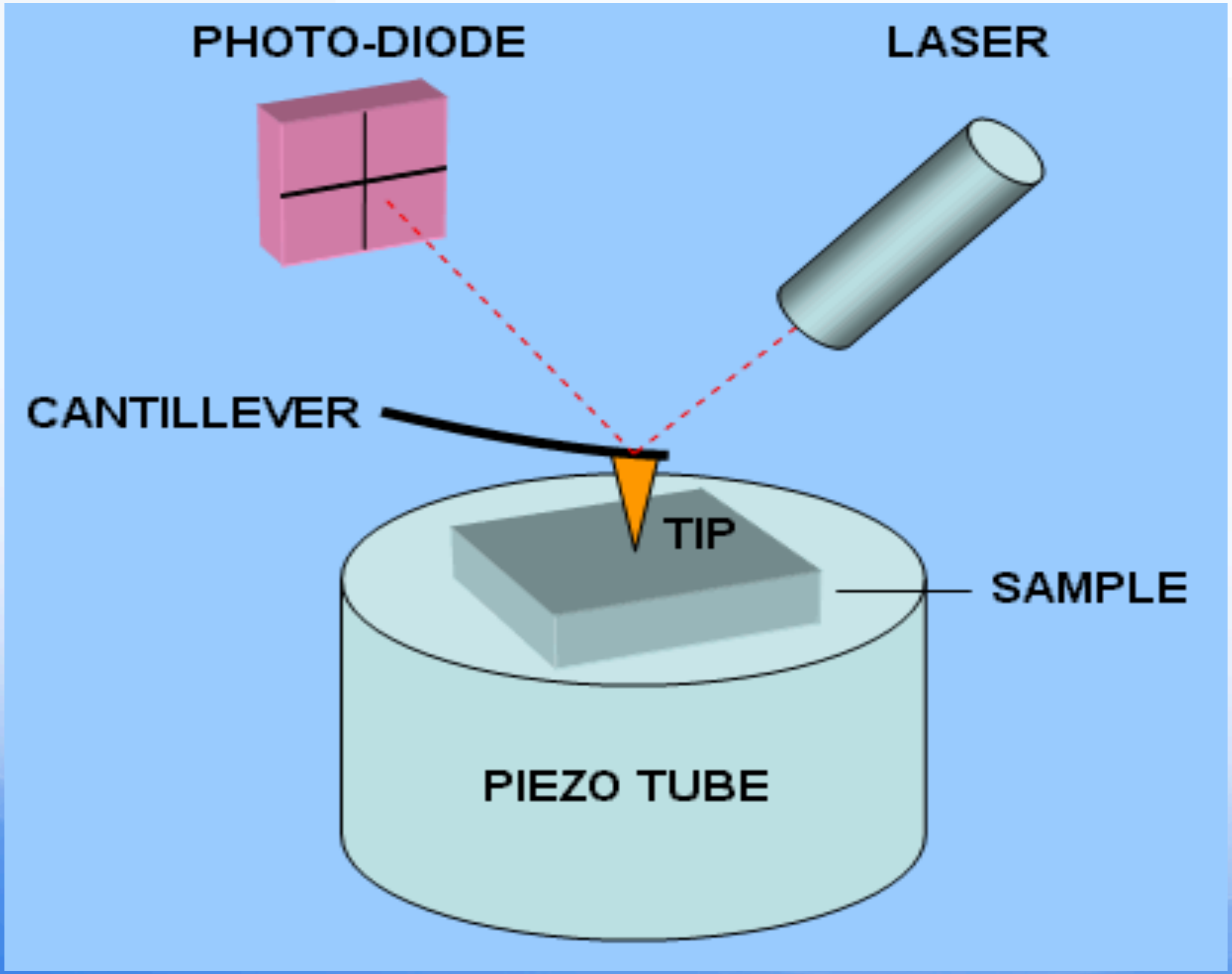
Frictional Forces

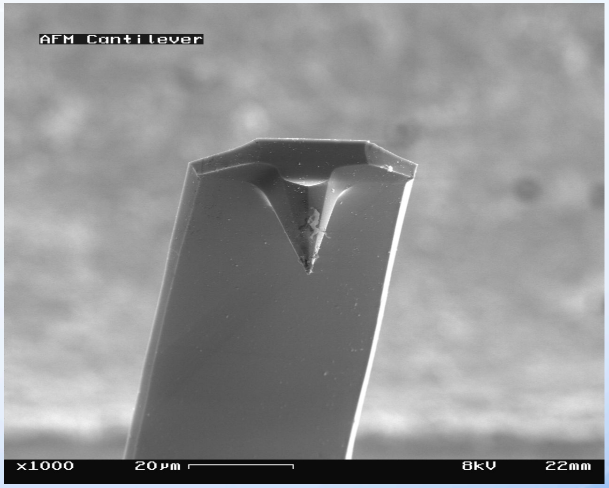
- *RESISTIVE* force between object and neighbors or the medium
- Examples:
  - Sliding a box
  - Rolling resistance
  - Air resistance

Image AFM of Cu Surface



How AFM (atomic force microscopy) works

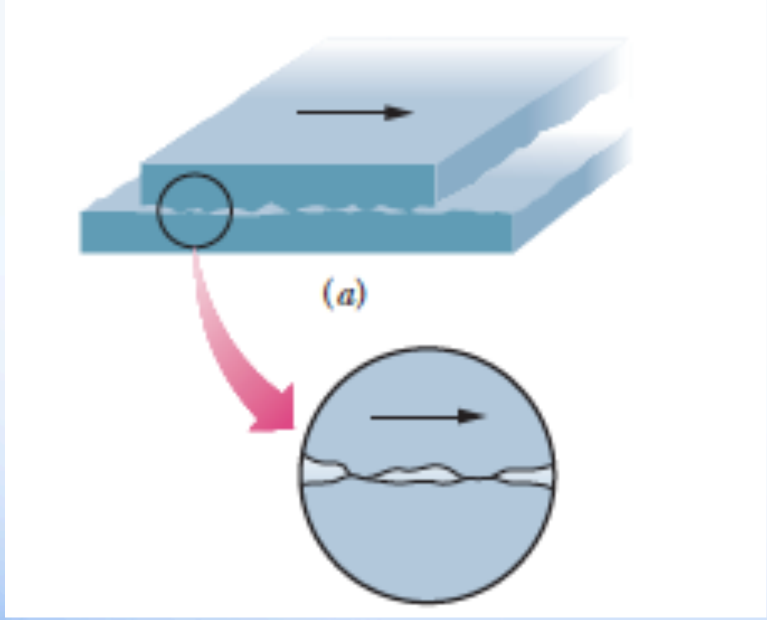


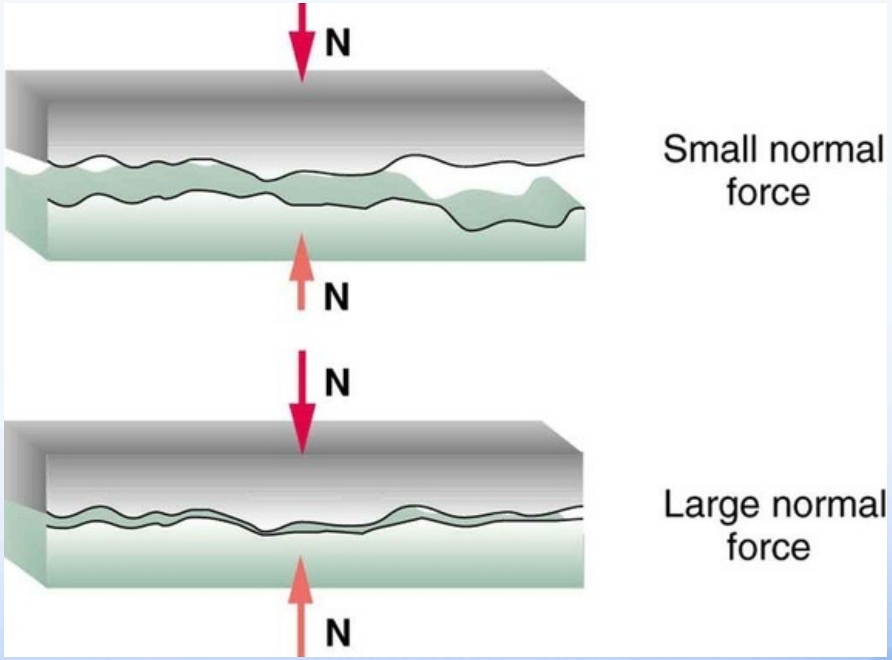


Atomic origin of friction:

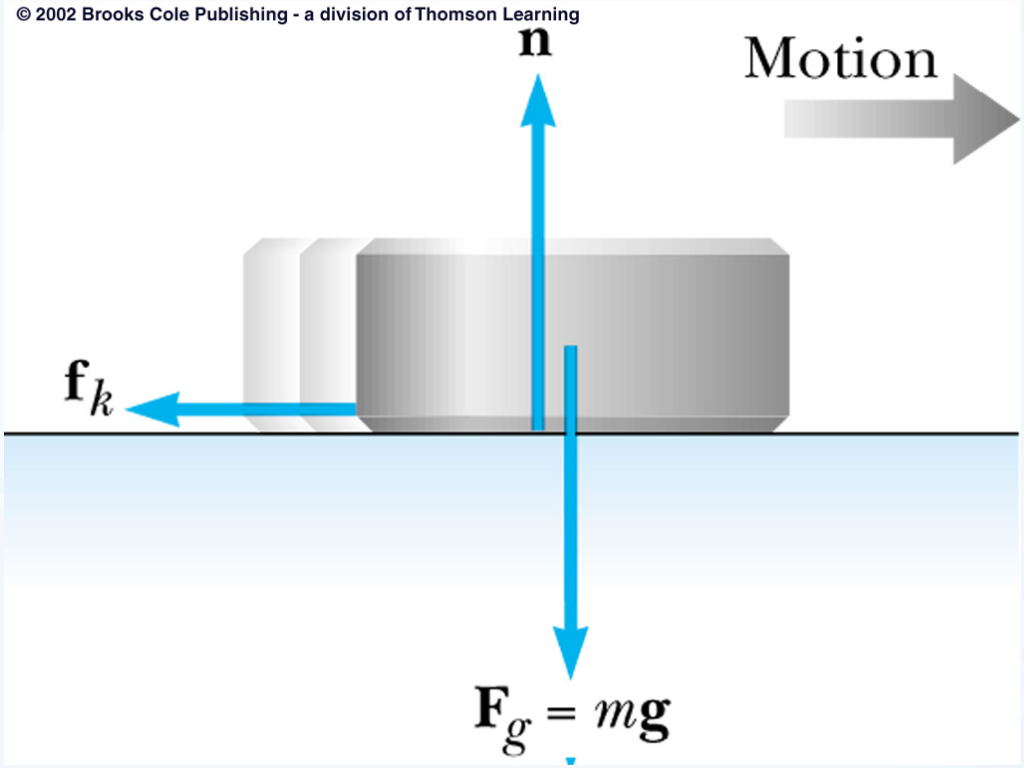
- J. Krim, Atomic-Scale Origins of Friction, Langmuir 12, 4564, 1996.

- http://www.physics.ncsu.edu/nanotribology/publications/ref60.pdf


(a)


N
Small normal force
N
N
Large normal force
N

Sliding Friction



- Parallel to surface,

  opposing direction of motion

- Depends on the surfaces in contact
  - Object at rest: Static friction
  - Object in motion: Kinetic friction

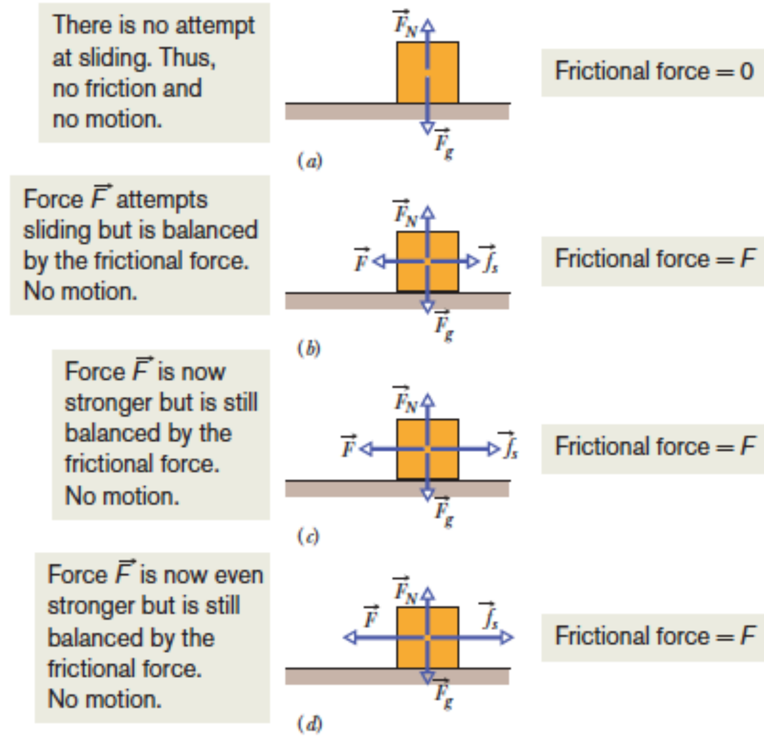


Figure 6-1 (*a*) The forces on a stationary block. (*b–d*) An external force $\vec{F}$, applied to the block, is balanced by a static frictional force $\vec{f}_s$. As F is increased, f_s also increases, until f_s reaches a certain maximum value. (*Figure continues*)

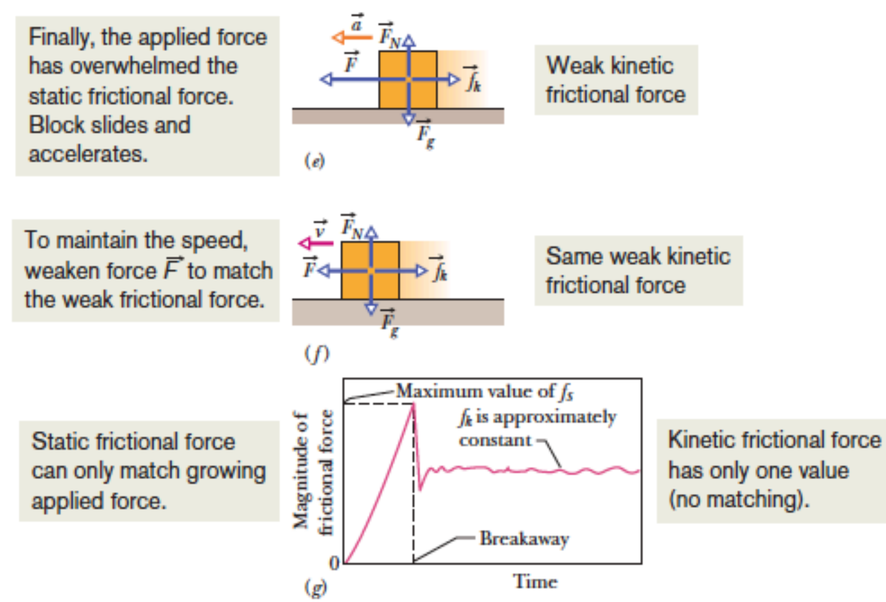
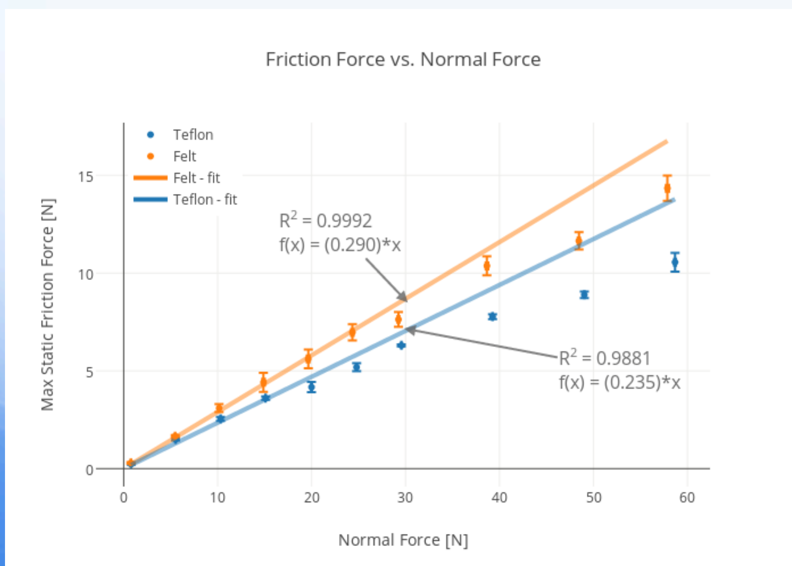


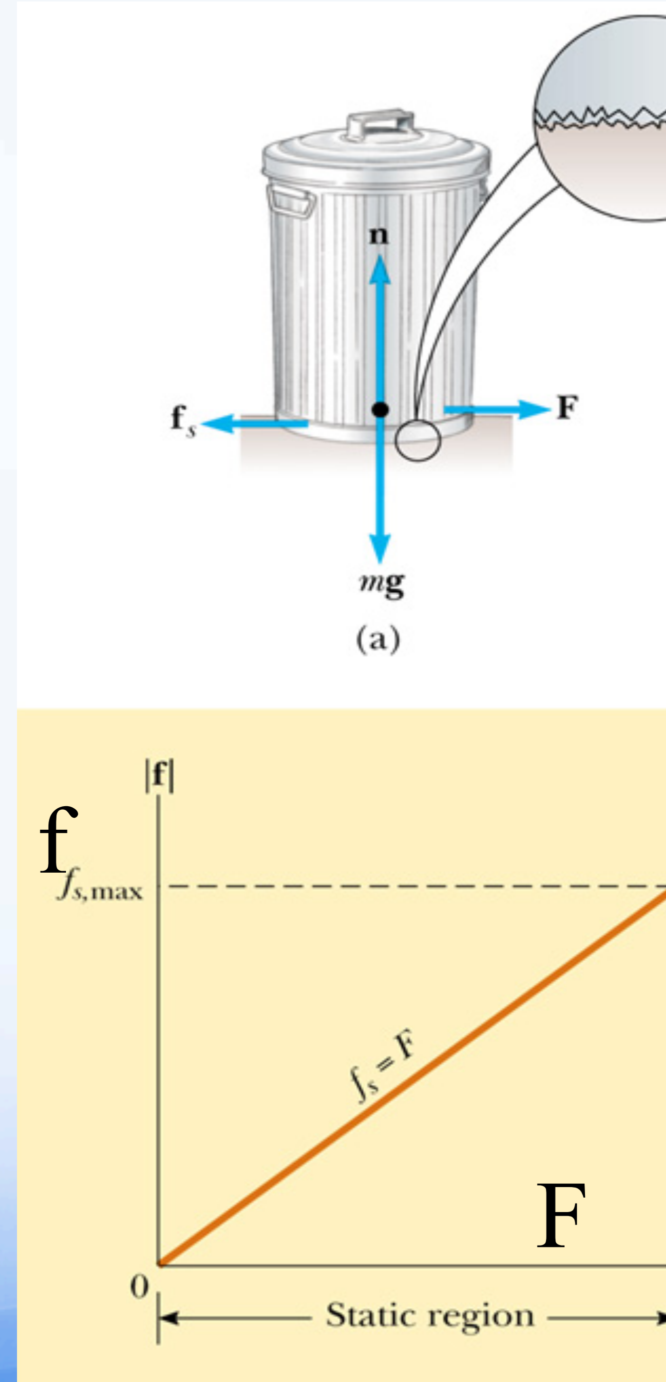
Figure 6-1 (*Continued*) (*e*) Once f_s reaches its maximum value, the block "breaks away," accelerating suddenly in the direction of $\vec{F}$. (*f*) If the block is now to move with constant velocity, F must be reduced from the maximum value it had just before the block broke away. (*g*) Some experimental results for the sequence (*a*) through (*f*). **In *WileyPLUS*, this figure is available as an animation with voiceover.**

Static Friction, f_s

$$f_s \leq \mu_s N$$

- Just enough force to keep object at rest.
- μ_s is coefficient of static friction
- *N* is the normal force

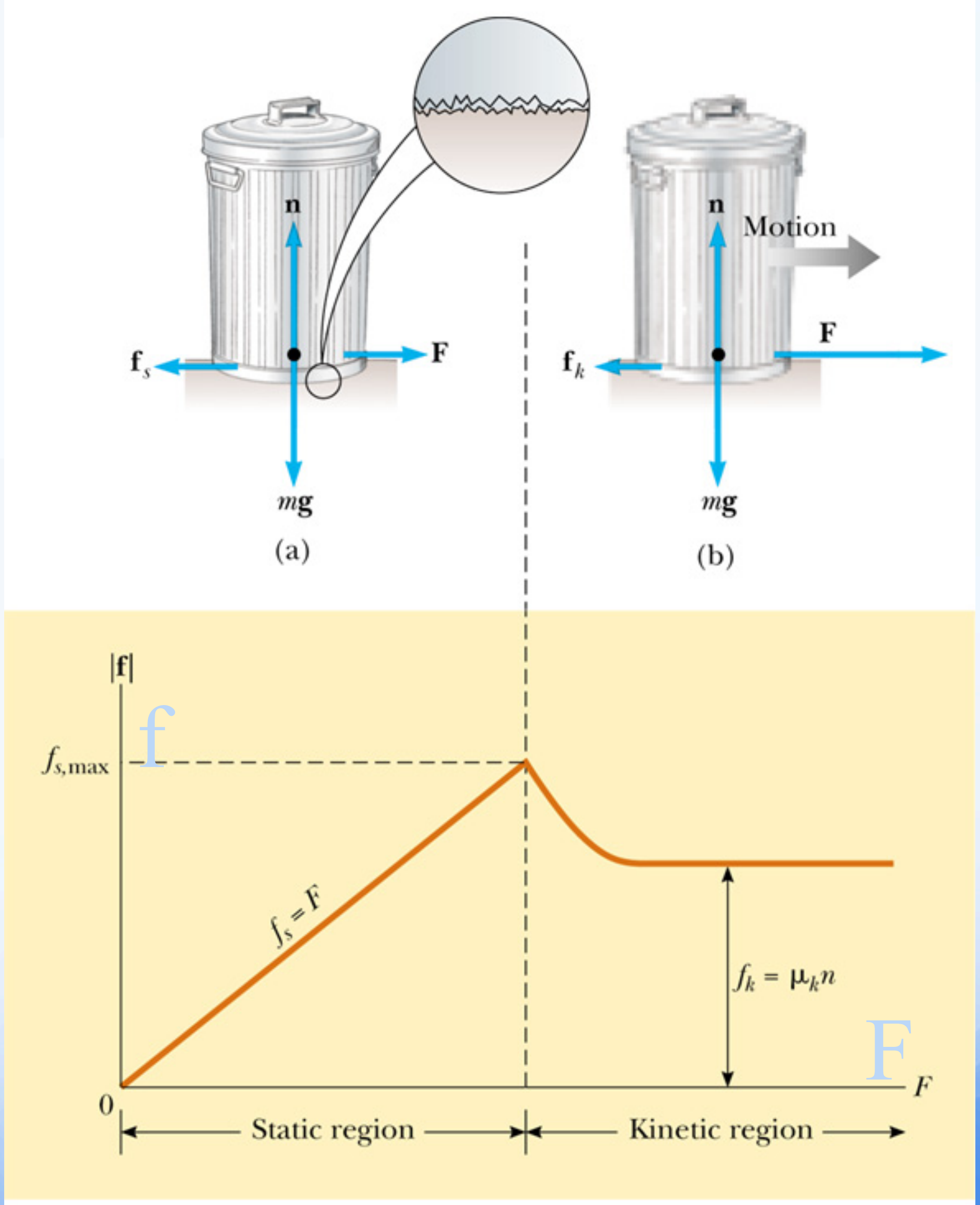




Kinetic Friction, f_k

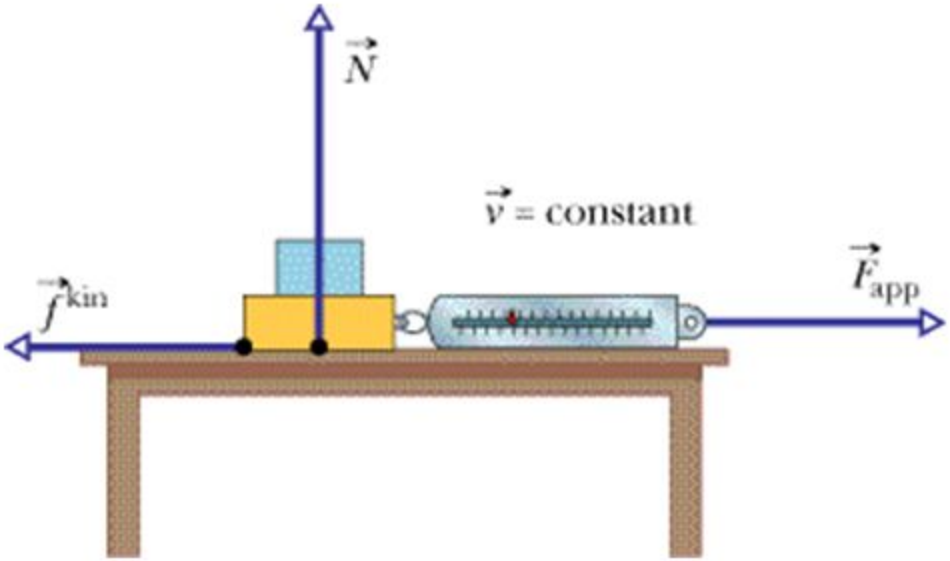
$$f_k = \mu_k N$$

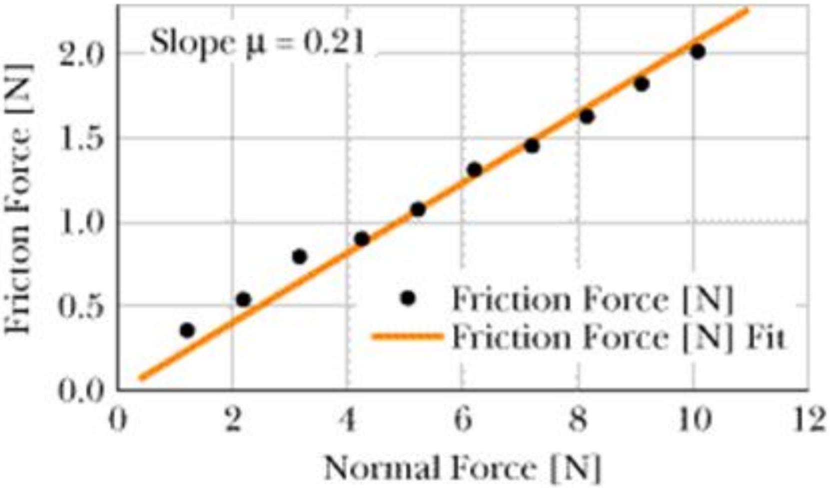
- μ_k is coefficient of kinetic friction
- Friction force opposes direction of motion
- N is the normal force



Kinetic Friction Forces

An object is experiencing a **kinetic friction force** when the object is moving relative to a surface.

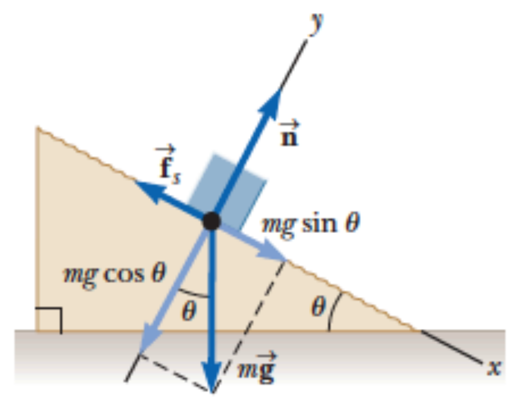




- *Kinetic friction forces is linearly proportional to the normal force.*
- *The slope of Kinetic friction forces vs. the normal force is changing for different surfaces.*

Example 5.11 Experimental Determination of μ_s and μ_k

The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in Active Figure 5.18. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.



ACTIVE FIGURE 5.18

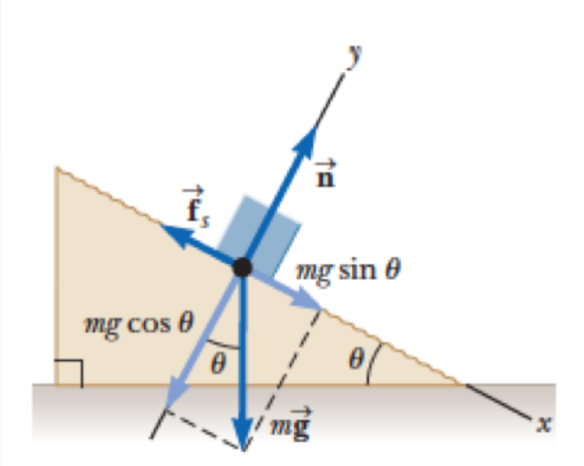
(Example 5.11) The external forces exerted on a block lying on a rough incline are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of friction $\vec{f}_s$. For convenience, the gravitational force is resolved into a component $mg \sin \theta$ along the incline and a component $mg \cos \theta$ perpendicular to the incline.

SOLUTION

Conceptualize Consider Active Figure 5.18 and imagine that the block tends to slide down the incline due to the gravitational force. To simulate the situation, place a coin on this book's cover and tilt the book until the coin begins to slide. Notice how this example differs from Example 5.6. When there is no friction on an incline, *any* angle of the incline will cause a stationary object to begin moving. When there is friction, however, there is no movement of the object for angles less than the critical angle.

Categorize The block is subject to various forces. Because we are raising the plane to the angle at which the block is just ready to begin to move but is not moving, we categorize the block as a particle in equilibrium.

Analyze The diagram in Active Figure 5.18 shows the forces on the block: the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of static friction $\vec{f}_s$. We choose x to be parallel to the plane and y perpendicular to it.



5.11 cont.

Apply Equation 5.8 to the block in both the x and y directions:

(1) $\sum F_x = mg \sin\theta - f_s = 0$

(2) $\sum F_y = n - mg\cos\theta = 0$

Substitute $mg = n/\cos\theta$ from Equation (2) into Equation (1):

(3) $f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s n$. The angle θ in this situation is the critical angle θ_c. Make these substitutions in Equation (3):

$\mu_s n = n \tan\theta_c$

$\mu_s = \tan\theta_c$

For example, if the block just slips at $\theta_c = 20.0°$, we find that $\mu_s = \tan 20.0° = 0.364$.

Finalize Once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k n$. If θ is reduced to a value less than θ_c, however, it may be possible to find an angle θ_c' such that the block moves down the incline with constant speed as a particle in equilibrium again ($a_x = 0$). In this case, use Equations (1) and (2) with f_s replaced by f_k to find μ_k: $\mu_k = \tan\theta_c'$, where $\theta_c' < \theta_c$.

Coefficients of Friction

$$\mu_s \geq \mu_k$$

TABLE 4.2 Coefficients of Friction[a]

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

[a] All values are approximate.

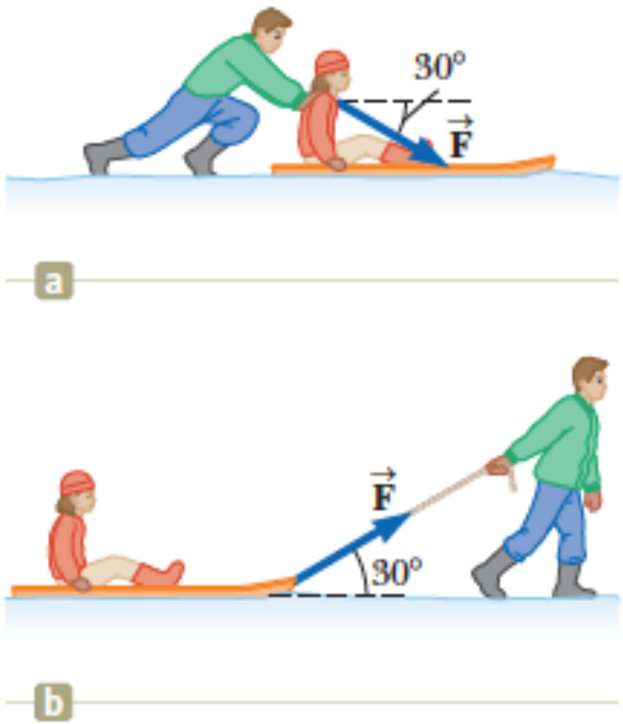


Figure 5.17 (Quick Quiz 5.7) A father slides his daughter on a sled either by (a) pushing down on her shoulders or (b) pulling up on a rope.

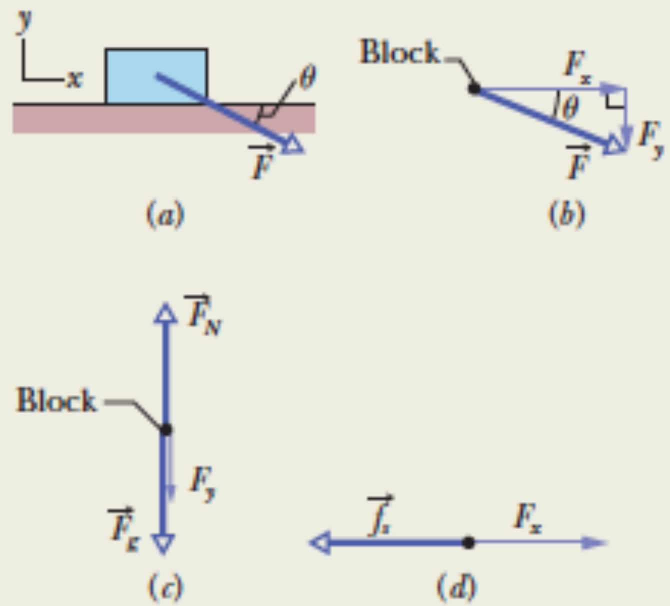


Figure 6-3 (*a*) A force is applied to an initially stationary block. (*b*) The components of the applied force. (*c*) The vertical force components. (*d*) The horizontal force components.

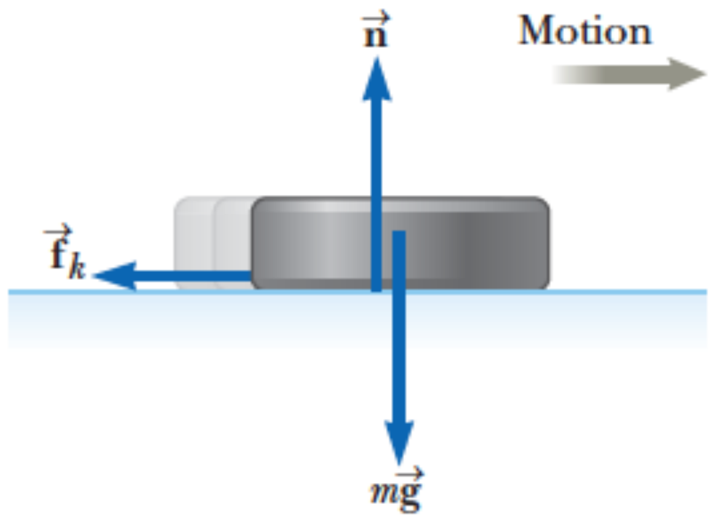


Figure 5.19 (Example 5.12) After the puck is given an initial velocity to the right, the only external forces acting on it are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of kinetic friction $\vec{f}_k$.

Example 5.12 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

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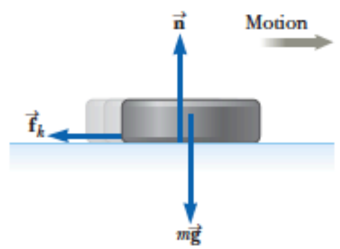


Figure 5.19 (Example 5.12) After the puck is given an initial velocity to the right, the only external forces acting on it are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of kinetic friction $\vec{f}_k$.

SOLUTION

Conceptualize Imagine that the puck in Figure 5.19 slides to the right and eventually comes to rest due to the force of kinetic friction.

Categorize The forces acting on the puck are identified in Figure 5.19, but the text of the problem provides kinematic variables. Therefore, we categorize the problem in two ways. First, it involves a particle under a net force: kinetic friction causes the puck to accelerate. Furthermore, because we model the force of kinetic friction as independent of speed, the acceleration of the puck is constant. So, we can also categorize this problem as one involving a particle under constant acceleration.

Analyze First, let's find the acceleration algebraically in terms of the coefficient of kinetic friction, using Newton's second law. Once we know the acceleration of the puck and the distance it travels, the equations of kinematics can be used to find the numerical value of the coefficient of kinetic friction. The diagram in Figure 5.19 shows the forces on the puck.

Apply the particle under a net force model in the x direction to the puck:

$$(1)\quad \sum F_x = -f_k = ma_x$$

Apply the particle in equilibrium model in the y direction to the puck:

$$(2)\quad \sum F_y = n - mg = 0$$

Substitute $n = mg$ from Equation (2) and $f_k = \mu_k n$ into Equation (1):

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left in Figure 5.19. Because the velocity of the puck is to the right, the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume μ_k remains constant.

continued

5.12 cont.

Apply the particle under constant acceleration model to the puck, using Equation 2.17, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_f = 0$:

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

Solve for the coefficient of kinetic friction:

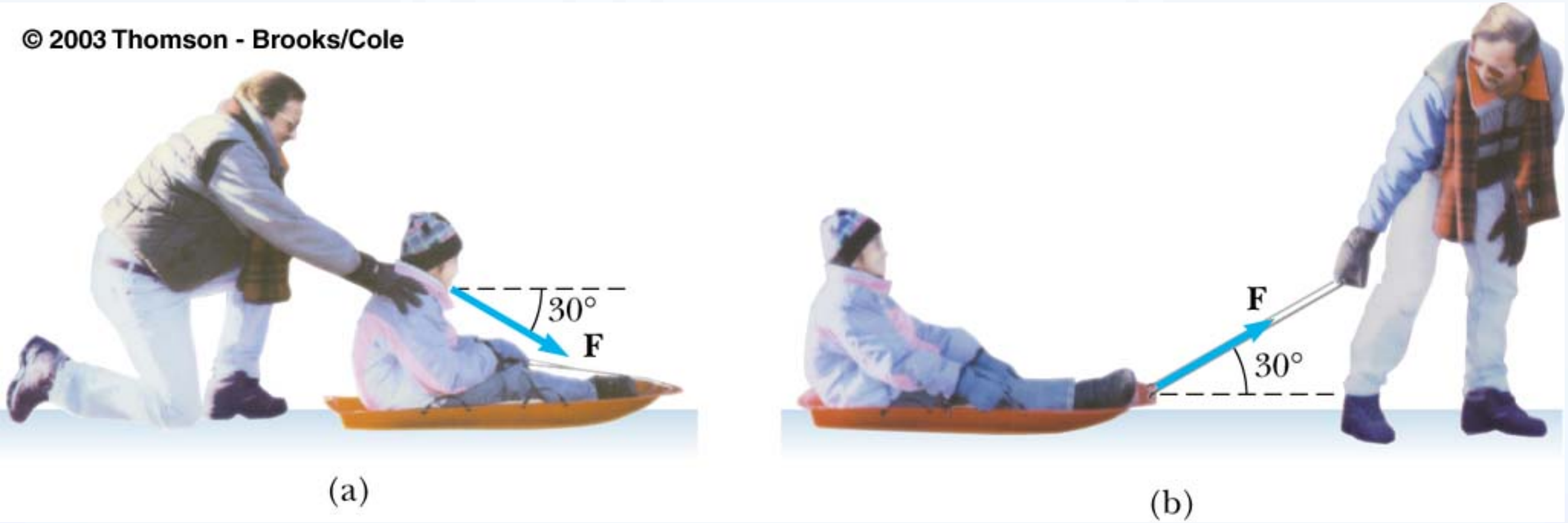
$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

Substitute the numerical values:

$$\mu_k = \frac{(20.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(115\text{ m})} = \boxed{0.177}$$

Finalize Notice that μ_k is dimensionless, as it should be, and that it has a low value, consistent with an object sliding on ice.

Example:



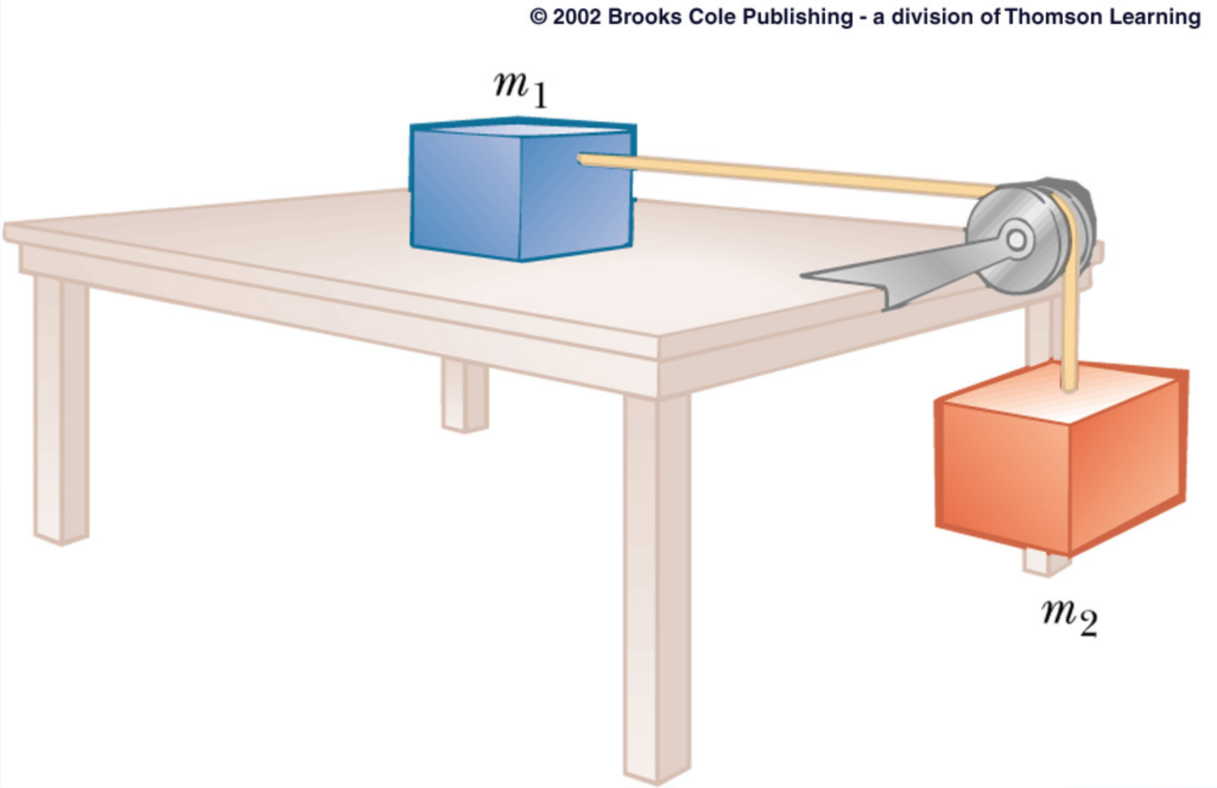
The man pushes/pulls with a force of 200 N. The child and sled combo has a mass of 30 kg and the coefficient of kinetic friction is 0.15. For each case:

What is the frictional force opposing his efforts?

What is the acceleration of the child?

f=59 N, a=3.80 m/s^2 / f=29.1 N, a=4.8 m/s^2

Example:



Given m_1 = 10 kg and m_2 = 5 kg:

a) What value of μ_s would stop the block from sliding?

b) If the box is sliding and μ_k = 0.2, what is the acceleration?

c) What is the tension of the rope?

a) $\mu_s = 0.5$ b) $a = 1.96\ m/s^2$ c) 39.25 N

Example 5.13 Acceleration of Two Connected Objects When Friction Is Present

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.

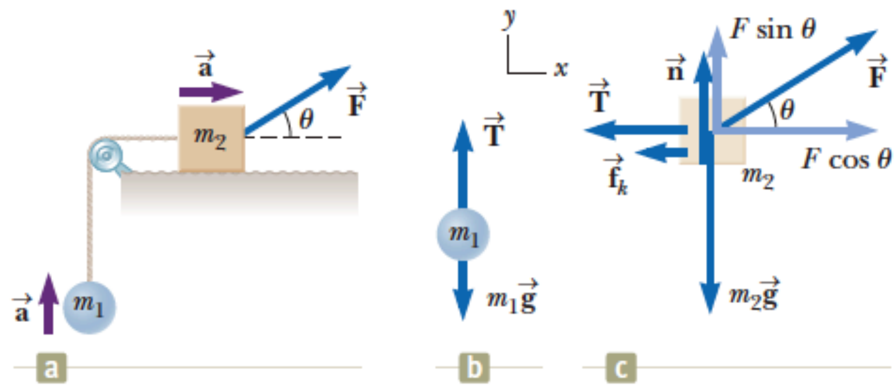


Figure 5.20 (Example 5.13) (a) The external force $\vec{F}$ applied as shown can cause the block to accelerate to the right. (b, c) Diagrams showing the forces on the two objects, assuming the block accelerates to the right and the ball accelerates upward.

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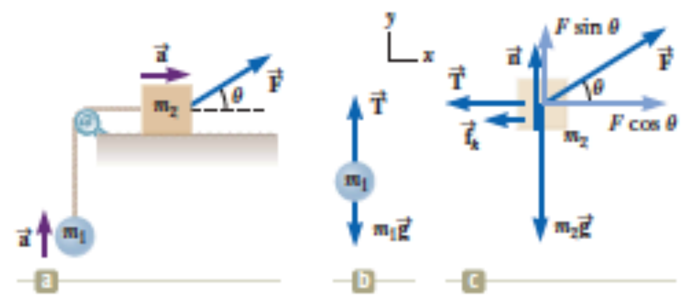

Figure 5.20 (Example 5.13) (a) The external force $\vec{F}$ applied as shown can cause the block to accelerate to the right. (b, c) Diagrams showing the forces on the two objects, assuming the block accelerates to the right and the ball accelerates upward.

SOLUTION

Conceptualize Imagine what happens as $\vec{F}$ is applied to the block. Assuming $\vec{F}$ is not large enough to lift the block, the block slides to the right and the ball rises.

Categorize We can identify forces and we want an acceleration, so we categorize this problem as one involving

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$

5.20b and 5.20c. Notice that the string exerts
components $F\cos\theta$ and $F\sin\theta$, respectively.
f the x component of the acceleration of the
oth a. Let us assume the motion of the block is

Apply the particle under a net force model to the block in the horizontal direction:

$$(1)\ \sum F_x = F\cos\theta - f_k - T = m_2 a_x = m_2 a$$

Because the block moves only horizontally, apply the particle in equilibrium model to the block in the vertical direction:

$$(2)\ \sum F_y = n + F\sin\theta - m_2 g = 0$$

Apply the particle under a net force model to the ball in the vertical direction:

$$(3)\ \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

Solve Equation (2) for n:

$$n = m_2 g - F\sin\theta$$

Substitute n into $f_k = \mu_k n$ from Equation 5.10:

$$(4)\ f_k = \mu_k(m_2 g - F\sin\theta)$$

Substitute Equation (4) and the value of T from Equation (3) into Equation (1):

$$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$$

Solve for a:

$$(5)\ a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$

Example 6.3 What Is the Maximum Speed of the Car?

A 1 500-kg car moving on a flat, horizontal road negotiates a curve as shown in Figure 6.4a. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.

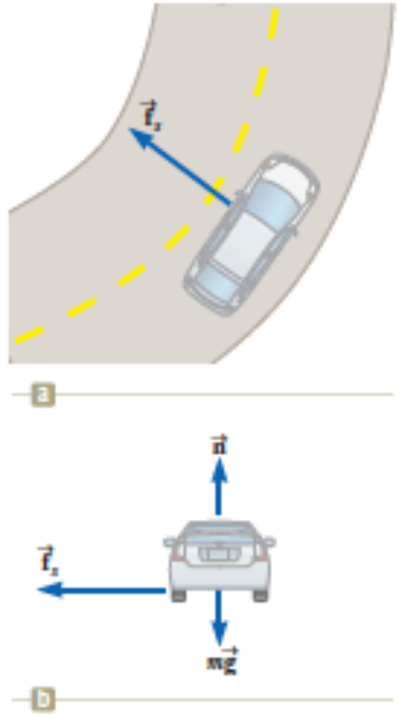

Figure 6.4 (Example 6.3) (a) The force of static friction directed toward the center of the curve keeps the car moving in a circular path. (b) The forces acting on the car.

SOLUTION

Conceptualize Imagine that the curved roadway is part of a large circle so that the car is moving in a circular path.

Categorize Based on the conceptualize step of the problem, we model the car as a particle in uniform circular motion in the horizontal direction. The car is not accelerating vertically, so it is modeled as a particle in equilibrium in the vertical direction.

Analyze The force that enables the car to remain in its circular path is the force of static friction. (It is *static* because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the curved road.) The maximum speed v_{max} the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,max} = \mu_s n$.

Apply Equation 6.1 in the radial direction for the maximum speed condition:

$$(1)\quad f_{s,max} = \mu_s n = m\frac{v_{max}^2}{r}$$

Apply the particle in equilibrium model to the car in the vertical direction:

$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

Solve Equation (1) for the maximum speed and substitute for n:

$$(2)\quad v_{max} = \sqrt{\frac{\mu_s n r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

Substitute numerical values:

$$v_{max} = \sqrt{(0.523)(9.80\text{ m/s}^2)(35.0\text{ m})} = \boxed{13.4\text{ m/s}}$$

Example

Example 6.5 Riding the Ferris Wheel

A child of mass *m* rides on a Ferris wheel as shown in Figure 6.6a. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.

(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, *mg*.

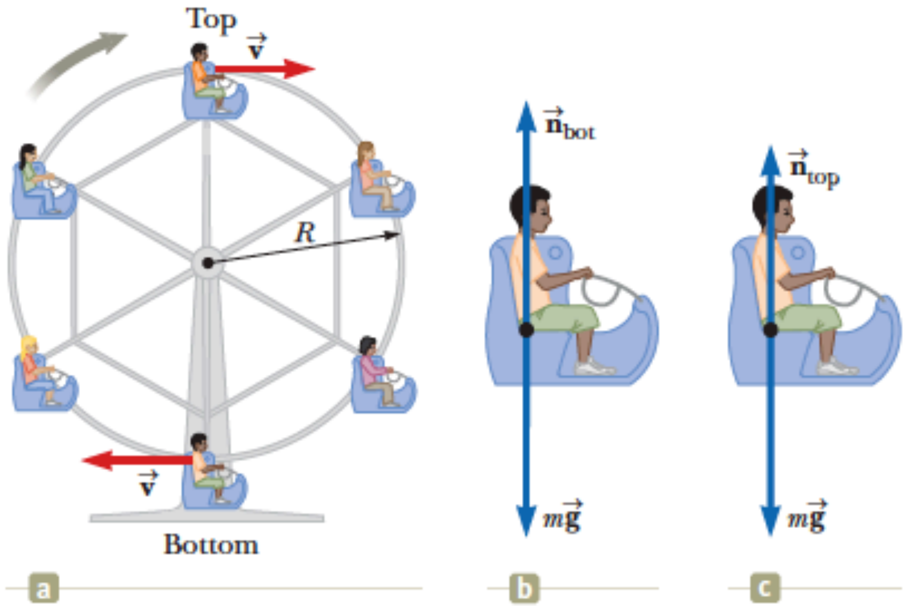

Figure 6.6 (Example 6.5) (a) A child rides on a Ferris wheel. (b) The forces acting on the child at the bottom of the path. (c) The forces acting on the child at the top of the path.

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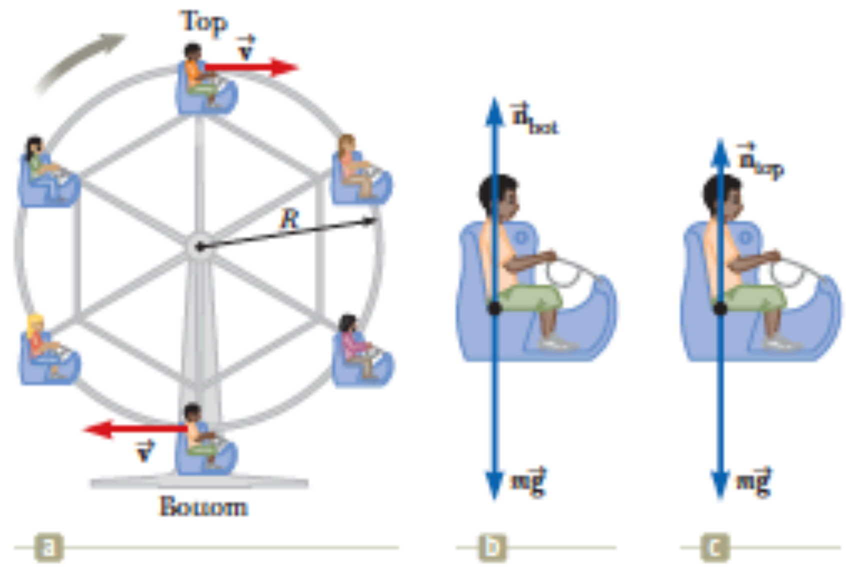


Figure 6.6 (Example 6.5) (a) A child rides on a Ferris wheel. (b) The forces acting on the child at the bottom of the path. (c) The forces acting on the child at the top of the path.

SOLUTION

Conceptualize Look carefully at Figure 6.6a. Based on experiences you may have had on a Ferris wheel or driving over small hills on a roadway, you would expect to feel lighter at the top of the path. Similarly, you would expect to feel heavier at the bottom of the path. At both the bottom of the path and the top, the normal and gravitational forces on the child act in *opposite* directions. The vector sum of these two forces gives a force of constant magnitude that keeps the child moving in a circular path at a constant speed. To yield net force vectors with the same magnitude, the normal force at the bottom must be greater than that at the top.

Categorize Because the speed of the child is constant, we can categorize this problem as one involving a particle (the child) in uniform circular motion, complicated by the gravitational force acting at all times on the child.

Analyze We draw a diagram of forces acting on the child at the bottom of the ride as shown in Figure 6.6b. The only forces acting on him are the downward gravitational force $\vec{F}_g = m\vec{g}$ and the upward force $\vec{n}_{bot}$ exerted by the seat. The net upward force on the child that provides his centripetal acceleration has a magnitude $n_{bot} - mg$.

Apply Newton's second law to the child in the radial direction:

$$\sum F = n_{bot} - mg = m\frac{v^2}{r}$$

Solve for the force exerted by the seat on the child:

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

6.5 cont.

Substitute the values given for the speed and radius:

$$n_{bot} = mg\left[1 + \frac{(3.00 \text{ m/s})^2}{(10.0 \text{ m})(9.80 \text{ m/s}^2)}\right]$$

$$= \boxed{1.09\, mg}$$

Hence, the magnitude of the force $\vec{\mathbf{n}}_{bot}$ exerted by the seat on the child is *greater* than the weight of the child by a factor of 1.09. So, the child experiences an apparent weight that is greater than his true weight by a factor of 1.09.

(B) Determine the force exerted by the seat on the child at the top of the ride.

SOLUTION

Analyze The diagram of forces acting on the child at the top of the ride is shown in Figure 6.6c. The net downward force that provides the centripetal acceleration has a magnitude $mg - n_{top}$.

Apply Newton's second law to the child at this position:

$$\sum F = mg - n_{top} = m\frac{v^2}{r}$$

Solve for the force exerted by the seat on the child:

$$n_{top} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

Substitute numerical values:

$$n_{top} = mg\left[1 - \frac{(3.00 \text{ m/s})^2}{(10.0 \text{ m})(9.80 \text{ m/s}^2)}\right]$$

$$= \boxed{0.908\, mg}$$

In this case, the magnitude of the force exerted by the seat on the child is *less* than his true weight by a factor of 0.908, and the child feels lighter.

Finalize The variations in the normal force are consistent with our prediction in the Conceptualize step of the problem.

WHAT IF? Suppose a defect in the Ferris wheel mechanism causes the speed of the child to increase to 10.0 m/s. What does the child experience at the top of the ride in this case?

Answer If the calculation above is performed with v = 10.0 m/s, the magnitude of the normal force at the top of the ride is negative, which is impossible. We interpret it to mean that the required centripetal acceleration of the child is larger than that due to gravity. As a result, the child will lose contact with the seat and will only stay in his circular path if there is a safety bar that provides a downward force on him to keep him in his seat. At the bottom of the ride, the normal force is 2.02 mg, which would be uncomfortable.

Example

Example 6.6 Keep Your Eye on the Ball

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed point O as illustrated in Figure 6.9. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

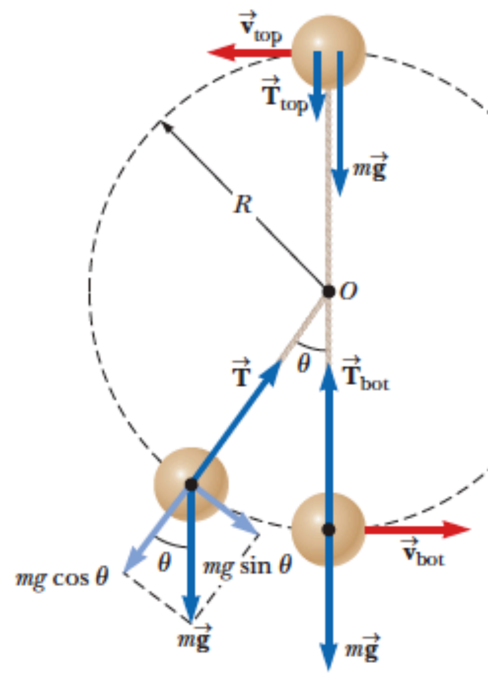


Figure 6.9 (Example 6.6) The forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O. Forces acting on the sphere are shown when the sphere is at the top and bottom of the circle and at an arbitrary location.

(A) What speed would the ball have as it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

(B) What if the ball is set in motion such that the speed at the top is less than this value? What happens?

Example 6.6 **Keep Your Eye on the Ball**

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed point O as illustrated in Figure 6.9. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

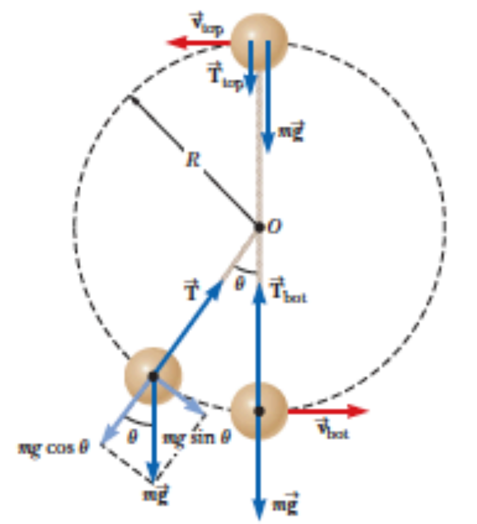


Figure 6.9 (Example 6.6) The forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O. Forces acting on the sphere are shown when the sphere is at the top and bottom of the circle and at an arbitrary location.

SOLUTION

Conceptualize Compare the motion of the sphere in Figure 6.9 with that of the child in Figure 6.6a associated with Example 6.5. Both objects travel in a circular path. Unlike the child in Example 6.5, however, the speed of the sphere is *not* uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere.

Categorize We model the sphere as a particle under a net force and moving in a circular path, but it is not a particle in *uniform* circular motion. We need to use the techniques discussed in this section on nonuniform circular motion.

Analyze From the force diagram in Figure 6.9, we see that the only forces acting on the sphere are the gravitational force $\vec{F}_g = m\vec{g}$ exerted by the Earth and the force $\vec{T}$ exerted by the cord. We resolve $\vec{F}_g$ into a tangential component $mg\sin\theta$ and a radial component $mg\cos\theta$.

Apply Newton's second law to the sphere in the tangential direction:

$$\sum F_t = mg\sin\theta = ma_t$$
$$a_t = \boxed{g\sin\theta}$$

Apply Newton's second law to the forces acting on the sphere in the radial direction, noting that both $\vec{T}$ and $\vec{a}_r$ are directed toward O:

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$
$$T = \boxed{mg\left(\frac{v^2}{Rg} + \cos\theta\right)}$$

Finalize Let us evaluate this result at the top and bottom of the circular path (Fig. 6.9):

$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right) \qquad T_{bot} = mg\left(\frac{v_{bot}^2}{Rg} + 1\right)$$

These results have similar mathematical forms as those for the normal forces n_{top} and n_{bot} on the child in Example 6.5, which is consistent with the normal force on the child playing a similar physical role in Example 6.5 as the tension in the string plays in this example. Keep in mind, however, that the normal force $\vec{n}$ on the child in Example 6.5 is always upward, whereas the force $\vec{T}$ in this example changes direction because it must always point inward along the string. Also note that v in the expressions above varies for different positions of the sphere, as indicated by the subscripts, whereas v in Example 6.5 is constant.

6.6 cont.

WHAT IF? What if the ball is set in motion with a slower speed?

(A) What speed would the ball have as it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

Answer Let us set the tension equal to zero in the expression for T_{top}:

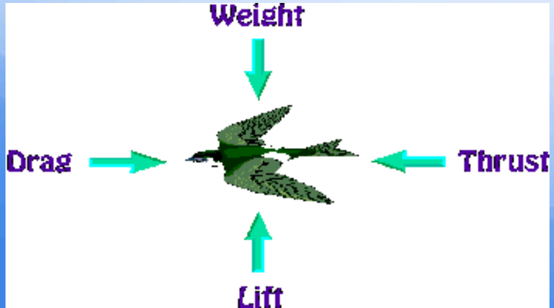
$$0 = mg\left(\frac{v_{top}^2}{Rg} - 1\right) \rightarrow \quad v_{top} = \sqrt{gR}$$

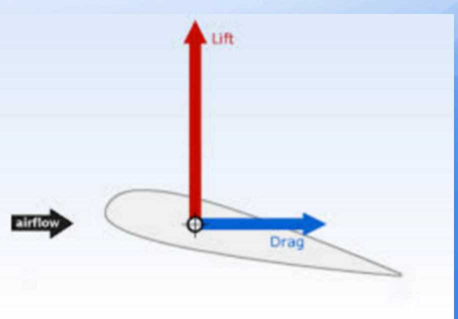
(B) What if the ball is set in motion such that the speed at the top is less than this value? What happens?

Answer In this case, the ball never reaches the top of the circle. At some point on the way up, the tension in the string goes to zero and the ball becomes a projectile. It follows a segment of a parabolic path over the top of its motion, rejoining the circular path on the other side when the tension becomes nonzero again.

Terminal Speed
Lift and Drag Forces

- Another type of friction is air resistance
- Air resistance is proportional to the (speed)^a of the object
- When the upward force (lift) of air resistance equals the downward force of gravity, the net force on the object is zero
- The constant speed of the object is the *terminal speed*





Resistive Force

- 1)

$$R = -b\,v$$

- 2)

$$R = D = -\frac{1}{2}C\rho A\,v^2$$

Model 1: Resistive Force Proportional to Object Velocity

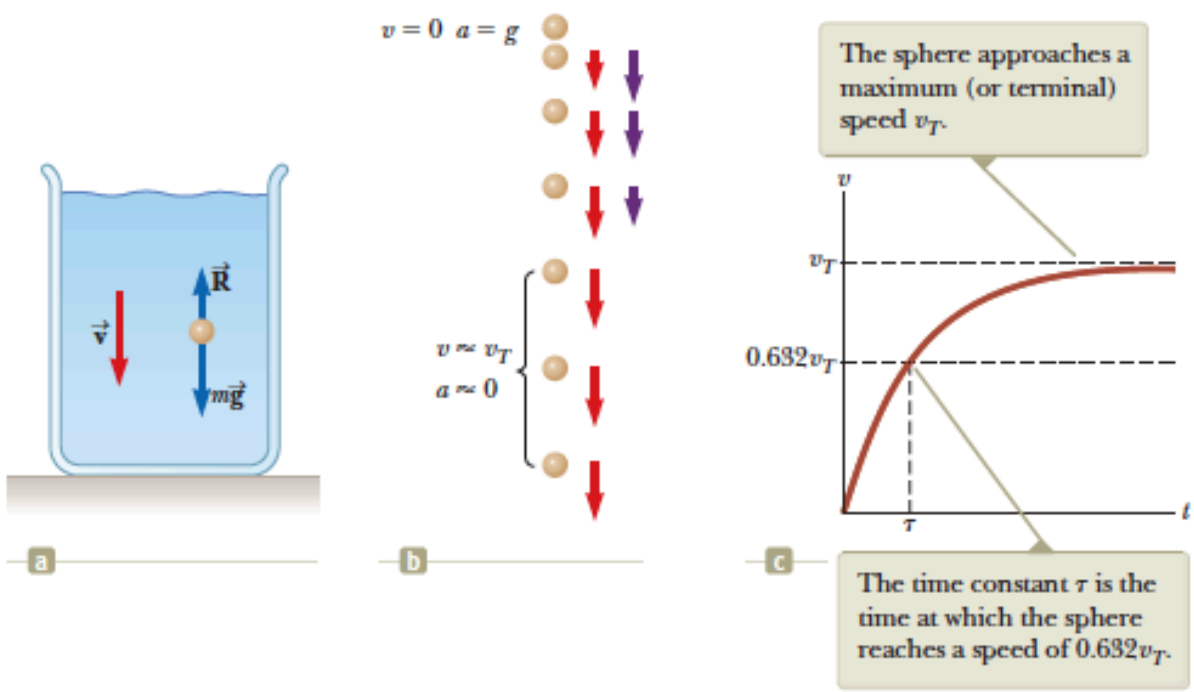
If we model the resistive force acting on an object moving through a liquid or gas as proportional to the object's velocity, the resistive force can be expressed as

$$\vec{\mathbf{R}} = -b\vec{\mathbf{v}} \quad (6.2)$$

where b is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object and $\vec{\mathbf{v}}$ is the velocity of the object relative to the medium. The negative sign indicates that $\vec{\mathbf{R}}$ is in the opposite direction to $\vec{\mathbf{v}}$.

Consider a small sphere of mass m released from rest in a liquid as in Active Figure 6.13a. Assuming the only forces acting on the sphere are the resistive force $\vec{\mathbf{R}} = -b\vec{\mathbf{v}}$ and the gravitational force $\vec{\mathbf{F}}_g$, let us describe its motion.[1] Applying Newton's second law to the vertical motion, choosing the downward direction to be positive, and noting that $\Sigma F_y = mg - bv$, we obtain

$$mg - bv = ma \quad (6.3)$$



where the acceleration of the sphere is downward. Noting that the acceleration a is equal to dv/dt gives

$$\frac{dv}{dt} = g - \frac{b}{m}v \tag{6.4}$$

This equation is called a *differential equation*, and the methods of solving it may not be familiar to you as yet. Notice, however, that initially when $v = 0$, the magnitude of the resistive force is also zero and the acceleration of the sphere is simply g. As t increases, the magnitude of the resistive force increases and the acceleration decreases. The acceleration approaches zero when the magnitude of the resistive force approaches the sphere's weight. In this situation, the speed of the sphere approaches its **terminal speed** v_T.

Terminal speed ▶

The terminal speed is obtained from Equation 6.4 by setting $dv/dt = 0$, which gives

$$mg - bv_T = 0 \quad \text{or} \quad v_T = \frac{mg}{b}$$

Because you may not be familiar with differential equations yet, we won't show the details of the solution that gives the expression for v for all times t. If $v = 0$ at $t = 0$, this expression is

$$v = \frac{mg}{b}(1 - e^{-bt/m}) = v_T(1 - e^{-t/\tau}) \tag{6.5}$$

$\tau = m/b$

$t \to \infty$

$v_T = \dfrac{mg}{b}$

Because you may not be familiar with differential equations yet, we won't show the details of the solution that gives the expression for v for all times t. If $v = 0$ at $t = 0$, this expression is

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right) \tag{6.5}$$

This function is plotted in Active Figure 6.13c. The symbol e represents the base of the natural logarithm and is also called *Euler's number:* $e = 2.718\,28$. The **time constant** $\tau = m/b$ (Greek letter tau) is the time at which the sphere released from rest at $t = 0$ reaches 63.2% of its terminal speed; when $t = \tau$, Equation 6.5 yields $v = 0.632v_T$. (The number 0.632 is $1 - e^{-1}$.)

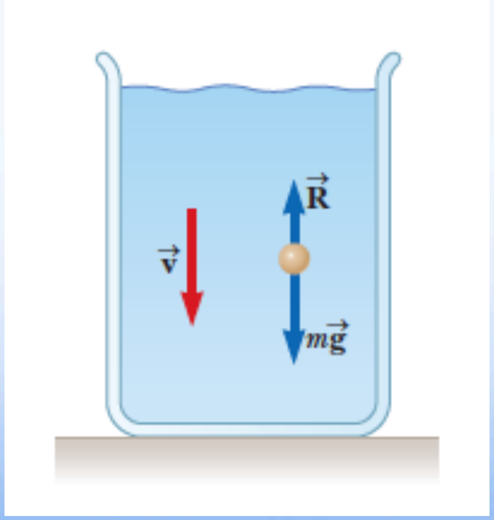
We can check that Equation 6.5 is a solution to Equation 6.4 by direct differentiation:

$$\frac{dv}{dt} = \frac{d}{dt}\left[\frac{mg}{b}\left(1 - e^{-bt/m}\right)\right] = \frac{mg}{b}\left(0 + \frac{b}{m}e^{-bt/m}\right) = ge^{-bt/m}$$

(See Appendix Table B.4 for the derivative of e raised to some power.) Substituting into Equation 6.4 both this expression for dv/dt and the expression for v given by Equation 6.5 shows that our solution satisfies the differential equation.

Example 6.8 Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time at which the sphere reaches 90.0% of its terminal speed.



Example 6.8 Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time at which the sphere reaches 90.0% of its terminal speed.

SOLUTION

Conceptualize With the help of Active Figure 6.13, imagine dropping the sphere into the oil and watching it sink to the bottom of the vessel. If you have some thick shampoo in a clear container, drop a marble in it and observe the motion of the marble.

Categorize We model the sphere as a particle under a net force, with one of the forces being a resistive force that depends on the speed of the sphere.

Analyze From $v_T = mg/b$, evaluate the coefficient b:

$$b = \frac{mg}{v_T} = \frac{(2.00\text{ g})(980\text{ cm/s}^2)}{5.00\text{ cm/s}} = 392\text{ g/s}$$

Evaluate the time constant τ:

$$\tau = \frac{m}{b} = \frac{2.00\text{ g}}{392\text{ g/s}} = \boxed{5.10 \times 10^{-3}\text{ s}}$$

6.8 cont.

Find the time t at which the sphere reaches a speed of $0.900v_T$ by setting $v = 0.900v_T$ in Equation 6.5 and solving for t:

$$0.900v_T = v_T(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = 0.900$$

$$e^{-t/\tau} = 0.100$$

$$-\frac{t}{\tau} = \ln(0.100) = -2.30$$

$$t = 2.30\tau = 2.30(5.10 \times 10^{-3}\text{ s}) = 11.7 \times 10^{-3}\text{ s}$$

$$= \boxed{11.7\text{ ms}}$$

Finalize The sphere reaches 90.0% of its terminal speed in a very short time interval. You should have also seen this behavior if you performed the activity with the marble and the shampoo. Because of the short time interval required to reach terminal velocity, you may not have noticed the time interval at all. The marble may have appeared to immediately begin moving through the shampoo at a constant velocity.

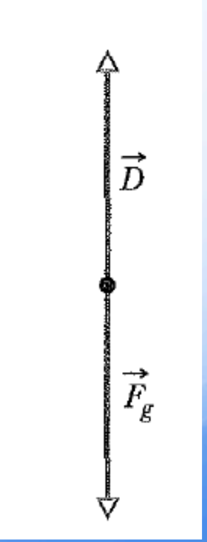
Other kind of Drag force

In such cases, the magnitude of the drag force $\vec{D}$ is related to the relative speed v

$$R = F_d = D = \frac{1}{2} C \rho A v^2$$

- Air resistance, F ~ Area v^2

$$F_g - D = ma$$



Terminal velocity:

$$\tfrac{1}{2} C \rho A v_t^2 - F_g = 0,$$

$$v_t = \sqrt{\frac{2F_g}{C \rho A}}.$$

6-2 THE DRAG FORCE AND TERMINAL SPEED

Learning Objectives

After reading this module, you should be able to . . .

6.04 Apply the relationship between the drag force on an object moving through air and the speed of the object.

6.05 Determine the terminal speed of an object falling through air.

Key Ideas

- When there is relative motion between air (or some other fluid) and a body, the body experiences a drag force $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of $\vec{D}$ is related to the relative speed v by an experimentally determined drag coefficient C according to

$$D = \tfrac{1}{2}C\rho A v^2,$$

where ρ is the fluid density (mass per unit volume) and A is the effective cross-sectional area of the body (the area of a cross section taken perpendicular to the relative velocity $\vec{v}$).

- When a blunt object has fallen far enough through air, the magnitudes of the drag force $\vec{D}$ and the gravitational force $\vec{F}_g$ on the body become equal. The body then falls at a constant terminal speed v_t given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

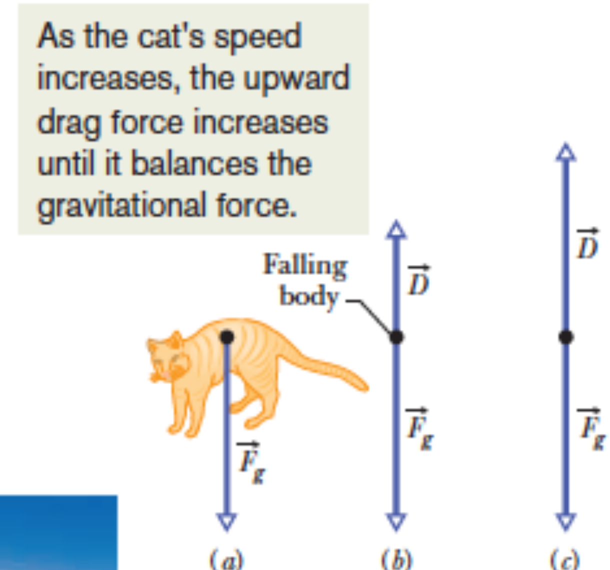


Figure 6.15 (Conceptual Example 6.9) A skysurfer.

Table 6-1 Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance[a] (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

Drag Coefficient:

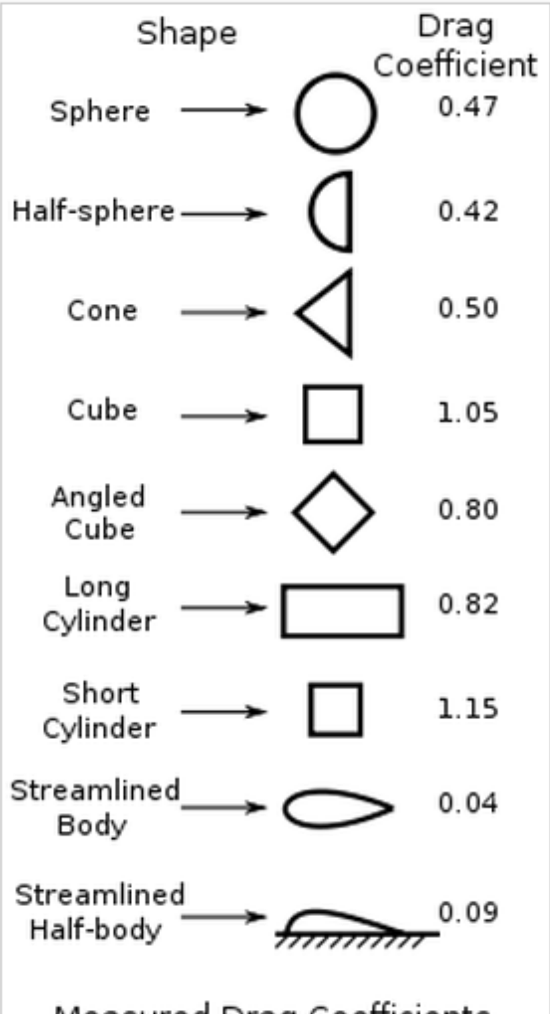
Definition

The drag coefficient C_d is defined as:

$$C = C_d = \frac{2F_d}{\rho v^2 A}$$

where:

- F_d is the drag force, which is by definition the force component in the direction of the flow velocity
- ρ is the mass density of the fluid,[7]
- v is the speed of the object relative to the fluid and
- A is the reference area.



Measured Drag Coefficients

- Audi A6: 2011-present (Cd 0.26) ...
- BMW i8: 2014 (Cd 0.26) ...
- Mazda3 Sedan: 2012-present (Cd 0.26) ...
- Mercedes-Benz B-Class: 2012-present (Cd 0.26) ...
- Nissan GT-R, 2011â€“present (Cd 0.26) ...
- Peugeot 508, 2011â€“present (Cd 0.25) ...
- Hyundai Sonata Hybrid, 2013-present (Cd 0.25) ...
- Toyota Prius, 2010-present (Cd 0.25)

- Audi A6: 2011-present (Cd 0.26) ...
- BMW i8: 2014 (Cd 0.26) ...
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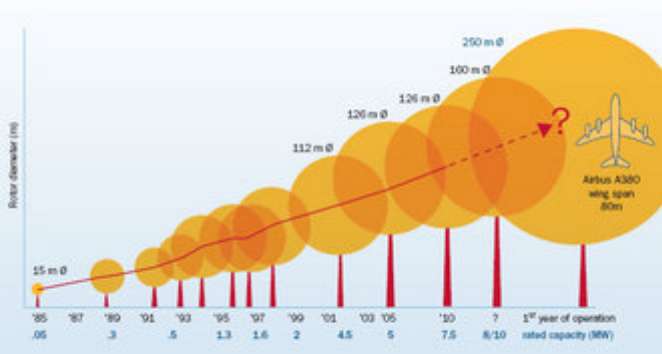
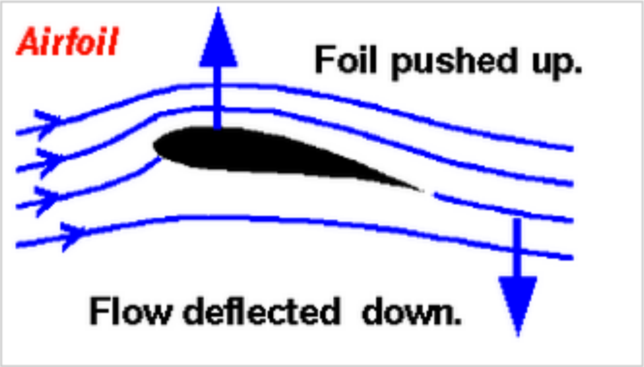
- Hyundai Sonata Hybrid, 2013-present (Cd 0.25) ...
- Toyota Prius, 2010-present (Cd 0.25)

$C_d \simeq 0.45!$

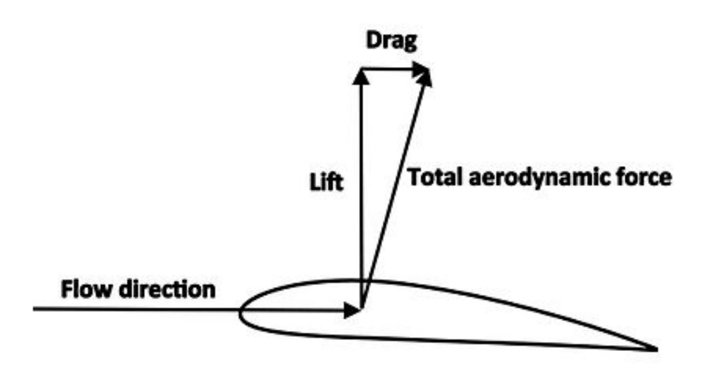
Lift force



Rotor diameter (m)
15 m Ø
112 m Ø
126 m Ø
126 m Ø
160 m Ø
250 m Ø
Airbus A380
wing span
80m
'85 '87 '89 '91 '93 '95 '97 '99 '01 '03 '05 '10 ?
1st year of operation
.05 .3 .5 1.3 1.6 2 4.5 5 7.5 8/10
rated capacity (MW)

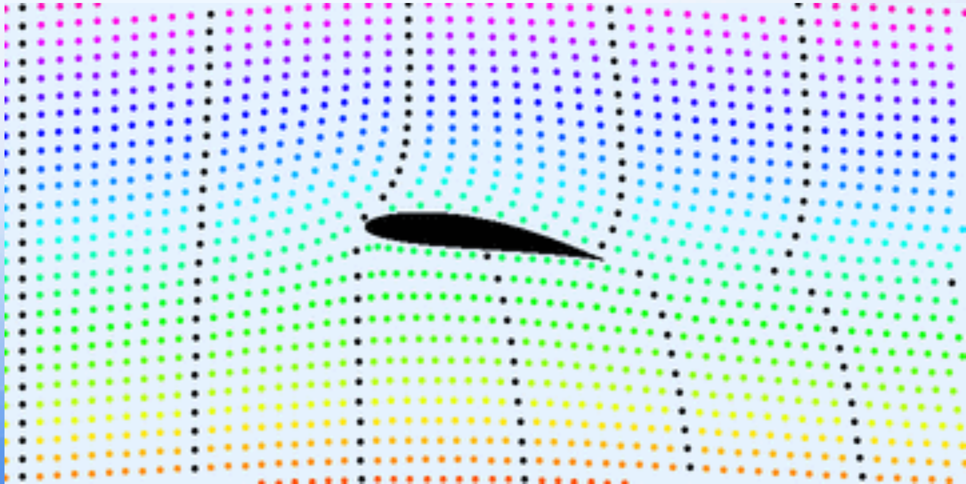


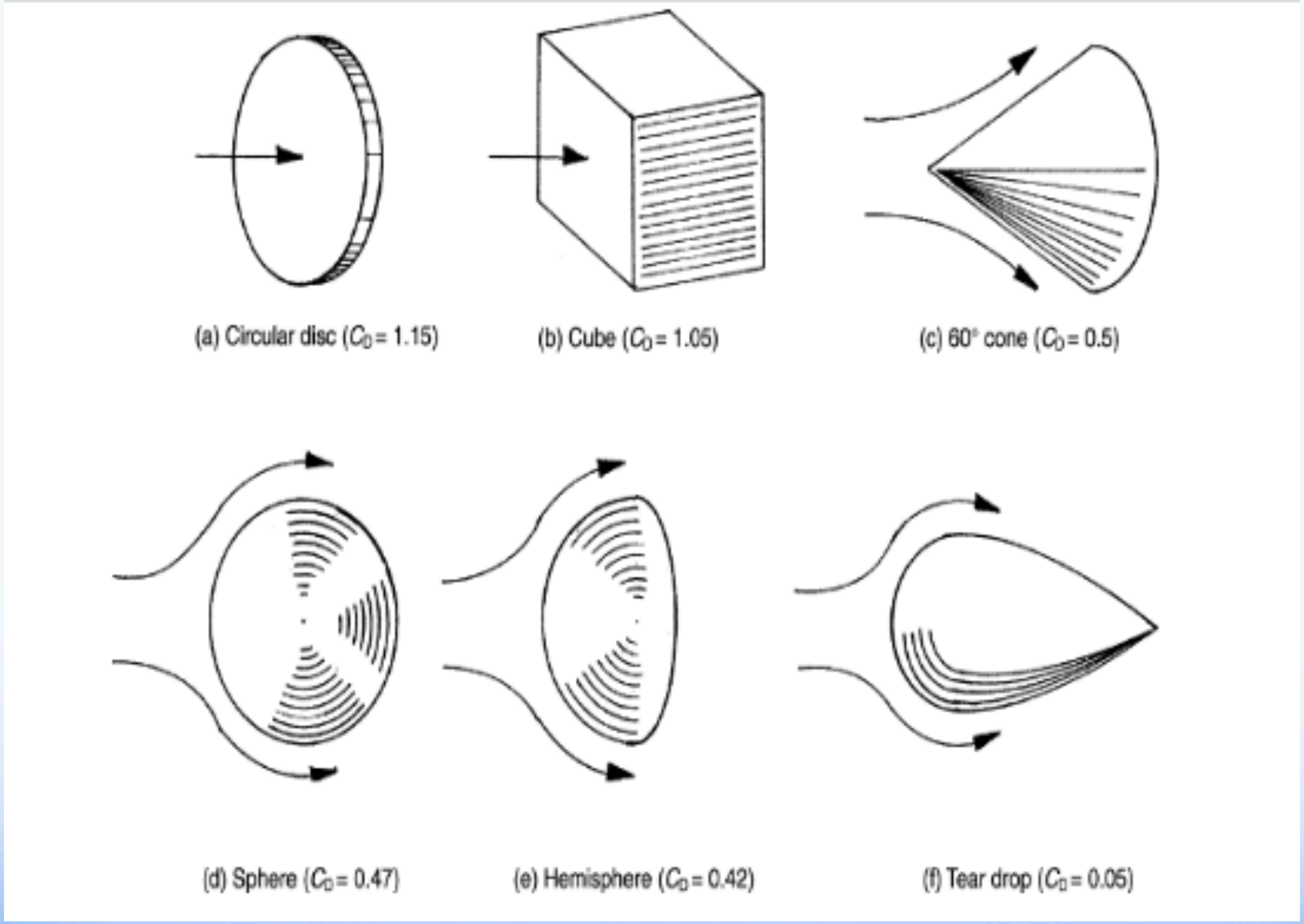
Airfoil
Foil pushed up.
Flow deflected down.



Drag
Lift
Total aerodynamic force
Flow direction

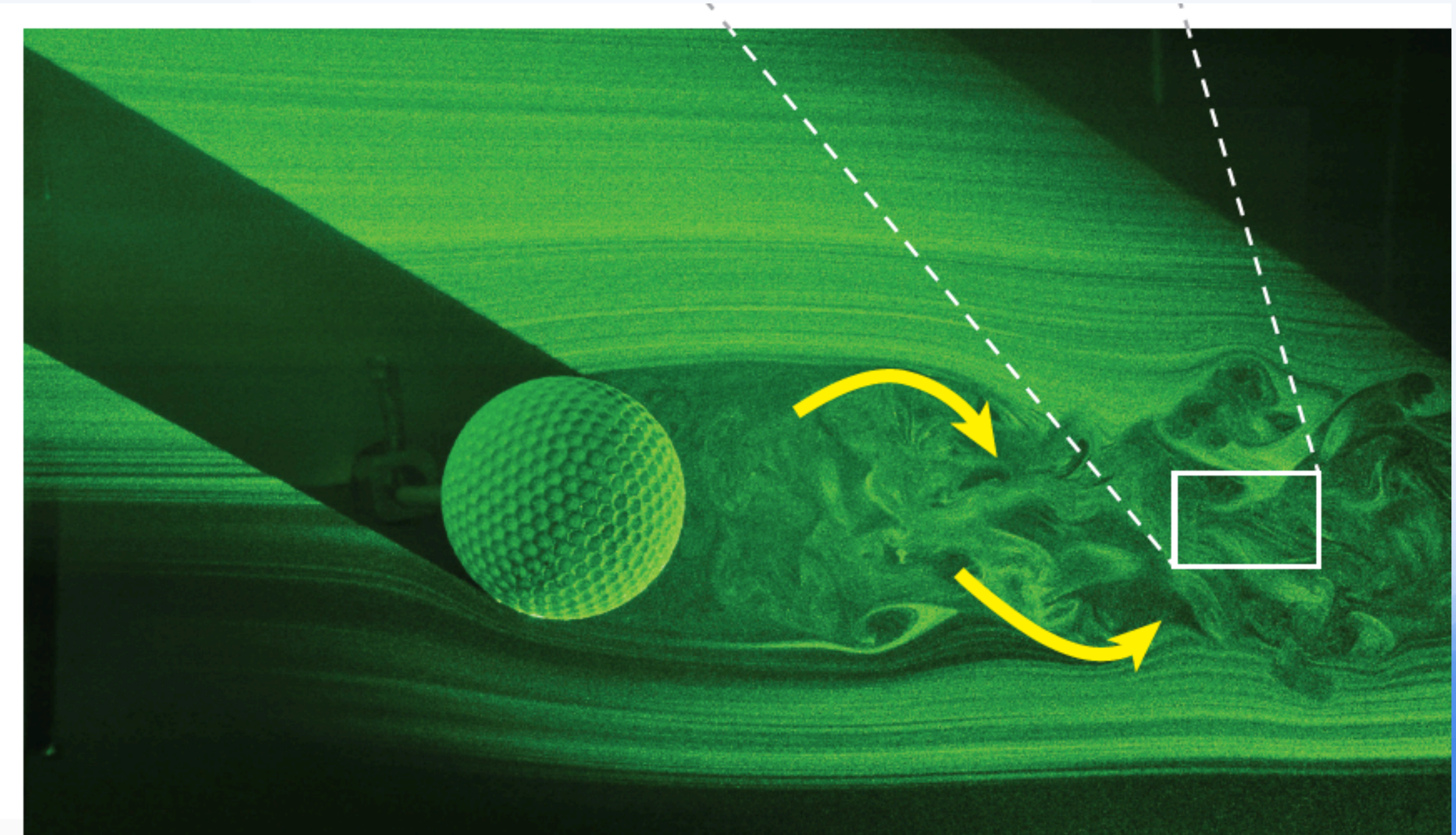
A380
AIRBUS
A380

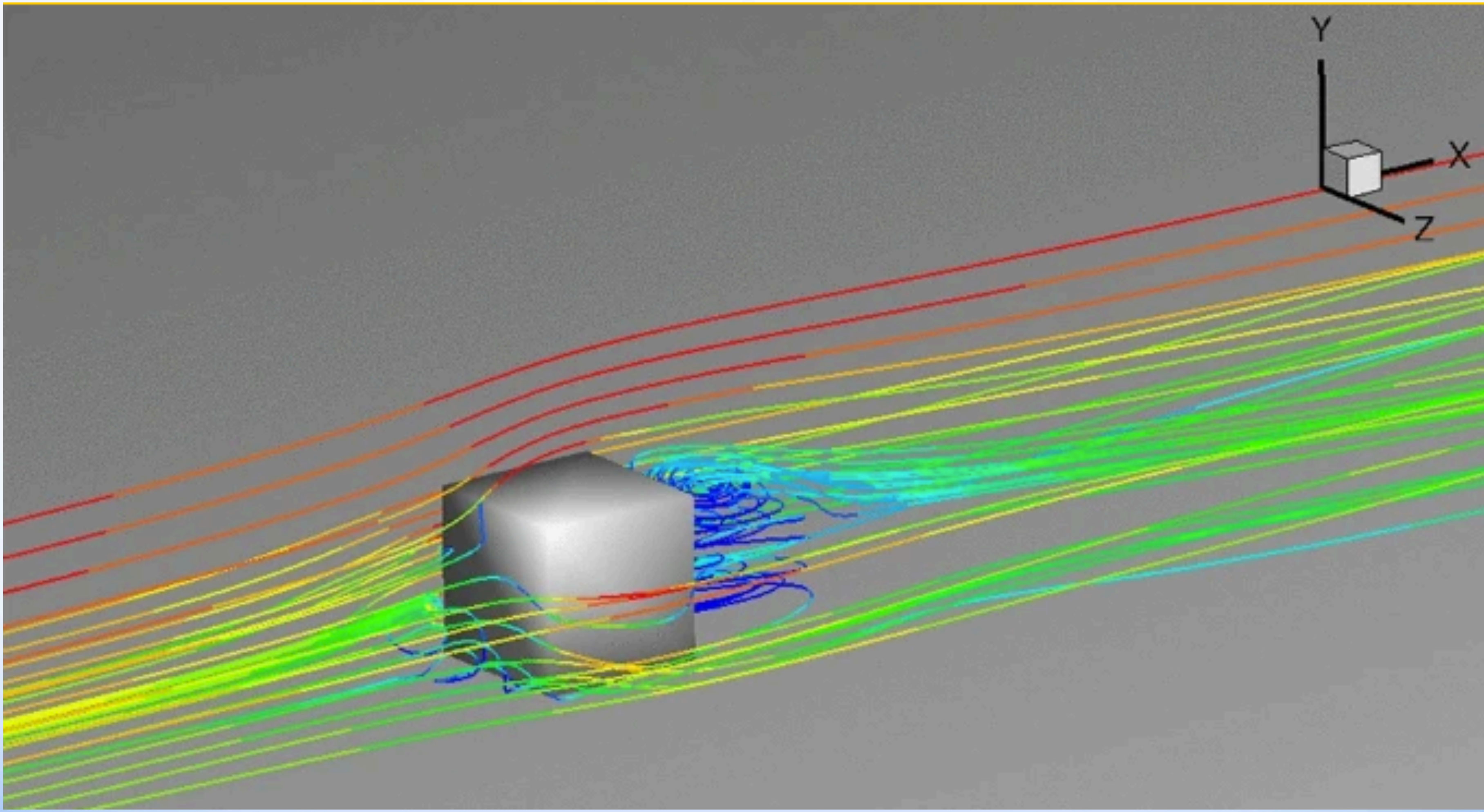




$$C = C_d = \frac{2F_d}{\rho v^2 A}$$

Particle image velocimetry (PIV)




Y
X
Z

Example

33. **S** Assume the resistive force acting on a speed skater is proportional to the square of the skater's speed v and is given by $f = -kmv^2$, where k is a constant and m is the skater's mass. The skater crosses the finish line of a straight-line race with speed v_i and then slows down by coasting on his skates. Show that the skater's speed at any time t after crossing the finish line is $v(t) = v_i/(1 + ktv_i)$.

Example

Sample Problem 6.03 Terminal speed of falling raindrop

A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

(a) As Table 6-1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

(a) As Table 6-1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

KEY IDEA

The drop reaches a terminal speed v_t when the gravitational force on it is balanced by the air drag force on it, so its acceleration is zero. We could then apply Newton's second law and the drag force equation to find v_t, but Eq. 6-16 does all that for us.

Calculations: To use Eq. 6-16, we need the drop's effective cross-sectional area A and the magnitude F_g of the gravitational force. Because the drop is spherical, A is the area of a circle (πR^2) that has the same radius as the sphere. To find F_g, we use three facts: (1) $F_g = mg$, where m is the drop's mass; (2) the (spherical) drop's volume is $V = \frac{4}{3}\pi R^3$; and (3) the density of the water in the drop is the mass per volume, or $\rho_w = m/V$. Thus, we find

$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g.$$

We next substitute this, the expression for A, and the given data into Eq. 6-16. Being careful to distinguish between the air density ρ_a and the water density ρ_w, we obtain

$$v_t = \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5 \times 10^{-3}\ \text{m})(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{(3)(0.60)(1.2\ \text{kg/m}^3)}}$$

$$= 7.4\ \text{m/s} \approx 27\ \text{km/h.} \qquad \text{(Answer)}$$

Note that the height of the cloud does not enter into the calculation.

(b) What would be the drop's speed just before impact if there were no drag force?

KEY IDEA

With no drag force to reduce the drop's speed during the fall, the drop would fall with the constant free-fall acceleration g, so the constant-acceleration equations of Table 2-1 apply.

Calculation: Because we know the acceleration is g, the initial velocity v_0 is 0, and the displacement $x - x_0$ is $-h$, we use Eq. 2-16 to find v:

$$v = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(1200\ \text{m})}$$

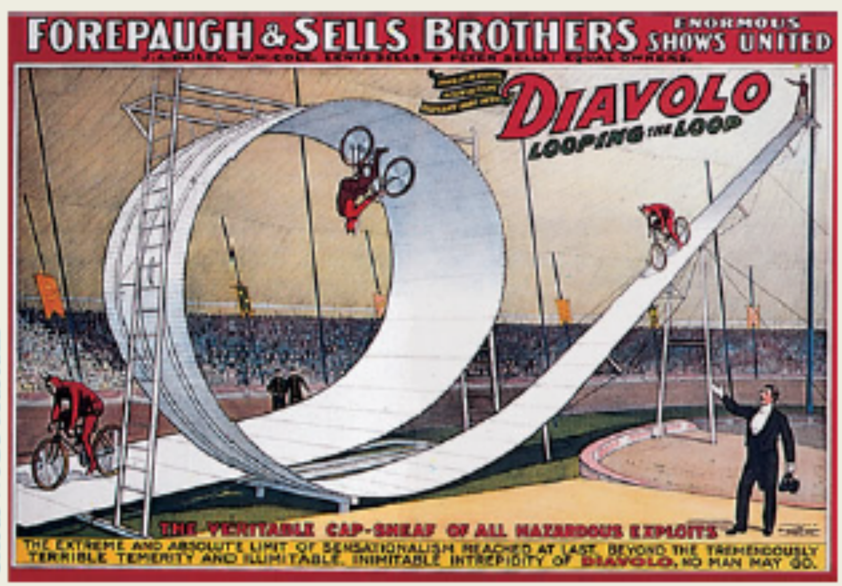
$$= 153\ \text{m/s} \approx 550\ \text{km/h.} \qquad \text{(Answer)}$$

Had he known this, Shakespeare would scarcely have written, "it droppeth as the gentle rain from heaven, upon the place beneath." In fact, the speed is close to that of a bullet from a large-caliber handgun!

Sample Problem 6.04 Vertical circular loop, Diavolo

Largely because of riding in cars, you are used to horizontal circular motion. Vertical circular motion would be a novelty. In this sample problem, such motion seems to defy the gravitational force.

In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6-9*a*). Assuming that the loop is a circle with radius $R = 2.7$ m, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?



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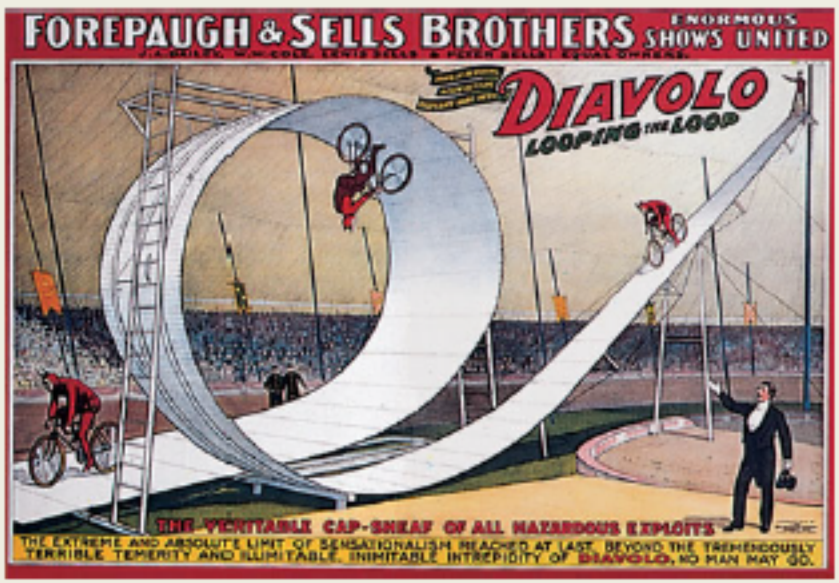
(*a*)



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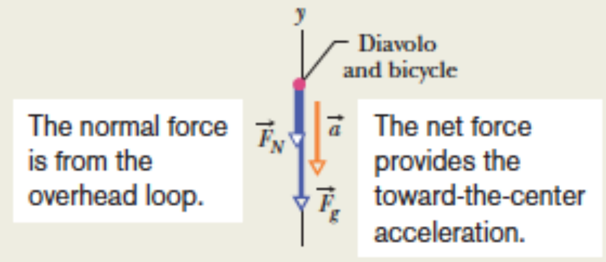
(*a*)



(*b*)

Figure 6-9 (*a*) Contemporary advertisement for Diavolo and (*b*) free-body diagram for the performer at the top of the loop.

Calculations: The forces on the particle when it is at the top of the loop are shown in the free-body diagram of Fig 6-9*b*. The gravitational force $\vec{F}_g$ is downward along a *y* axis; so is the normal force $\vec{F}_N$ on the particle from the loop (the loop can push down, not pull up); so also is the centripetal acceleration of the particle. Thus, Newton's second law for *y* components ($F_{net,y} = ma_y$) gives us

$$-F_N - F_g = m(-a)$$

and

$$-F_N - mg = m\left(-\frac{v^2}{R}\right). \qquad (6\text{-}19)$$

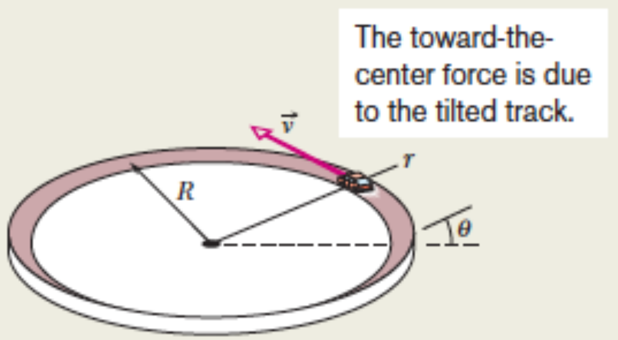
If the particle has the *least speed* *v* needed to remain in contact, then it is on the *verge of losing contact* with the loop (falling away from the loop), which means that $F_N = 0$ at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for F_N in Eq. 6-19, solving for *v*, and then substituting known values give us

$$v = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(2.7\ \text{m})}$$
$$= 5.1\ \text{m/s}. \qquad \text{(Answer)}$$

Kyle Busch, driver of the #18 Snickers Toyota, leads Jeff Gordon, driver of the #24 Dupont Chevrolet, during the NASCAR Sprint Cup Series Kobalt Tools 500 at the Atlanta Motor Speedway on March 9, 2008 in Hampton, Georgia. The cars travel on a banked roadway to help them undergo circular motion on the turns. *(Chris Graythen/Getty Images for NASCAR)*

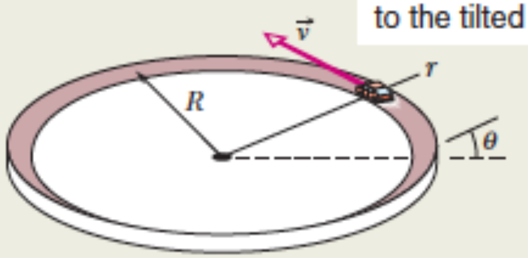
Car in banked circular turn

Kyle Busch, driver of the #18 Snickers Toyota, leads Jeff Gordon, driver of the #24 Dupont Chevrolet, during the NASCAR Sprint Cup Series Kobalt Tools 500 at the Atlanta Motor Speedway on March 9, 2008 in Hampton, Georgia. The cars travel on a banked roadway to help them undergo circular motion on the turns. *(Chris Graythen/Getty Images for NASCAR)*

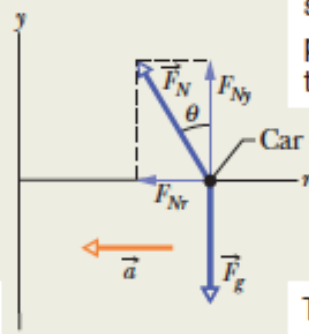


(*a*)

a car of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius $R = 190$ m. (It is a normal car, rather than a race car, which means that any vertical force from the passing air is negligible.) If the frictional force from the track is negligible, what bank angle θ prevents sliding?



(a)



(b)

Radial calculation: As Fig. 6-11*b* shows (and as you should verify), the angle that force $\vec{F}_N$ makes with the vertical is equal to the bank angle θ of the track. Thus, the radial component F_{Nr} is equal to $F_N \sin\theta$. We can now write Newton's second law for components along the r axis ($F_{net,r} = ma_r$) as

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right). \qquad (6\text{-}23)$$

We cannot solve this equation for the value of θ because it also contains the unknowns F_N and m.

Vertical calculations: We next consider the forces and acceleration along the y axis in Fig. 6-11*b*. The vertical component of the normal force is $F_{Ny} = F_N \cos\theta$, the gravitational force $\vec{F}_g$ on the car has the magnitude mg, and the acceleration of the car along the y axis is zero. Thus we can write Newton's second law for components along the y axis ($F_{net,y} = ma_y$) as

$$F_N \cos\theta - mg = m(0),$$

from which

$$F_N \cos\theta = mg. \qquad (6\text{-}24)$$

Combining results: Equation 6-24 also contains the unknowns F_N and m, but note that dividing Eq. 6-23 by Eq. 6-24 neatly eliminates both those unknowns. Doing so, replacing $(\sin\theta)/(\cos\theta)$ with $\tan\theta$, and solving for θ then yield

$$\theta = \tan^{-1}\frac{v^2}{gR}$$

$$= \tan^{-1}\frac{(20\text{ m/s})^2}{(9.8\text{ m/s}^2)(190\text{ m})} = 12°. \qquad \text{(Answer)}$$

Example :

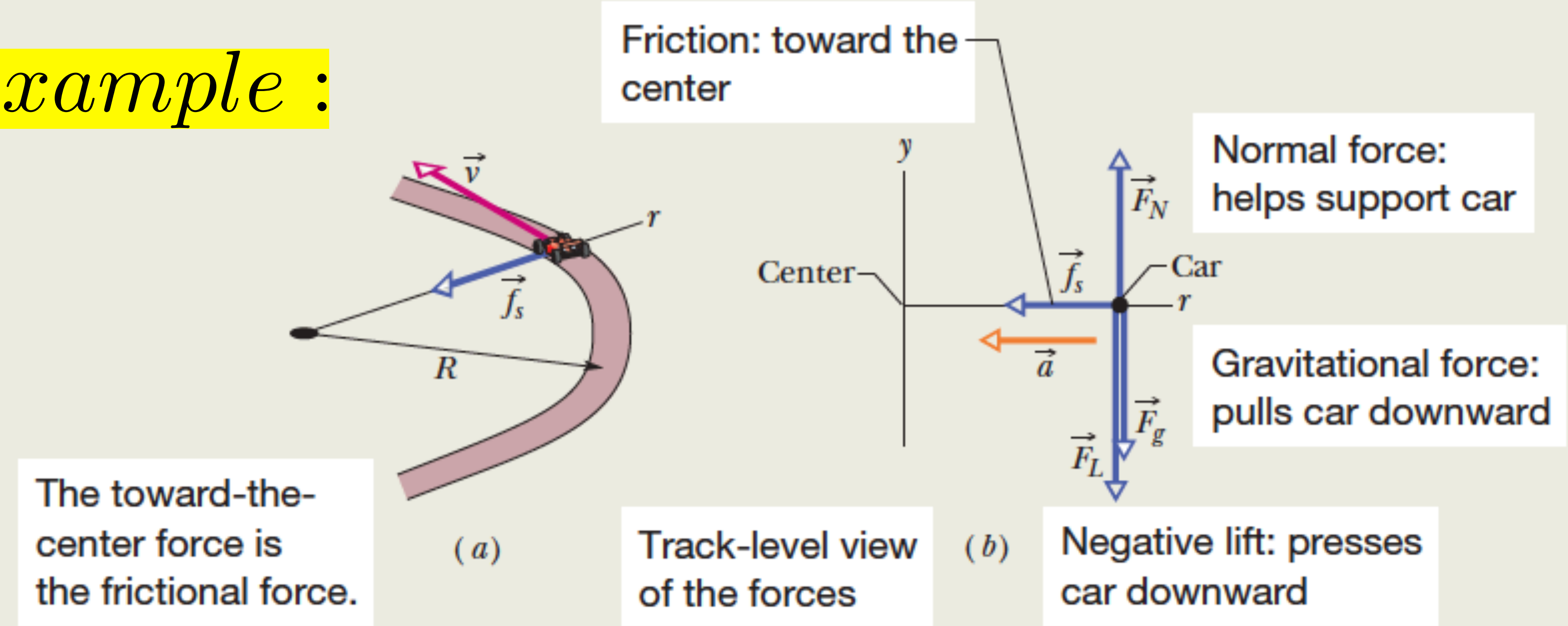
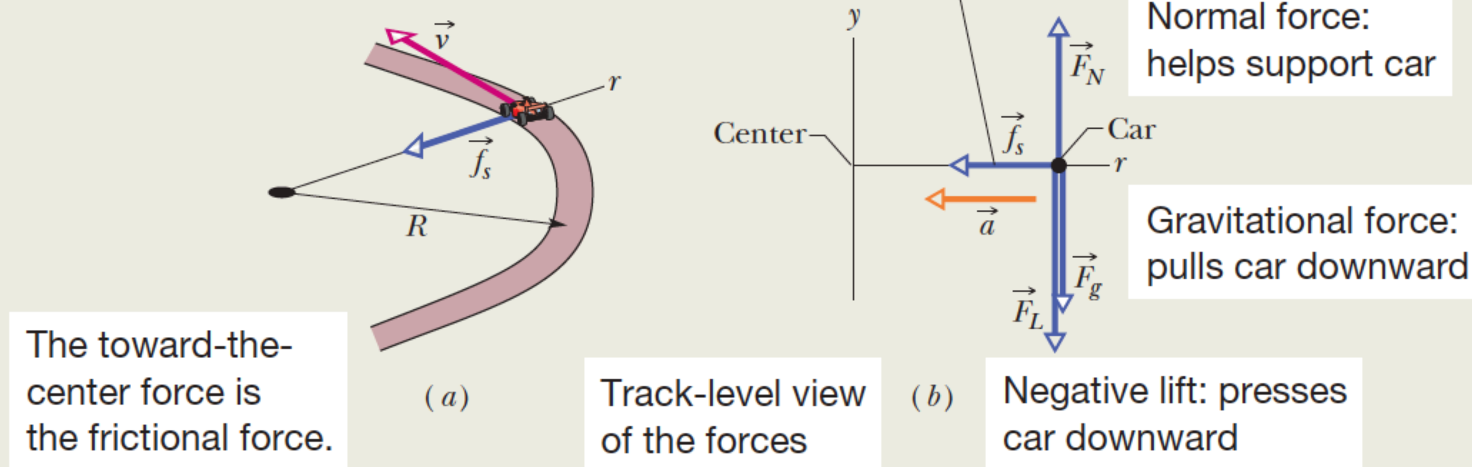


Figure 6-10 (*a*) A race car moves around a flat curved track at constant speed *v*. The frictional force $\vec{f}_s$ provides the necessary centripetal force along a radial axis *r*. (*b*) A free-body diagram (not to scale) for the car, in the vertical plane containing *r*.

Figure 6-10*a* represents a Grand Prix race car of mass $m = 600$ kg as it travels on a flat track in a circular arc of radius $R = 100$ m. Because of the shape of the car and the wings on it, the passing air exerts a negative lift $\vec{F}_L$ downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

(a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of the negative lift $\vec{F}_L$ acting downward on the car?

center



$$-f_s = m\left(-\frac{v^2}{R}\right).$$

$f_{s,max} = \mu_s F_N$ for f_s leads us to

$$\mu_s F_N = m\left(\frac{v^2}{R}\right).$$

$$F_N - mg - F_L = 0,$$
$$F_N = mg + F_L.$$

$$F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$$
$$= (600\text{ kg})\left(\frac{(28.6\text{ m/s})^2}{(0.75)(100\text{ m})} - 9.8\text{ m/s}^2\right)$$
$$= 663.7\text{ N} \approx 660\text{ N}. \qquad \text{(Answer)}$$

(b) The magnitude F_L of the negative lift on a car depends on the square of the car's speed v^2, just as the drag force does (Eq. 6-14). Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

F_L is proportional to v^2.

Calculations: Thus we can write a ratio of the negative lift $F_{L,90}$ at $v = 90$ m/s to our result for the negative lift F_L at $v = 28.6$ m/s as

$$\frac{F_{L,90}}{F_L} = \frac{(90 \text{ m/s})^2}{(28.6 \text{ m/s})^2}.$$

Substituting our known negative lift of $F_L = 663.7$ N and solving for $F_{L,90}$ give us

$$F_{L,90} = 6572 \text{ N} \approx 6600 \text{ N}. \qquad \text{(Answer)}$$

Example

63. **S** A crate of weight F_g is pushed by a force $\vec{\mathbf{P}}$ on a horizontal floor as shown in Figure P5.63. The coefficient of static friction is μ_s, and $\vec{\mathbf{P}}$ is directed at angle θ below the horizontal. (a) Show that the minimum value of P that will move the crate is given by

$$P = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}$$

(b) Find the condition on θ in terms of μ_s for which motion of the crate is impossible for any value of P.

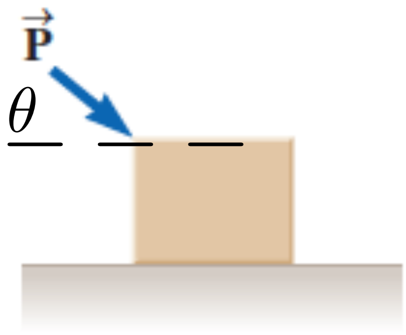


Figure P5.63

Example

59. Q|C S An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P6.59). The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R. (a) Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R\mu_s/g)^{1/2}$. (b) If the rate of revolution of the cylinder is made to be somewhat larger, what happens to the magnitude of each one of the forces acting on the person? What happens in the motion of the person? (c) If the rate of revolution of the cylinder is instead made to be somewhat smaller, what happens to the magnitude of each one of the forces acting on the person? How does the motion of the person change?

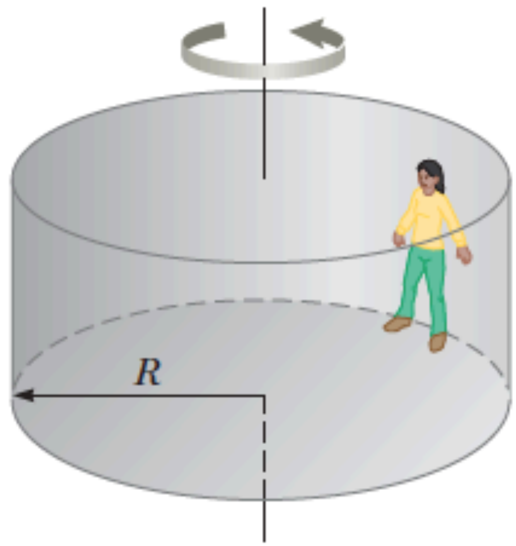


Figure P6.59

Example

69. M A car accelerates down a hill (Fig. P5.69), going from rest to 30.0 m/s in 6.00 s. A toy inside the car hangs by a string from the car's ceiling. The ball in the figure represents the toy, of mass 0.100 kg. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle θ and (b) the tension in the string.

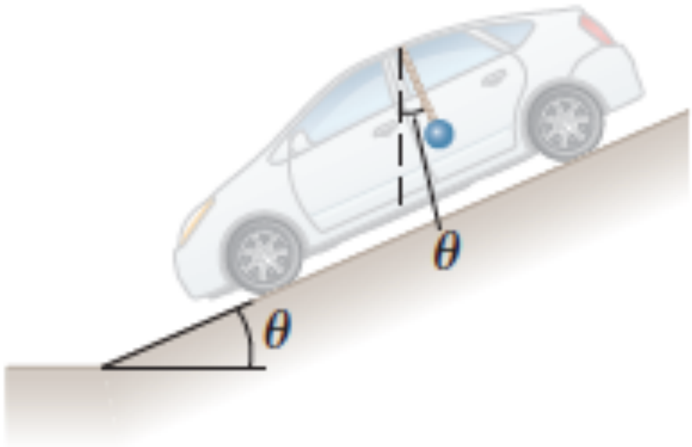


Figure P5.69

Example

67 In Fig. 6-51, a crate slides down an inclined right-angled trough. The coefficient of kinetic friction between the crate and the trough is μ_k. What is the acceleration of the crate in terms of μ_k, θ, and g?

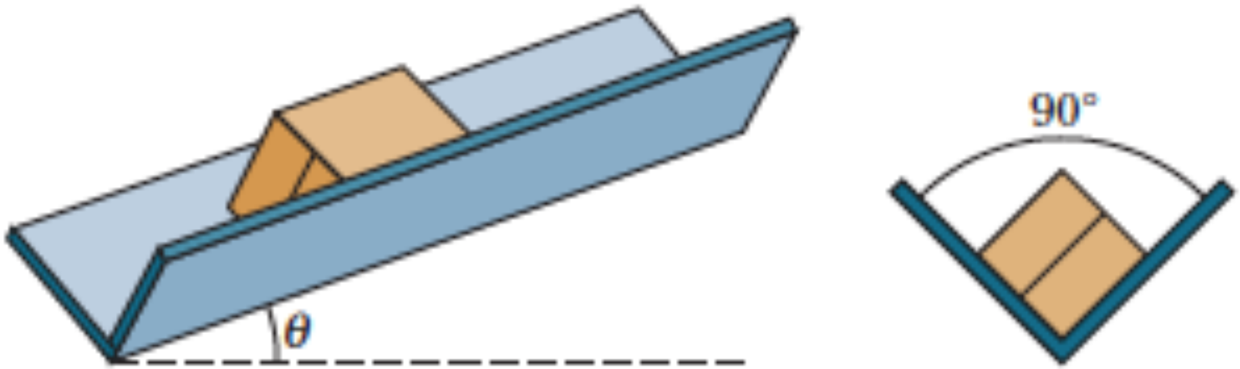


Figure 6-51 Problem 67.

$$g(\sin\theta - \sqrt{2}\ \mu_k\ \cos\theta)$$

Example

42. **S** A child's toy consists of a small wedge that has an acute angle θ (Fig. P6.42). The sloping side of the wedge is frictionless, and an object of mass m on it remains at constant height if the wedge is spun at a certain constant speed. The wedge is spun by rotating, as an axis, a vertical rod that is firmly attached to the wedge at the bottom end. Show that, when the object sits at rest at a point at distance L up along the wedge, the speed of the object must be $v = (gL \sin \theta)^{1/2}$.

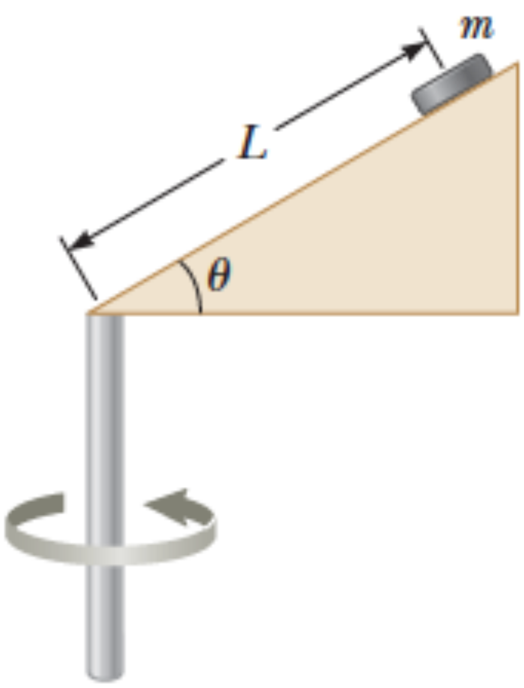


Figure P6.42

Example

98 In Fig. 6-62, a 5.0 kg block is sent sliding up a plane inclined at $\theta = 37°$ while a horizontal force $\vec{F}$ of magnitude 50 N acts on it. The coefficient of kinetic friction between block and plane is 0.30. What are the (a) magnitude and (b) direction (up or down the plane) of the block's acceleration? The block's initial speed is 4.0 m/s. (c) How far up the plane does the block go? (d) When it reaches its highest point, does it remain at rest or slide back down the plane?

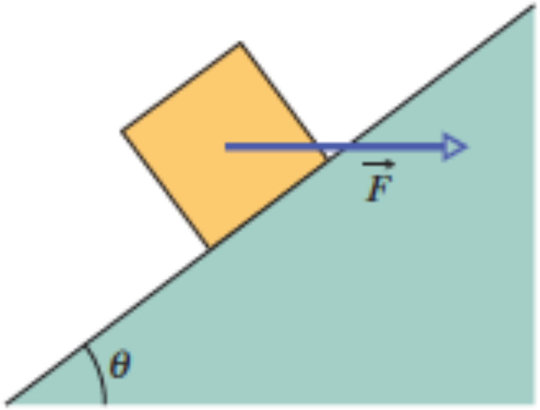


Figure 6-62 Problem 98.

Example

61. Raindrops make an angle θ with the vertical when viewed through a moving train window (Fig. 3–52). If the speed of the train is v_T, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

FIGURE 3–52
Problem 61.

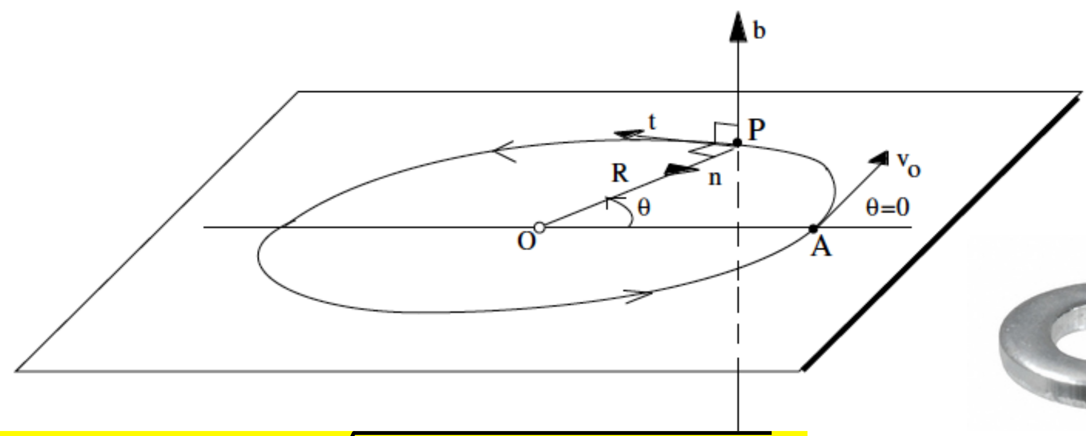
Example

78. A jet aircraft is accelerating at $3.8\ \mathrm{m/s^2}$ as it climbs at an angle of 18° above the horizontal (Fig. 4–67). What is the total force that the cockpit seat exerts on the 75-kg pilot?

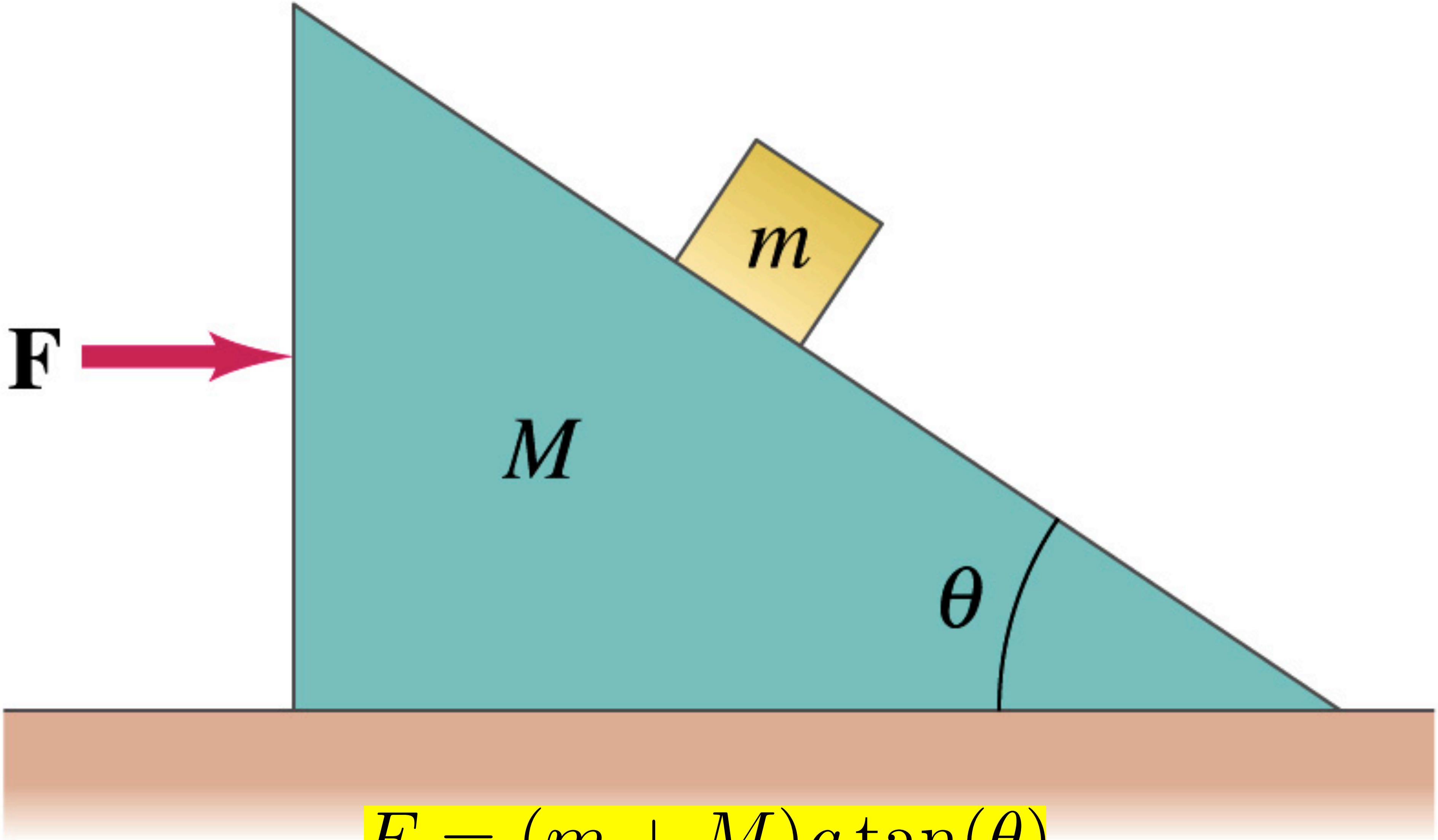
FIGURE 4–67
Problem 78.

Circular motion with friction

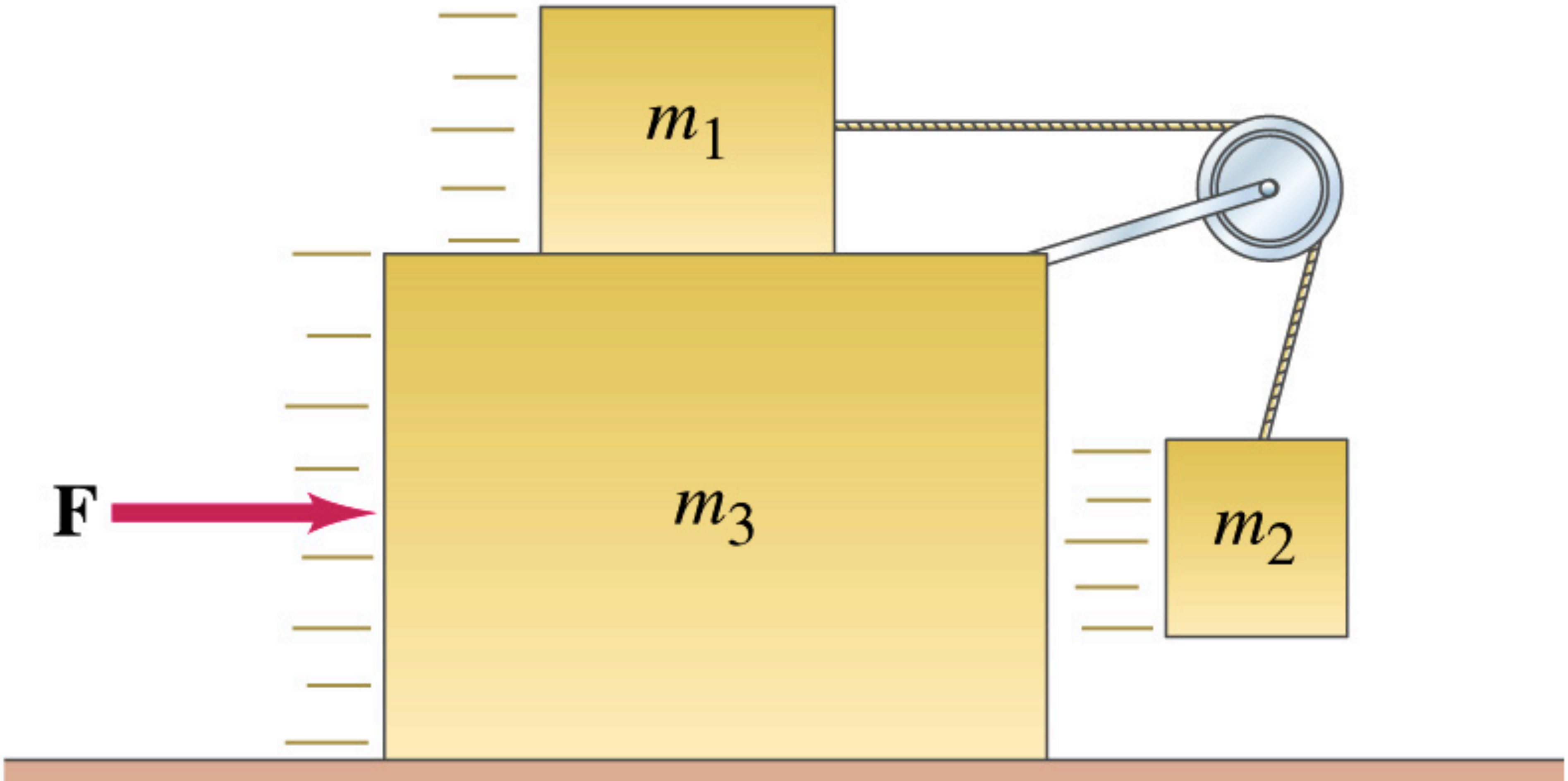
- A small bead of mass m is given an initial velocity of magnitude v_0 on a horizontal circular wire. If the coefficient of kinetic friction is μ_k, determine the distance travelled before the collar comes to rest.



$$L = \frac{R}{2\mu_k} \ln \left[\frac{v_0^2 + \sqrt{R^2 g^2 + v_0^4}}{Rg} \right]$$



$$F = (m + M) g \tan(\theta)$$



$$\sin(\theta) = m_2/m_1$$

$$a = g\tan(\theta) = \frac{m_2 g}{(m_1^2 - m_2^2)^{1/2}}$$

$$F = \frac{(m_1 + m_2 + m_3)m_2 g}{(m_1^2 - m_2^2)^{1/2}}$$