# General Physics I

## chapter 12

Sharif University of Technology
Mehr 1401 (2022-2023)

M. Reza Rahimi Tabar

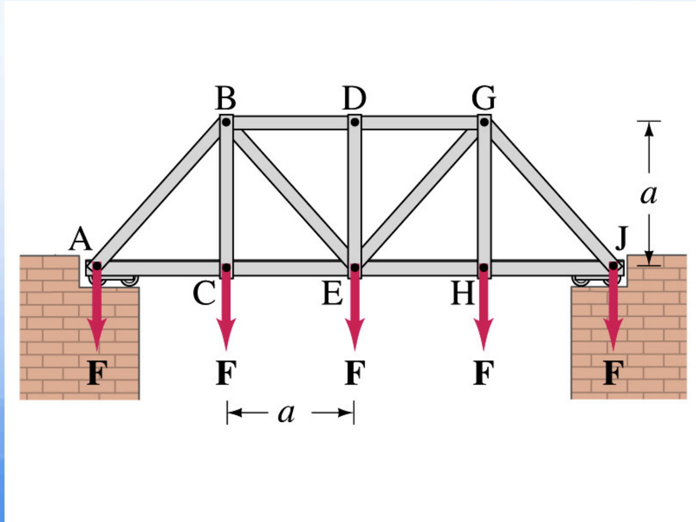
# Chapter 12
# Equilibrium and Elasticity

- What Is Physics?
- Equilibrium
- The Requirements of Equilibrium
- The Center of Gravity
- Some Examples of Static Equilibrium
- Indeterminate Structures
- Elasticity

# What is Physics?

- Human buildings should be stable in spite of the gravitational force and wind forces on it, and a bridge should be stable in spite of the gravitational force pulling it downward and the repeated jolting it receives from cars and trucks.

- In this chapter we examine the two main aspects of stability: the equilibrium of the forces and torques acting on rigid objects and the elasticity of non-rigid objects, a property that governs how such objects can deform.

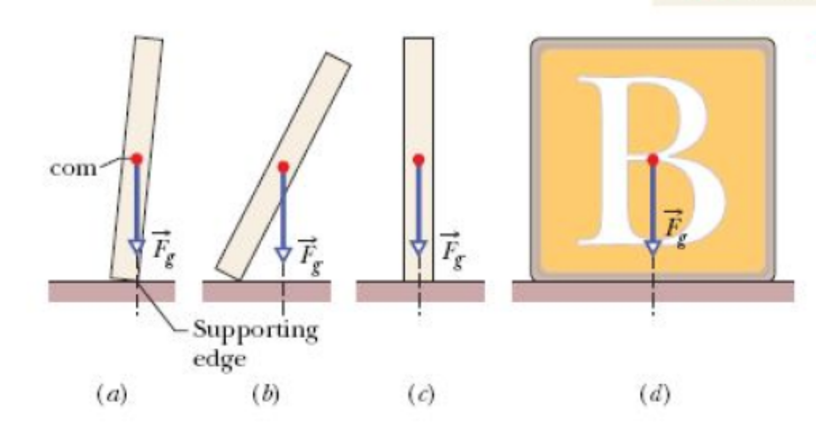


# Equilibrium

- Equilibrium: ( Force and torque are zero) ,

$$\vec{P} = \text{a constant} \quad \text{and} \quad \vec{L} = \text{a constant.}$$

- Static equilibrium: ( constant = 0 )



# The Requirements of Equilibrium

1. The net external force on the object must equal zero:

$$\sum \vec{\mathbf{F}}_{\text{ext}} = 0$$

✔

2. The net external torque on the object about *any* axis must be zero:

$$\sum \vec{\boldsymbol{\tau}}_{\text{ext}} = 0$$

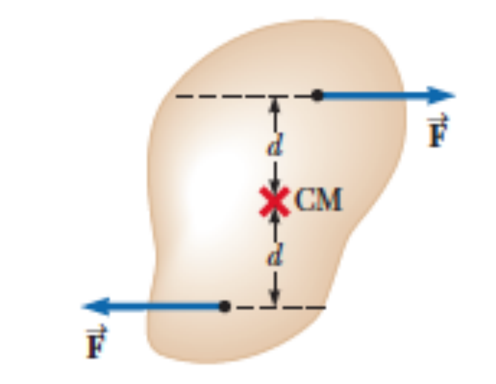
✔

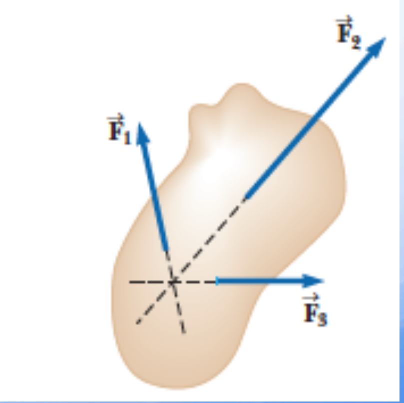
(that is, $v_{\text{CM}} = 0$ and $\omega = 0$).

- Examples: with zero net

External force

and zero net external torque



# Coplanar problems:

1. The net external force on the object must equal zero:

$$\sum \vec{\mathbf{F}}_{ext} = 0$$

2. The net external torque on the object about *any* axis must be zero:

$$\sum \vec{\boldsymbol{\tau}}_{ext} = 0$$

| Balance of forces | Balance of torques |
| --- | --- |
| $F_{net,x} = 0$ | $\tau_{net,x} = 0$ |
| $F_{net,y} = 0$ | $\tau_{net,y} = 0$ |
| $F_{net,z} = 0$ | $\tau_{net,z} = 0$ |

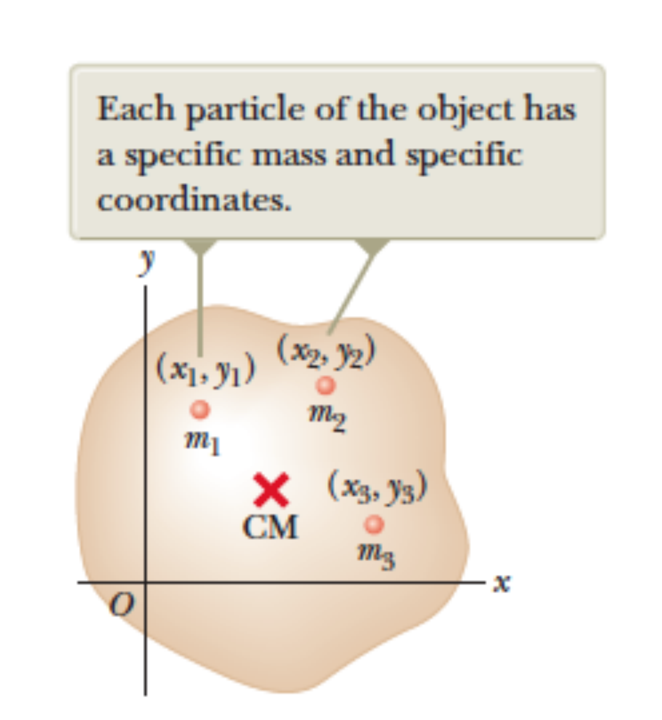
✔ $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum \tau_z = 0$

where the location of the axis of the torque equation is arbitrary.

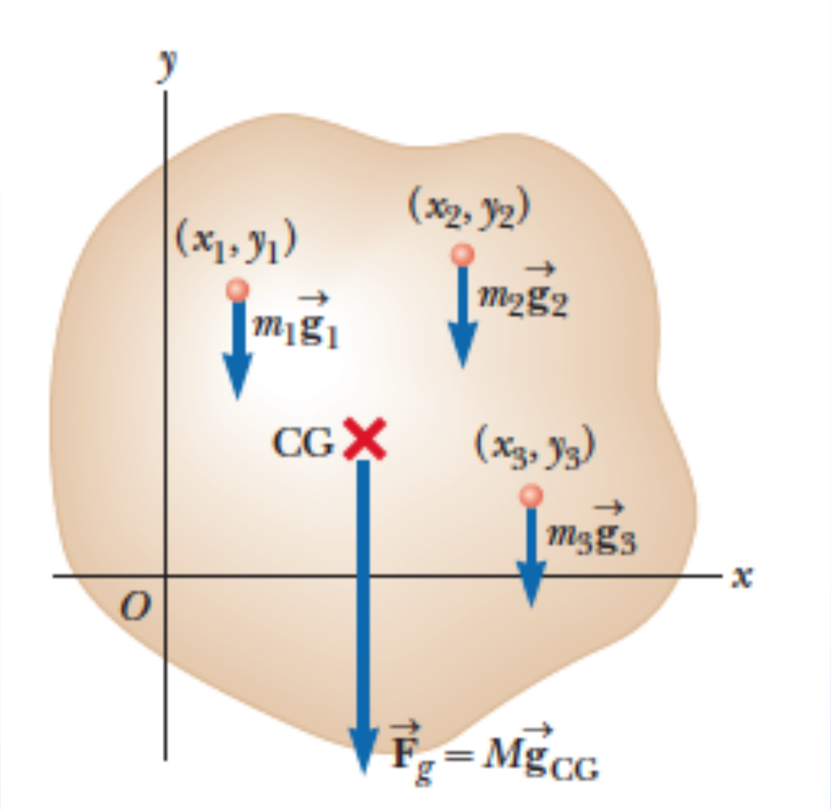
# The Center of Gravity

- Center of mass:

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$



- Center of gravity:




$$(m_1 + m_2 + m_3 + \cdots)g_{CG}\, x_{CG} = m_1 g_1 x_1 + m_2 g_2 x_2 + m_3 g_3 x_3 + \cdots$$

# Center of gravity:

Equating the torque resulting from $M\vec{g}_{CG}$ acting at the center of gravity to the sum of the torques acting on the individual particles gives

$$(m_1 + m_2 + m_3 + \cdots)g_{CG}\, x_{CG} = m_1 g_1 x_1 + m_2 g_2 x_2 + m_3 g_3 x_3 + \cdots$$

This expression accounts for the possibility that the value of $g$ can in general vary over the object. If we assume uniform $g$ over the object (as is usually the case), the $g$ factors cancel and we obtain

$$x_{CG} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

# Example:

A seesaw consisting of a uniform board of mass $M$ and length $\ell$ supports at rest a father and daughter with masses $m_f$ and $m_d$, respectively, as shown in Figure 12.7. The support (called the *fulcrum*) is under the center of gravity of the board, the father is a distance $d$ from the center, and the daughter is a distance $\ell/2$ from the center.

**(A)** Determine the magnitude of the upward force $\vec{n}$ exerted by the support on the board.

**(B)** Determine where the father should sit to balance the system at rest.

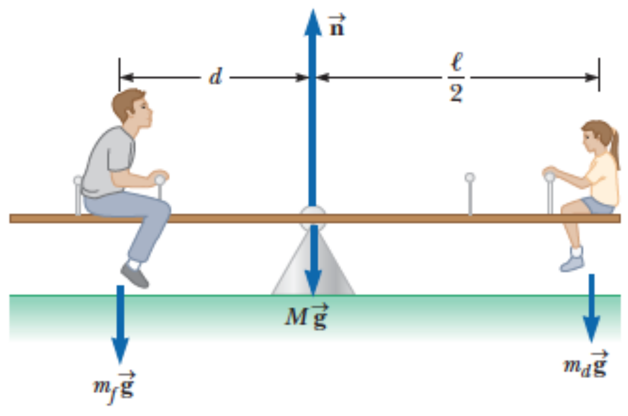



**Figure 12.7** (Example 12.1) A balanced system.

$$n - m_f g - m_d g - Mg = 0$$

$$n = m_f g + m_d g + Mg = (m_f + m_d + M)g$$

$$(m_f g)(d) - (m_d g)\frac{\ell}{2} = 0$$

$$d = \left(\frac{m_d}{m_f}\right)\frac{\ell}{2}$$

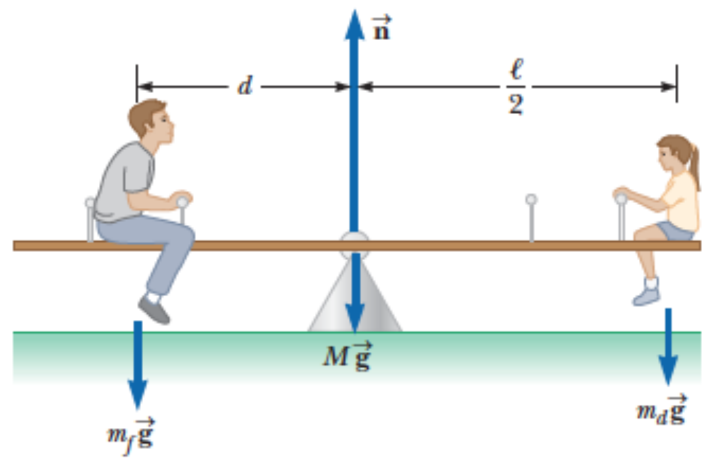
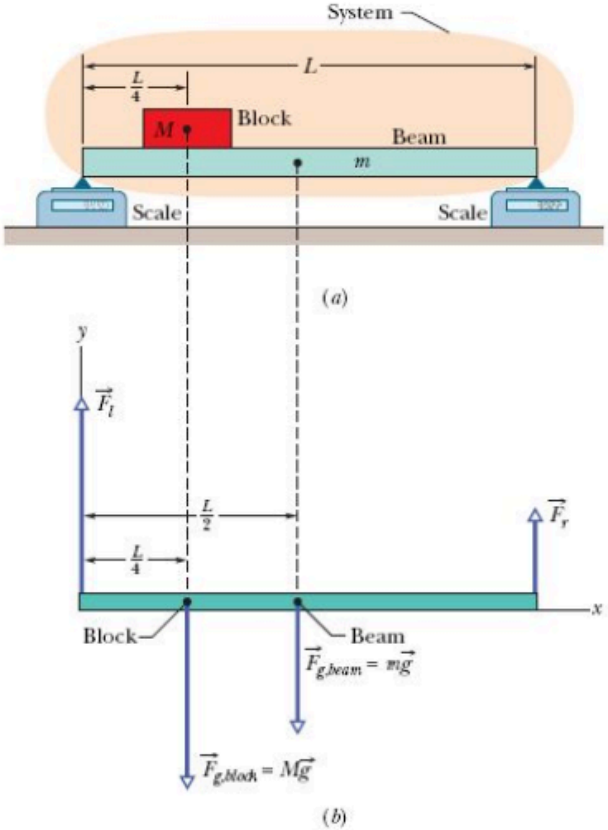



**Figure 12.7** (Example 12.1) A balanced system.

# Example:

In Fig. 12-5*a*, a uniform beam, of length $L$ and mass $m$ = 1.8 kg, is at rest on two scales. A uniform block, with mass $M$ = 2.7 kg, is at rest on the beam, with its center a distance $L/4$ from the beam's left end. What do the scales read?

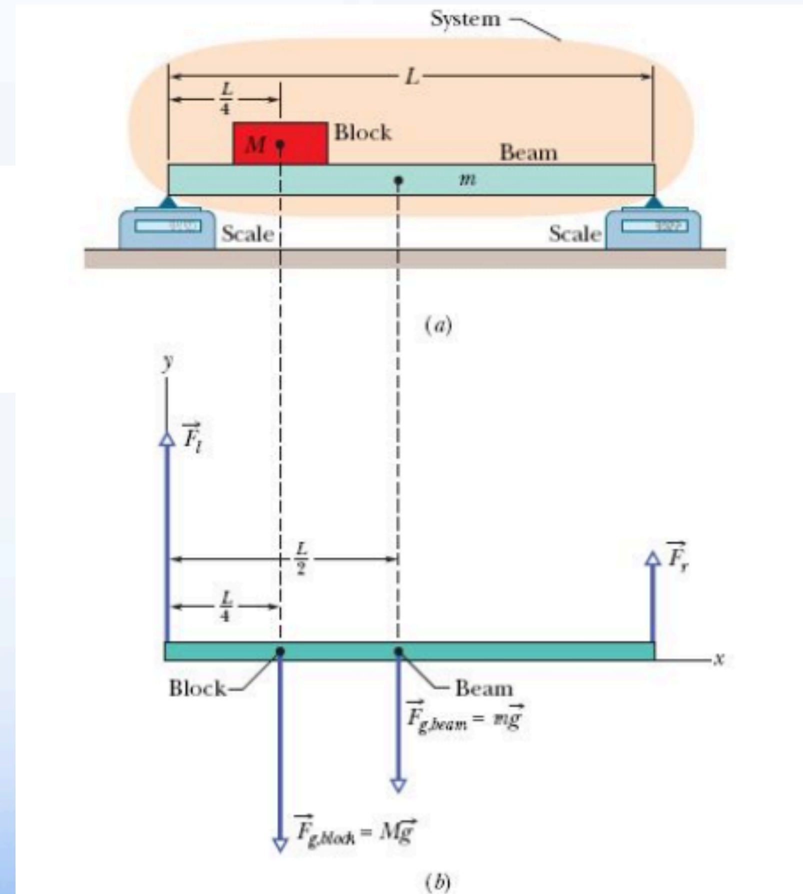


$$F_l + F_r - Mg - mg = 0.$$

$$(0)(F_l) - (L/4)(Mg) - (L/2)(mg) + (L)(F_r) = 0,$$

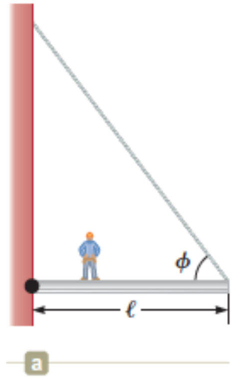
$$F_r = \tfrac{1}{4}Mg + \tfrac{1}{2}mg$$
$$= \tfrac{1}{4}(2.7\ \text{kg})(9.8\ \text{m/s}^2) + \tfrac{1}{2}(1.8\ \text{kg})(9.8\ \text{m/s}^2)$$
$$= 15.44\ \text{N} \approx 15\ \text{N}. \qquad \text{(Answer)}$$

$$F_l = (M + m)g - F_r$$
$$= (2.7\ \text{kg} + 1.8\ \text{kg})(9.8\ \text{m/s}^2) - 15.44\ \text{N}$$
$$= 28.66\ \text{N} \approx 29\ \text{N}. \qquad \text{(Answer)}$$



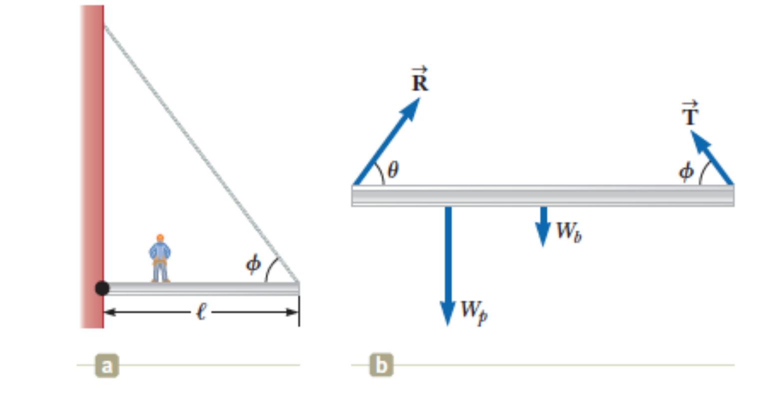
# Example:

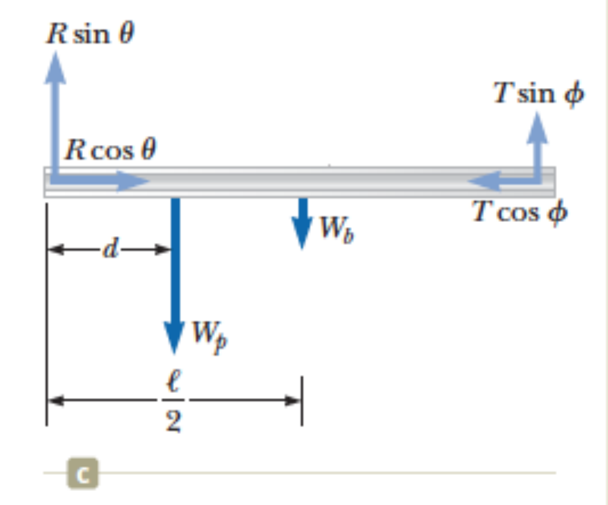
A uniform horizontal beam with a length of $\ell = 8.00$ m and a weight of $W_b = 200$ N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi = 53.0°$ with the beam (Fig. 12.8a). A person of weight $W_p = 600$ N stands a distance $d = 2.00$ m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.

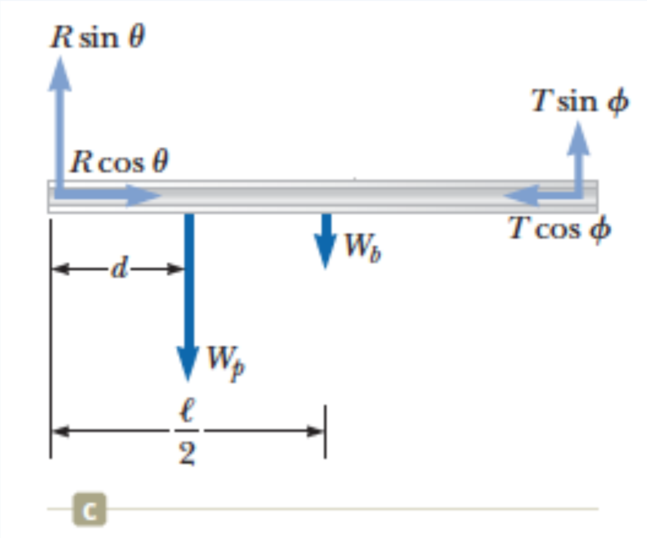


# Example:

A uniform horizontal beam with a length of $\ell$ = 8.00 m and a weight of $W_b$ = 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi$ = 53.0° with the beam (Fig. 12.8a). A person of weight $W_p$ = 600 N stands a distance $d$ = 2.00 m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.






$$(1)\ \sum F_x = R\cos\theta - T\cos\phi = 0$$

$$(2)\ \sum F_y = R\sin\theta + T\sin\phi - W_p - W_b = 0$$

$$\sum \tau_z = (T\sin\phi)(\ell) - W_p d - W_b\left(\frac{\ell}{2}\right) = 0$$

$$T = \frac{W_p d + W_b(\ell/2)}{\ell\sin\phi} = \frac{(600\ \text{N})(2.00\ \text{m}) + (200\ \text{N})(4.00\ \text{m})}{(8.00\ \text{m})\sin 53.0^\circ} = \boxed{313\ \text{N}}$$

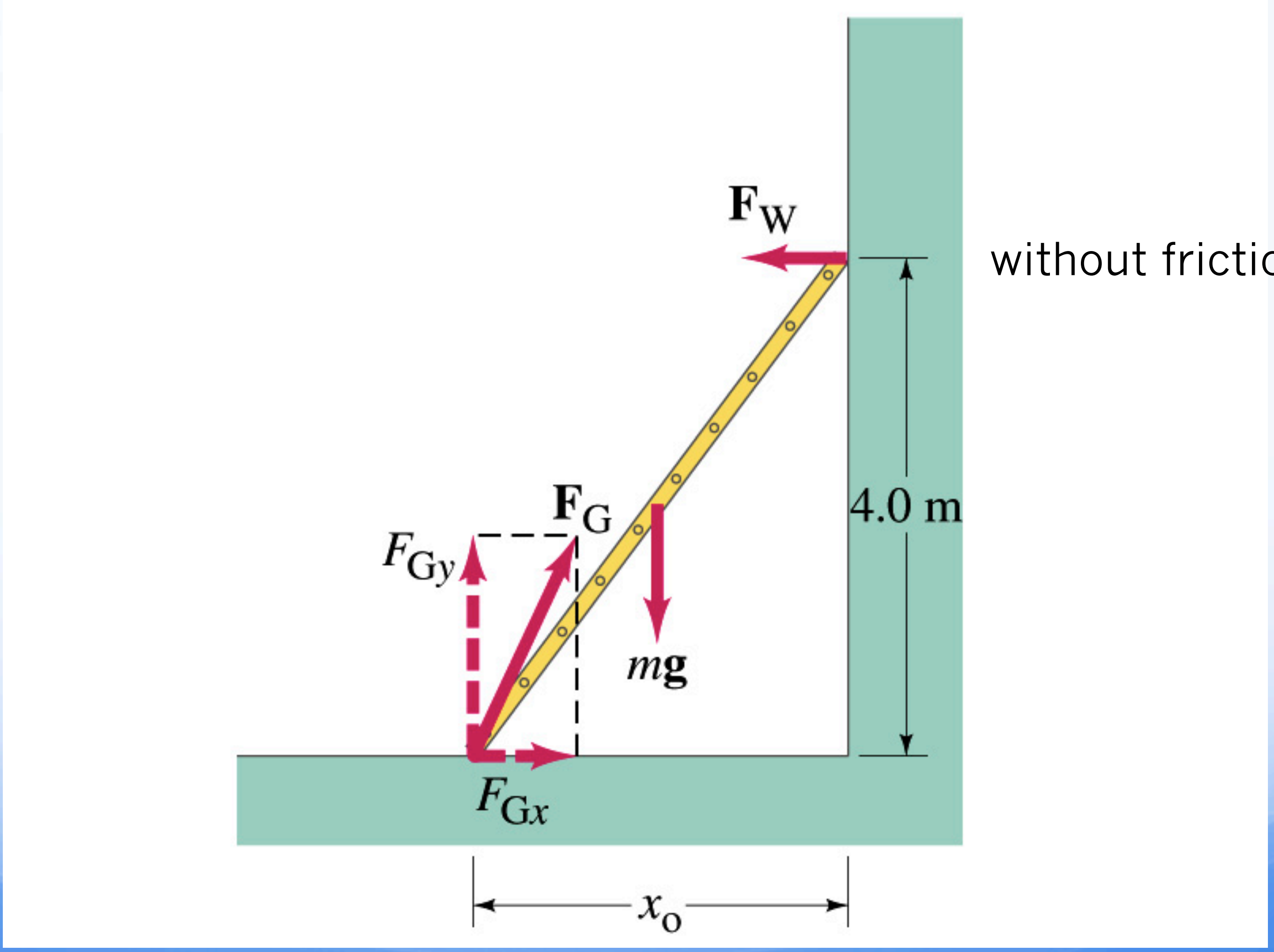
$$\frac{R\sin\theta}{R\cos\theta} = \tan\theta = \frac{W_p + W_b - T\sin\phi}{T\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{W_p + W_b - T\sin\phi}{T\cos\phi}\right)$$

$$= \tan^{-1}\left[\frac{600\ \text{N} + 200\ \text{N} - (313\ \text{N})\sin 53.0^\circ}{(313\ \text{N})\cos 53.0^\circ}\right] = \boxed{71.1^\circ}$$

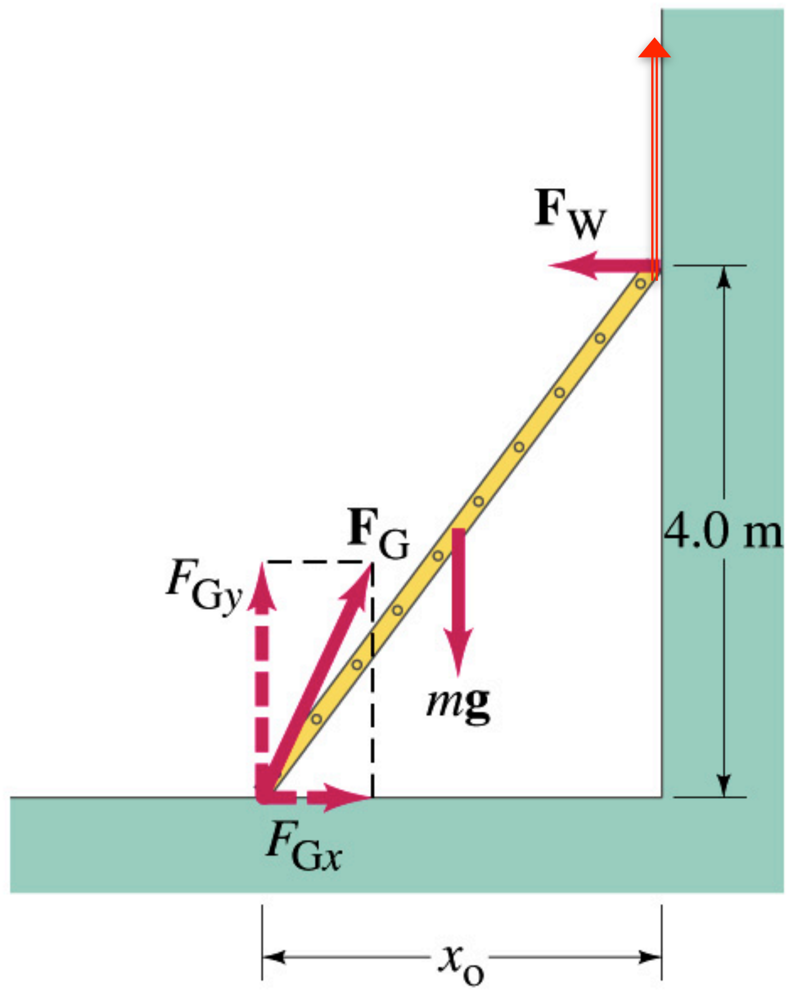
$$R = \frac{T\cos\phi}{\cos\theta} = \frac{(313\ \text{N})\cos 53.0^\circ}{\cos 71.1^\circ} = \boxed{581\ \text{N}}$$

# Example:




without friction

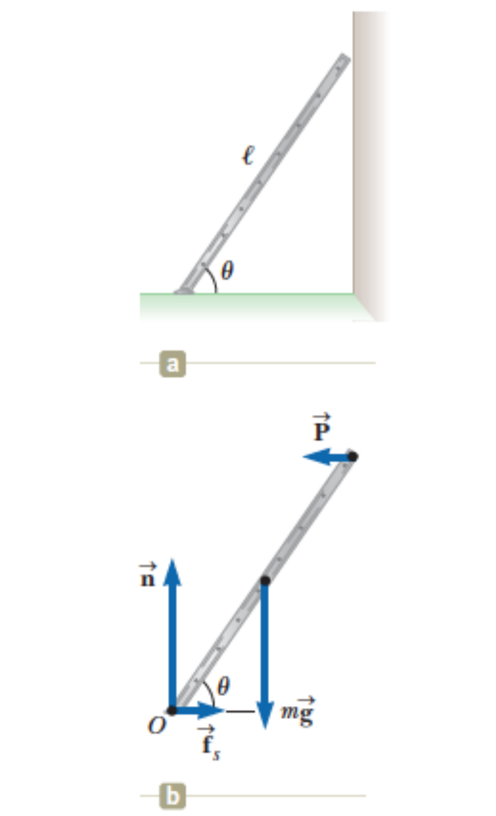
# Example:




with friction

# Example:

A uniform ladder of length $\ell$ rests against a smooth, vertical wall (Fig. 12.9a). The mass of the ladder is $m$, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle $\theta_{min}$ at which the ladder does not slip.



(1) $\sum F_x = f_s - P = 0$

(2) $\sum F_y = n - mg = 0$

(3) $P = f_s$

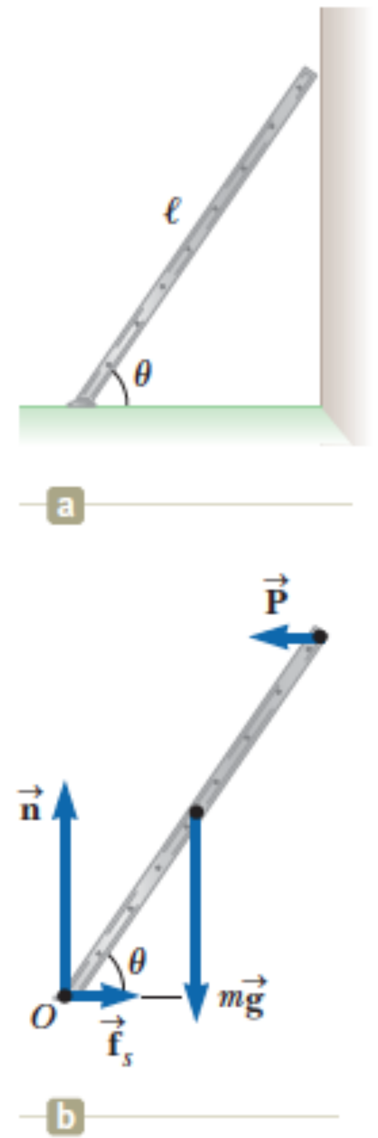
(4) $n = mg$

(5) $P = f_{s,\max} = \mu_s n = \mu_s mg$

$$\sum \tau_O = P\ell \sin\theta_{\min} - mg\frac{\ell}{2}\cos\theta_{\min} = 0$$

$$\frac{\sin\theta_{\min}}{\cos\theta_{\min}} = \tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s}$$

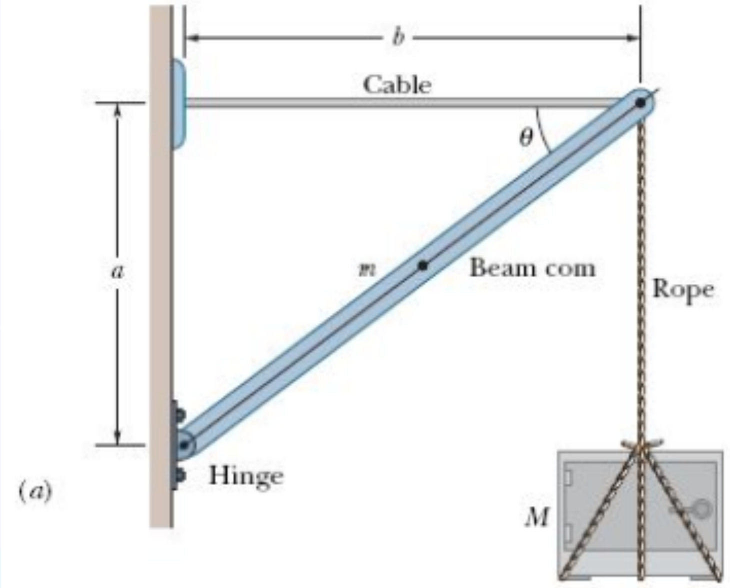
$$\theta_{\min} = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = \tan^{-1}\left[\frac{1}{2(0.40)}\right] = \boxed{51^\circ}$$



# Example:

Figure 12-7*a* shows a safe, of mass $M = 430$ kg, hanging by a rope from a boom with dimensions $a = 1.9$ m and $b = 2.5$ m. The boom consists of a hinged beam and a horizontal cable. The uniform beam has a mass $m$ of 85 kg; the masses of the cable and rope are negligible.

(a) What is the tension $T_c$ in the cable? In other words, what is the magnitude of the force $\vec{T}_c$ on the beam from the cable?



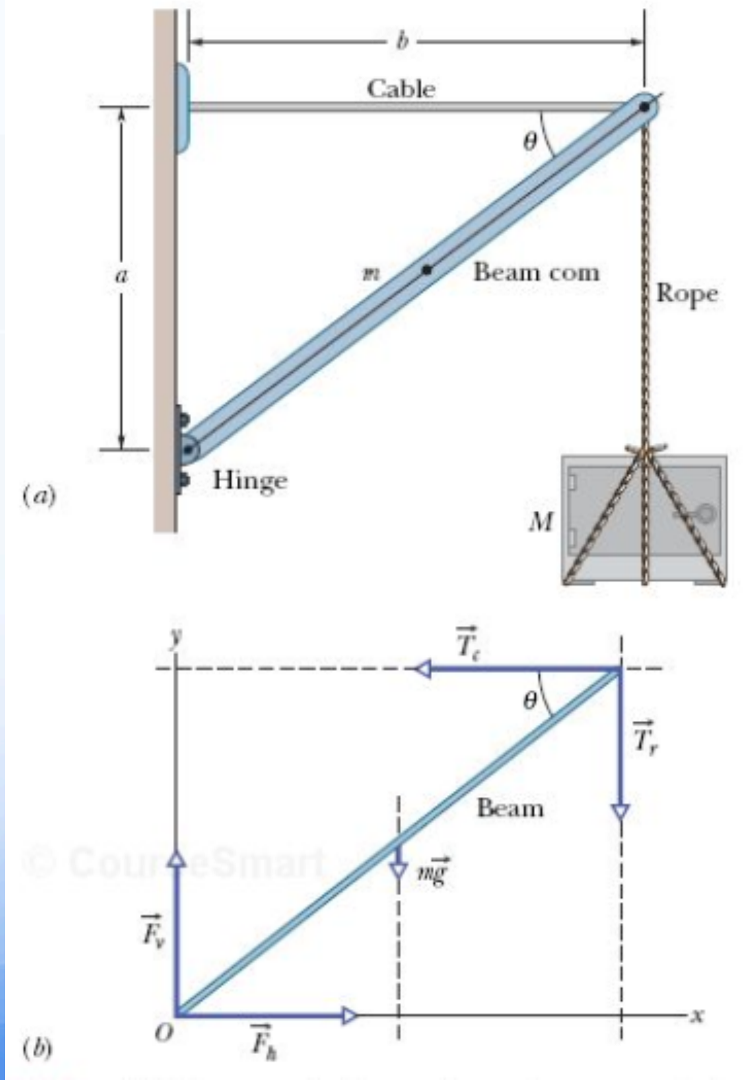

(b) Find the magnitude $F$ of the net force on the beam from the hinge.

# Example:

Figure 12-7*a* shows a safe, of mass $M = 430$ kg, hanging by a rope from a boom with dimensions $a = 1.9$ m and $b = 2.5$ m. The boom consists of a hinged beam and a horizontal cable. The uniform beam has a mass $m$ of 85 kg; the masses of the cable and rope are negligible.

(a) What is the tension $T_c$ in the cable? In other words, what is the magnitude of the force $\vec{T}_c$ on the beam from the cable?

(b) Find the magnitude $F$ of the net force on the beam from the hinge.



Writing torques in the form of $r_\perp F$ and using our rule about signs for torques, the balancing equation $\tau_{net,z} = 0$ becomes

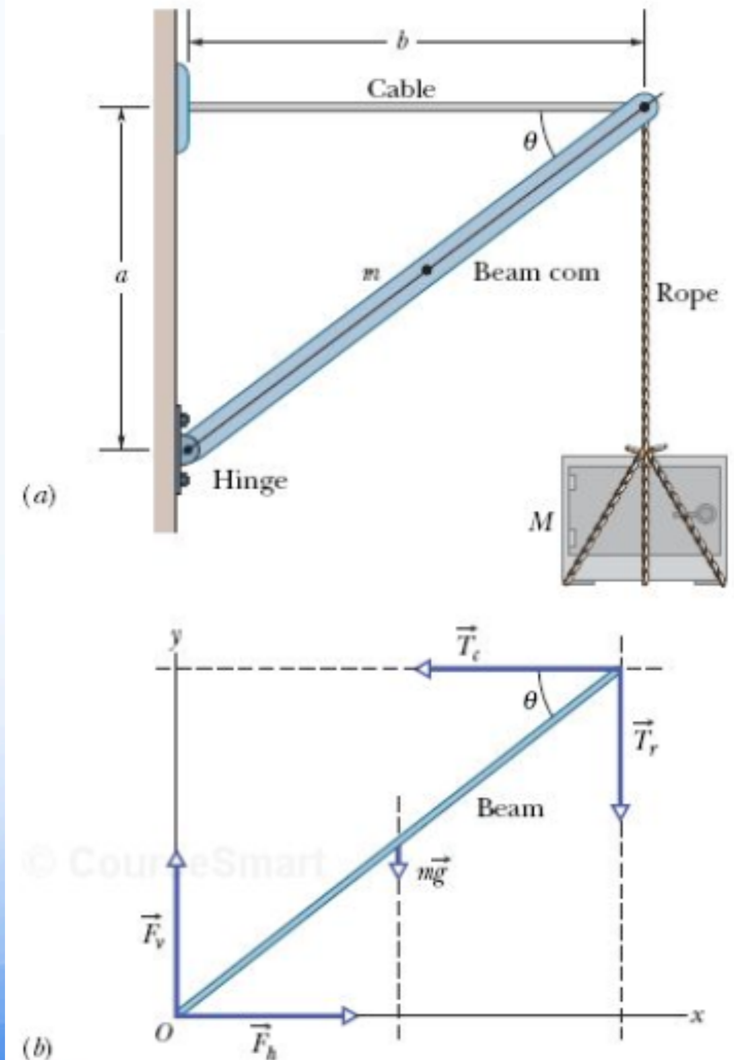
$$(a)(T_c) - (b)(T_r) - (\tfrac{1}{2}b)(mg) = 0.$$

Substituting $Mg$ for $T_r$ and solving for $T_c$, we find that

$$T_c = \frac{gb(M + \frac{1}{2}m)}{a}$$

$$= \frac{(9.8\ \text{m/s}^2)(2.5\ \text{m})(430\ \text{kg} + 85/2\ \text{kg})}{1.9\ \text{m}}$$

$$= 6093\ \text{N} \approx 6100\ \text{N}. \qquad \text{(Answer)}$$



(b) Find the magnitude $F$ of the net force on the beam from the hinge.

**Calculations:** For the horizontal balance, we write $F_{\text{net},x} = 0$ as

$$F_h - T_c = 0,$$

and so $$F_h = T_c = 6093\ \text{N}.$$

For the vertical balance, we write $F_{\text{net},y} = 0$ as

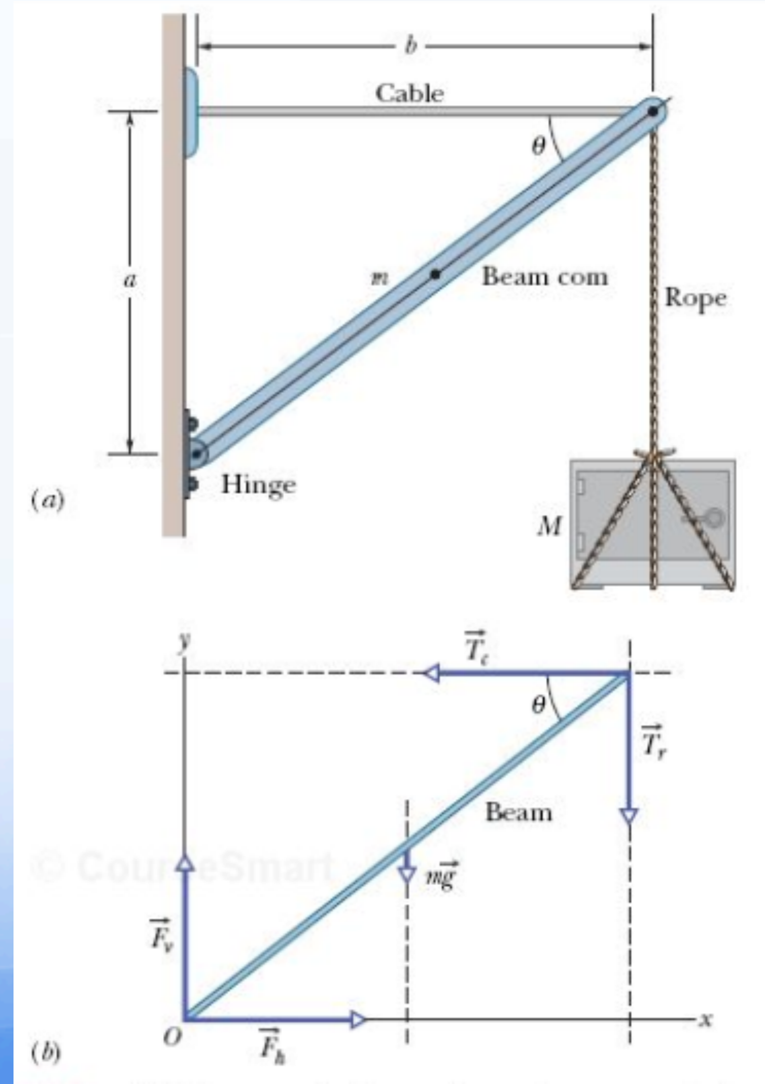
$$F_v - mg - T_r = 0.$$

Substituting $Mg$ for $T_r$ and solving for $F_v$, we find that

$$F_v = (m + M)g = (85\ \text{kg} + 430\ \text{kg})(9.8\ \text{m/s}^2) = 5047\ \text{N}.$$

From the Pythagorean theorem, we now have

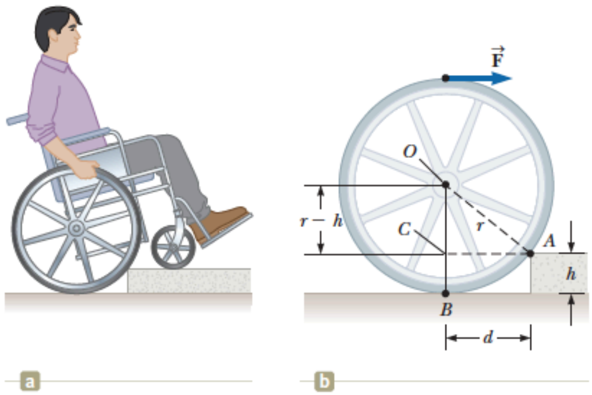
$$F = \sqrt{F_h^2 + F_v^2} = \sqrt{(6093\ \text{N})^2 + (5047\ \text{N})^2} \approx 7900\ \text{N}. \quad \text{(Answer)}$$



# Example:

**(A)** Estimate the magnitude of the force $\vec{F}$ a person must apply to a wheelchair's main wheel to roll up over a sidewalk curb (Fig. 12.10a). This main wheel that comes in contact with the curb has a radius $r$, and the height of the curb is $h$.

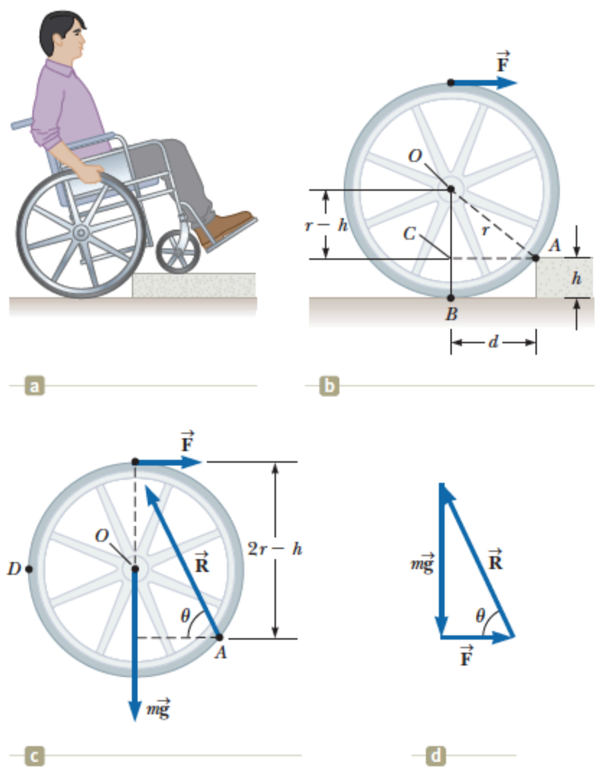
**(B)** Determine the magnitude and direction of $\vec{R}$.



# Example:

**(A)** Estimate the magnitude of the force $\vec{F}$ a person must apply to a wheelchair's main wheel to roll up over a sidewalk curb (Fig. 12.10a). This main wheel that comes in contact with the curb has a radius $r$, and the height of the curb is $h$.

**(B)** Determine the magnitude and direction of $\vec{R}$.



$$(1)\quad d = \sqrt{r^2 - (r-h)^2} = \sqrt{2rh - h^2}$$

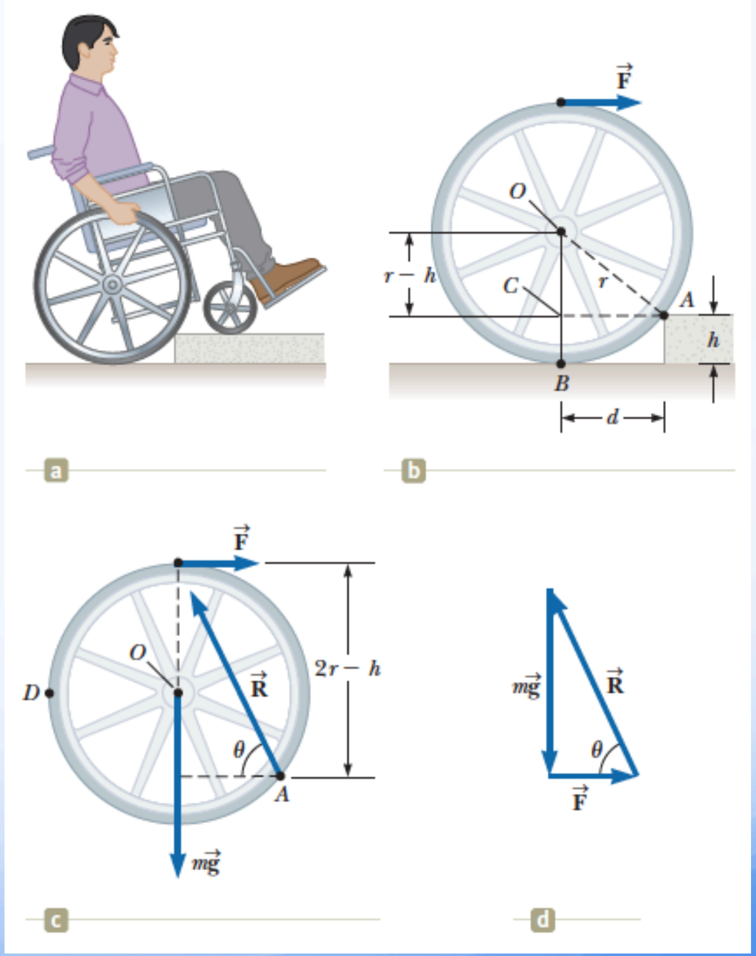
$$(2)\quad \sum \tau_A = mgd - F(2r-h) = 0$$

$$mg\sqrt{2rh - h^2} - F(2r-h) = 0$$

$$F = \frac{mg\sqrt{2rh - h^2}}{2r - h}$$

$$F = \frac{(700\ \text{N})\sqrt{2(0.3\ \text{m})(0.1\ \text{m}) - (0.1\ \text{m})^2}}{2(0.3\ \text{m}) - 0.1\ \text{m}}$$

$$= 3 \times 10^2\ \text{N}$$



**(B)** Determine the magnitude and direction of $\vec{\mathbf{R}}$.

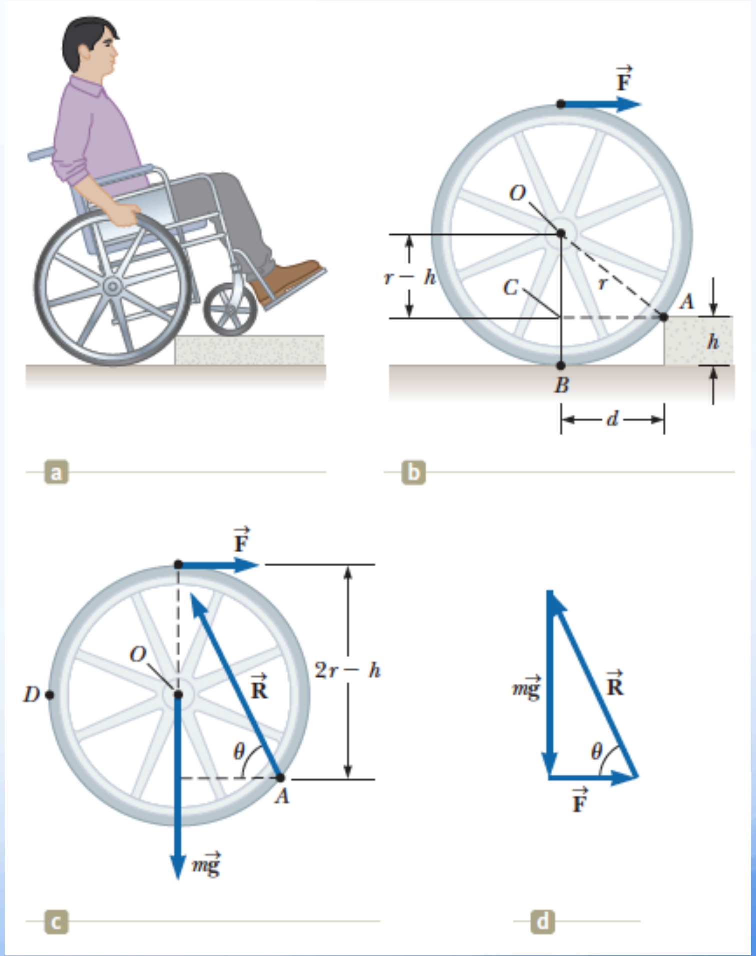
$$(3) \quad \sum F_x = F - R\cos\theta = 0$$

$$(4) \quad \sum F_y = R\sin\theta - mg = 0$$

$$\frac{R\sin\theta}{R\cos\theta} = \tan\theta = \frac{mg}{F}$$

$$\theta = \tan^{-1}\left(\frac{mg}{F}\right) = \tan^{-1}\left(\frac{700\text{ N}}{300\text{ N}}\right) = \boxed{70°}$$

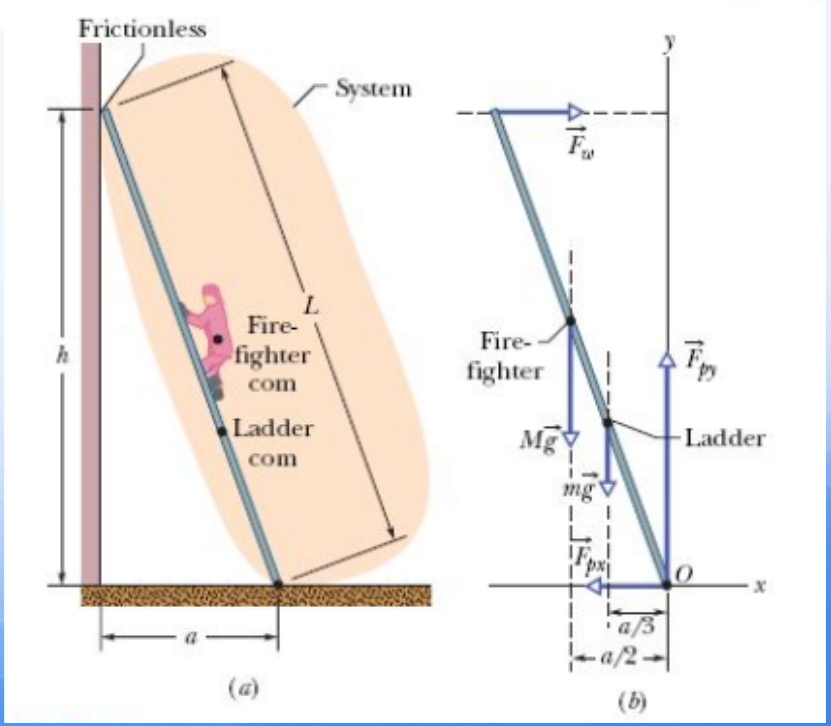
$$R = \frac{mg}{\sin\theta} = \frac{700\text{ N}}{\sin 70°} = \boxed{8 \times 10^2\text{ N}}$$



# Example:

In Fig. 12-6*a*, a ladder of length $L = 12$ m and mass $m =$ 45 kg leans against a slick (frictionless) wall. The ladder's upper end is at height $h = 9.3$ m above the pavement on which the lower end rests (the pavement is not frictionless). The ladder's center of mass is $L/3$ from the lower end. A firefighter of mass $M = 72$ kg climbs the ladder until her center of mass is $L/2$ from the lower end. What then are the magnitudes of the forces on the ladder from the wall and the pavement?





$$-(h)(F_w) + (a/2)(Mg) + (a/3)(mg) + (0)(F_{px}) + (0)(F_{py}) = 0.$$

Using the Pythagorean theorem, we find that

$$a = \sqrt{L^2 - h^2} = 7.58 \text{ m}.$$

Then Eq. 12-19 gives us

$$F_w = \frac{ga(M/2 + m/3)}{h}$$

$$= \frac{(9.8 \text{ m/s}^2)(7.58 \text{ m})(72/2 \text{ kg} + 45/3 \text{ kg})}{9.3 \text{ m}}$$

$$= 407 \text{ N} \approx 410 \text{ N}. \qquad \text{(Answer)}$$

Now we need to use the force balancing equations. The equation $F_{net,x} = 0$ gives us

$$F_w - F_{px} = 0,$$

so

$$F_{px} = F_w = 410 \text{ N}. \qquad \text{(Answer)}$$

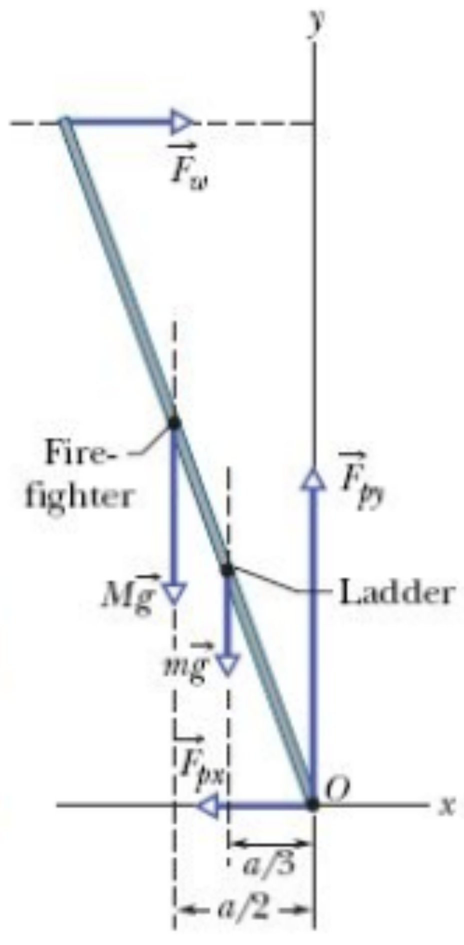
The equation $F_{net,y} = 0$ gives us

$$F_{py} - Mg - mg = 0,$$

so

$$F_{py} = (M + m)g = (72 \text{ kg} + 45 \text{ kg})(9.8 \text{ m/s}^2)$$

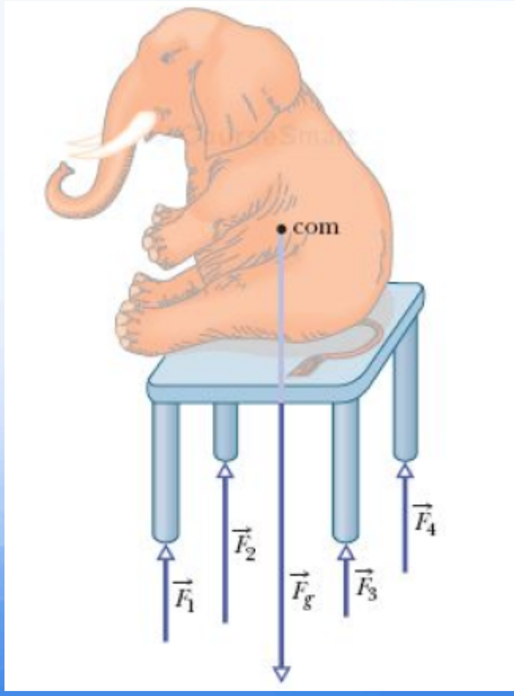
$$= 1146.6 \text{ N} \approx 1100 \text{ N}. \qquad \text{(Answer)}$$



(b)

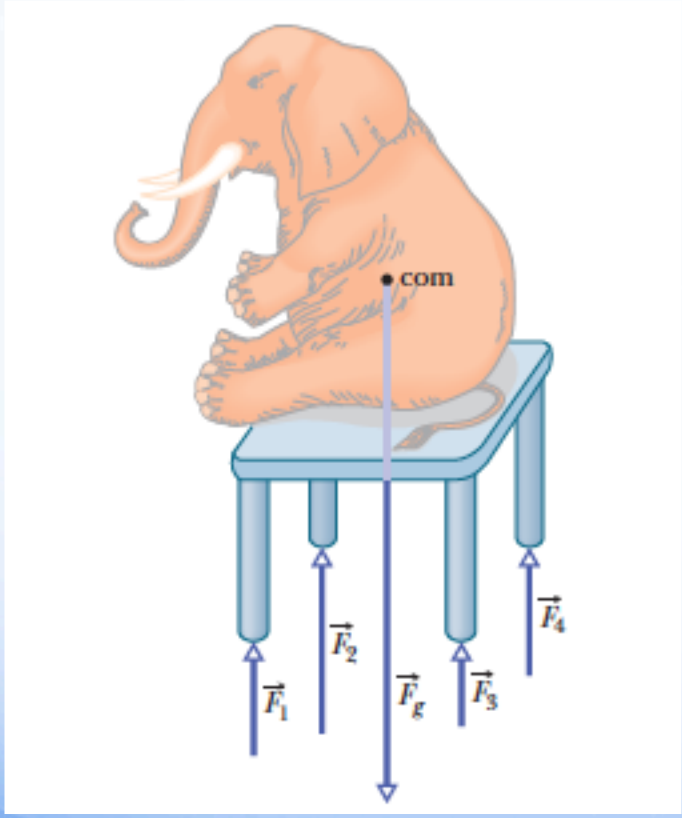
# Indeterminate Structures

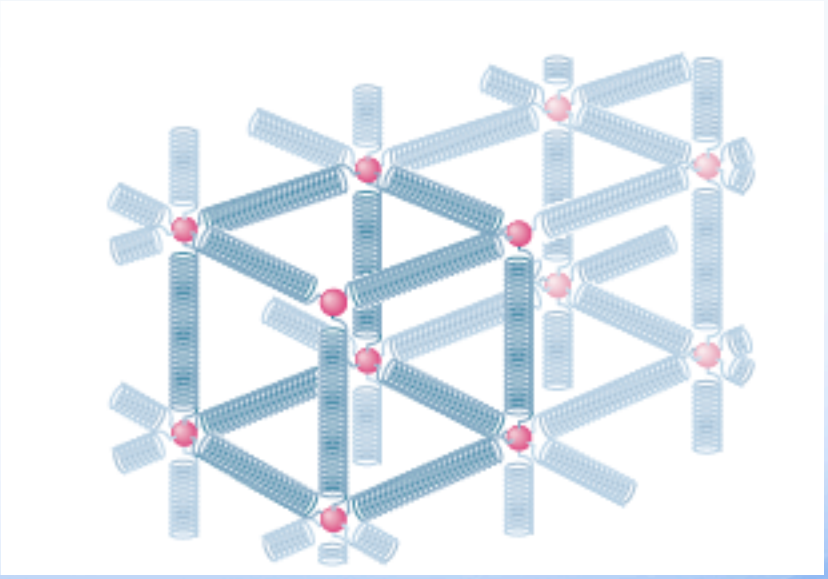
- For the problems of this chapter, we have only three independent equations at our disposal, usually two balance of forces equations and one balance of torques equation about a given rotation axis. **Thus, if a problem has more than three unknowns, we cannot solve it.**

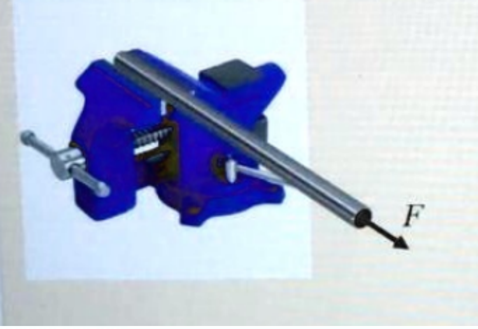


- Consider an unsymmetrically loaded car. What are the forces-all different on the four tires?
- We can solve an equilibrium problem for a table with three legs but not for one with four legs.
- Problems like these, in which there are more unknowns than equations, are called indeterminate.
- To solve such indeterminate equilibrium problems, we must supplement equilibrium equations with some knowledge of elasticity.

# Elasticity

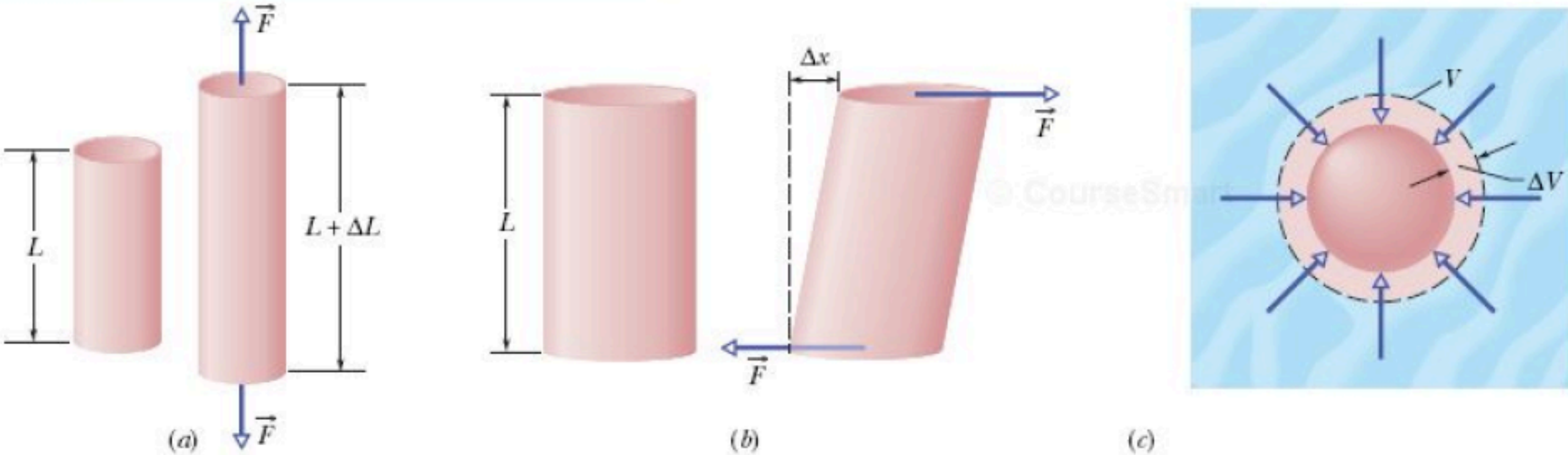






(a)



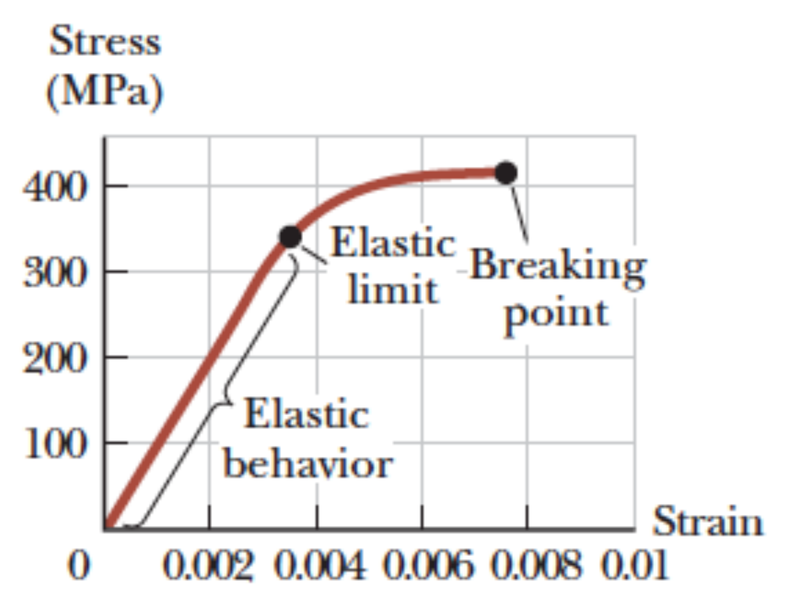


(a) (b) (c)

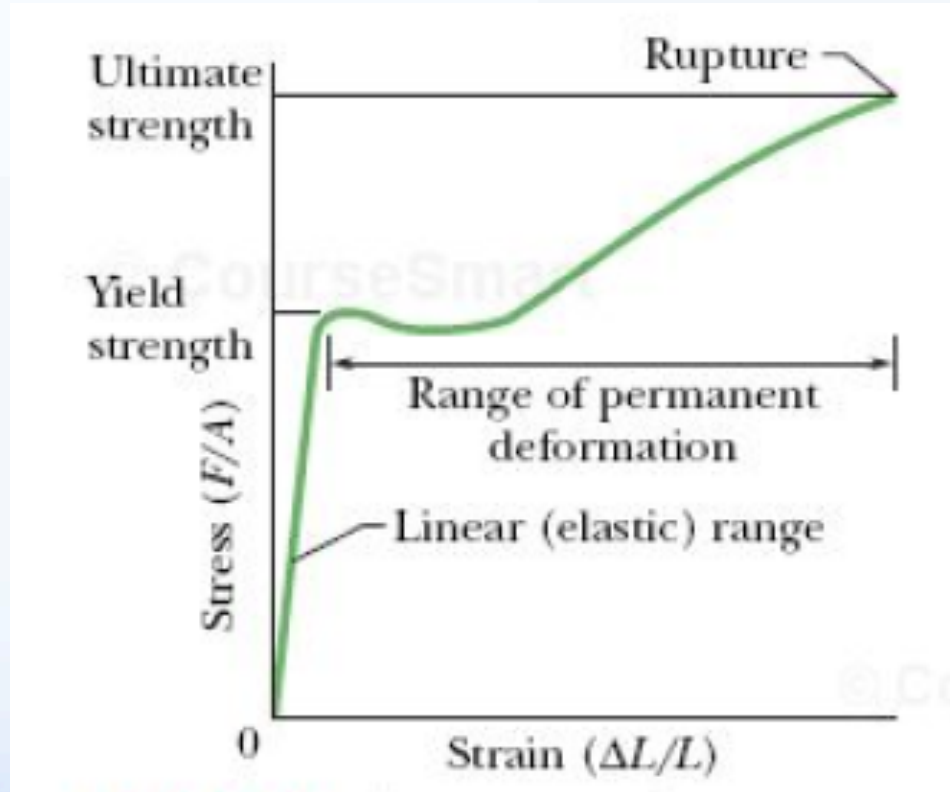
stress = modulus × strain.

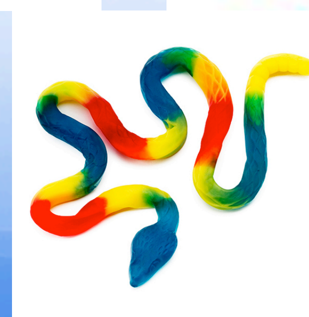
$$\frac{F}{A} = E\frac{\Delta L}{L}.$$

# Stress-Strain curve




Stress-versus-strain
curve for an elastic solid.

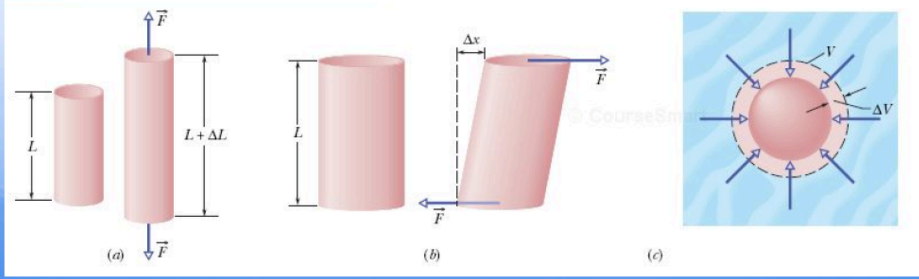


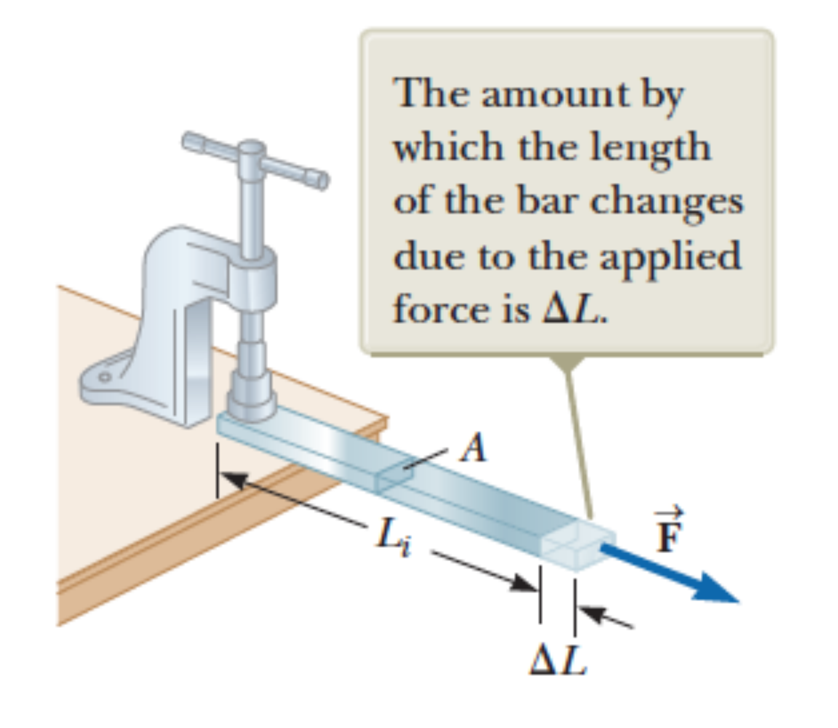
# Elastic modulus

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}}$$

1. **Young's modulus** measures the resistance of a solid to a change in its length.
2. **Shear modulus** measures the resistance to motion of the planes within a solid parallel to each other.
3. **Bulk modulus** measures the resistance of solids or liquids to changes in their volume.



# Young's Modulus: Elasticity in Length (E or Y)



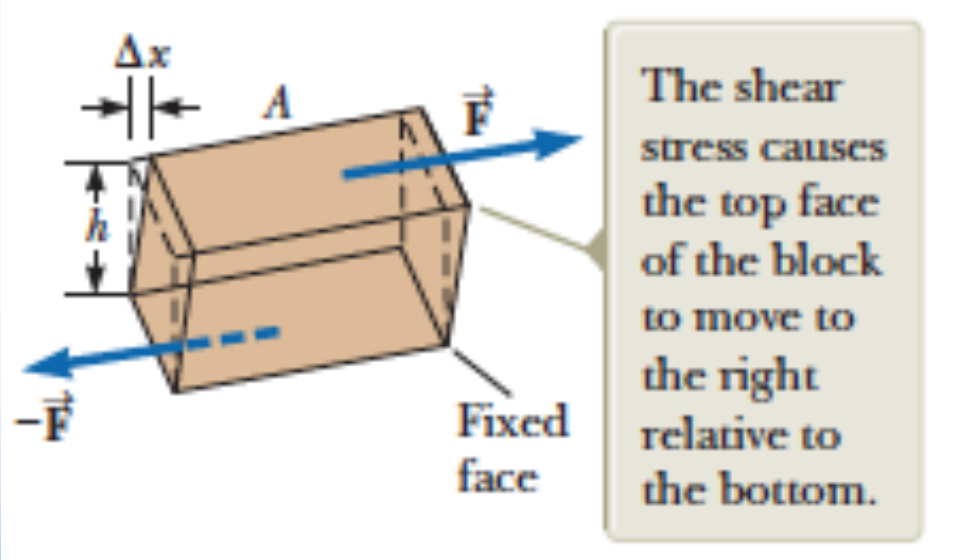

$$E \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

| Material | Density $\rho$ ($kg/m^3$) | Young's Modulus $E$ ($10^9$ $N/m^2$) | Ultimate Strength $S_u$ ($10^6$ $N/m^2$) | Yield Strength $S_y$ ($10^6$ $N/m^2$) |
|---|---|---|---|---|
| Steel[a] | 7860 | 200 | 400 | 250 |
| Aluminum | 2710 | 70 | 110 | 95 |
| Glass | 2190 | 65 | 50[b] | — |
| Concrete[c] | 2320 | 30 | 40[b] | — |
| Wood[d] | 525 | 13 | 50[b] | — |
| Bone | 1900 | 9[b] | 170[b] | — |
| Polystyrene | 1050 | 3 | 48 | — |

# Shear Modulus: Elasticity of Shape

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$

◀ **Shear modulus**

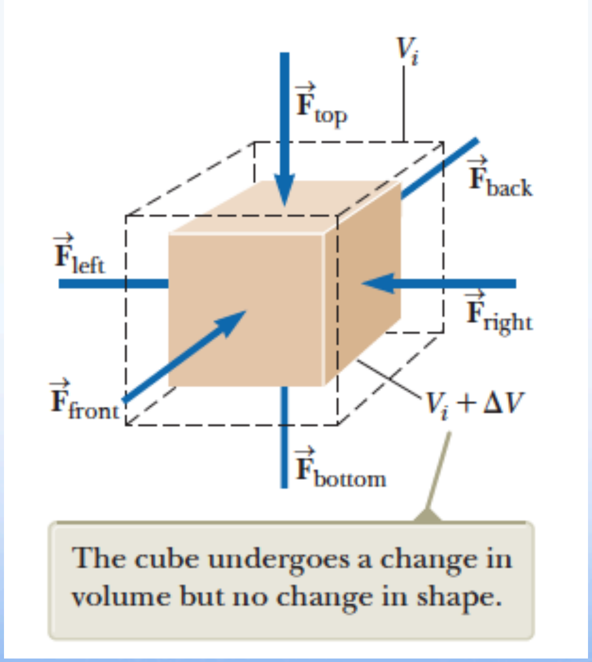



(a)

# Bulk Modulus: Volume Elasticity

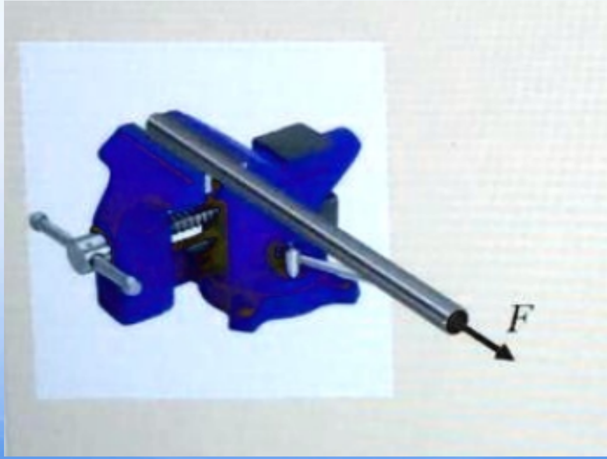
**Bulk modulus** ▶

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$



# Example:

One end of a steel rod of radius $R = 9.5$ mm and length $L = 81$ cm is held in a vise. A force of magnitude $F = 62$ kN is then applied perpendicularly to the end face (uniformly across the area) at the other end. What are the stress on the rod and the elongation $\Delta L$ and strain of the rod?



$$\text{stress} = \frac{F}{A} = \frac{F}{\pi R^2} = \frac{6.2 \times 10^4\ \text{N}}{(\pi)(9.5 \times 10^{-3}\ \text{m})^2}$$

$$= 2.2 \times 10^8\ \text{N/m}^2. \qquad \text{(Answer)}$$

The yield strength for structural steel is $2.5 \times 10^8\ \text{N/m}^2$, so this rod is dangerously close to its yield strength.

We find the value of Young's modulus for steel in Table 12-1. Then from Eq. 12-23 we find the elongation:

$$\Delta L = \frac{(F/A)L}{E} = \frac{(2.2 \times 10^8\ \text{N/m}^2)(0.81\ \text{m})}{2.0 \times 10^{11}\ \text{N/m}^2}$$

$$= 8.9 \times 10^{-4}\ \text{m} = 0.89\ \text{mm}. \qquad \text{(Answer)}$$

For the strain, we have

$$\frac{\Delta L}{L} = \frac{8.9 \times 10^{-4}\ \text{m}}{0.81\ \text{m}}$$

$$= 1.1 \times 10^{-3} = 0.11\%. \qquad \text{(Answer)}$$

# Example:

A table has three legs that are 1.00 m in length and a fourth leg that is longer by $d = 0.50$ mm, so that the table wobbles slightly. A steel cylinder with mass $M = 290$ kg is placed on the table (which has a mass much less than $M$) so that all four legs are compressed but unbuckled and the table is level but no longer wobbles. The legs are wooden cylinders with cross-sectional area $A = 1.0\ \text{cm}^2$; Young's modulus is $E = 1.3 \times 10^{10}\ \text{N/m}^2$. What are the magnitudes of the forces on the legs from the floor?

**KEY IDEAS** We take the table plus steel cylinder as our system. The situation is like that in Fig. 12-9, except now we have a steel cylinder on the table. If the tabletop remains level, the legs must be compressed in the following ways: Each of the short legs must be compressed by the same amount (call it $\Delta L_3$) and thus by the same force of magnitude $F_3$. The single long leg must be compressed by a larger amount $\Delta L_4$ and thus by a force with a larger magnitude $F_4$. In other words, for a level tabletop, we must have

$$\Delta L_4 = \Delta L_3 + d. \tag{12-26}$$

From Eq. 12-23, we can relate a change in length to the force causing the change with $\Delta L = FL/AE$, where $L$ is the original length of a leg. We can use this relation to replace $\Delta L_4$ and $\Delta L_3$ in Eq. 12-26. However, note that we can approximate the original length $L$ as being the same for all four legs.

**Calculations:** Making those replacements and that approximation gives us

$$\frac{F_4 L}{AE} = \frac{F_3 L}{AE} + d. \qquad (12\text{-}27)$$

We cannot solve this equation because it has two unknowns, $F_4$ and $F_3$.

To get a second equation containing $F_4$ and $F_3$, we can use a vertical $y$ axis and then write the balance of vertical forces ($F_{net,y} = 0$) as

$$3F_3 + F_4 - Mg = 0, \qquad (12\text{-}28)$$

where $Mg$ is equal to the magnitude of the gravitational force on the system. (*Three* legs have force $\vec{F}_3$ on them.) To solve the simultaneous equations 12-27 and 12-28 for, say, $F_3$, we first use Eq. 12-28 to find that $F_4 = Mg - 3F_3$. Substituting that into Eq. 12-27 then yields, after some algebra,

$$\begin{aligned} F_3 &= \frac{Mg}{4} - \frac{dAE}{4L} \\ &= \frac{(290\text{ kg})(9.8\text{ m/s}^2)}{4} \\ &\quad - \frac{(5.0 \times 10^{-4}\text{ m})(10^{-4}\text{ m}^2)(1.3 \times 10^{10}\text{ N/m}^2)}{(4)(1.00\text{ m})} \\ &= 548\text{ N} \approx 5.5 \times 10^2\text{ N}. \qquad \text{(Answer)} \end{aligned}$$

From Eq. 12-28 we then find

$$F_4 = Mg - 3F_3 = (290\ \text{kg})(9.8\ \text{m/s}^2) - 3(548\ \text{N})$$
$$\approx 1.2\ \text{kN}. \qquad \text{(Answer)}$$

You can show that to reach their equilibrium configuration, the three short legs are each compressed by 0.42 mm and the single long leg by 0.92 mm.

# Example:

**17.** A flexible chain weighing 40.0 N hangs between two hooks located at the same height (Fig. P12.17). At each hook, the tangent to the chain makes an angle $\theta = 42.0°$ with the horizontal. Find (a) the magnitude of the force each hook exerts on the chain and (b) the tension in the chain at its midpoint. *Suggestion:* For part (b), make a force diagram for half of the chain.


**Figure P12.17**

16. Q|C S A uniform beam of length $L$ and mass $m$ shown in Figure P12.16 is inclined at an angle $\theta$ to the horizontal. Its upper end is connected to a wall by a rope, and its lower end rests on a rough, horizontal surface. The coefficient of static friction between the beam and surface is $\mu_s$. Assume the angle $\theta$ is such that the static friction force is at its *maximum* value. (a) Draw a force diagram for the beam. (b) Using the

# Example:

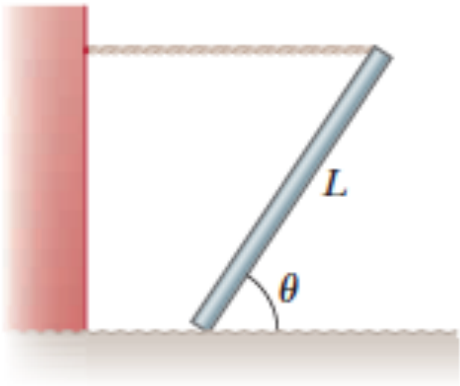



**Figure P12.16**

condition of rotational equilibrium, find an expression for the tension $T$ in the rope in terms of $m$, $g$, and $\theta$. (c) Using the condition of translational equilibrium, find a second expression for $T$ in terms of $\mu_s$, $m$, and $g$. (d) Using the results from parts (a) through (c), obtain an expression for $\mu_s$ involving only the angle $\theta$. (e) What happens if the ladder is lifted upward and its base is placed back on the ground slightly to the left of its position in Figure P12.16? Explain.

# Example:

23. One end of a uniform 4.00-m-long rod of weight $F_g$ is supported by a cable at an angle of $\theta = 37°$ with the rod. The other end rests against the wall, where it is held by friction as shown in Figure P12.23. The coefficient of static friction between the wall and the rod is $\mu_s = 0.500$. Determine the minimum distance $x$ from point $A$ at which an additional object, also with the same weight $F_g$, can be hung without causing the rod to slip at point $A$.

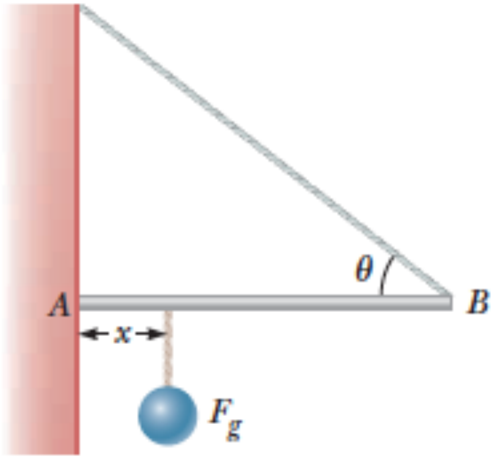



Figure P12.23

# Example:

24. **Q|C** A 10.0-kg monkey climbs a uniform ladder with weight $1.20 \times 10^2$ N and length $L = 3.00$ m as shown in Figure P12.24. The ladder rests against the wall and makes an angle of $\theta = 60.0°$ with the ground. The upper and lower ends of the ladder rest on frictionless surfaces. The lower end is connected to the wall by a horizontal rope that is frayed and can support a maximum tension of only 80.0 N. (a) Draw a force diagram for the ladder. (b) Find the normal force exerted on the bottom of the ladder. (c) Find the tension in the rope when the monkey is two-thirds of the way up the ladder. (d) Find the maximum distance $d$ that the monkey can climb up the ladder before the rope breaks. (e) If the horizontal surface were rough and the rope were removed, how would your analysis of the problem change? What other information would you need to answer parts (c) and (d)?

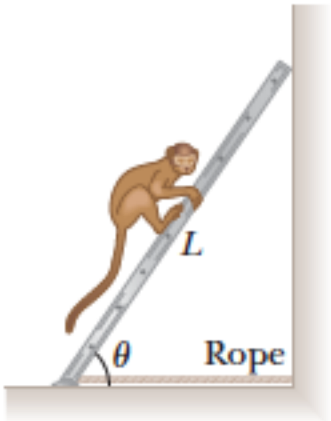


**Figure P12.24**

# Example:

45. **S** A uniform sign of weight $F_g$ and width $2L$ hangs from a light, horizontal beam hinged at the wall and supported by a cable (Fig. P12.45). Determine (a) the tension in the cable and (b) the components of the reaction force exerted by the wall on the beam in terms of $F_g$, $d$, $L$, and $\theta$.

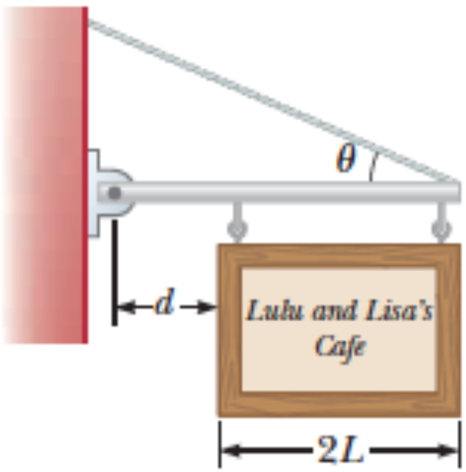



**Figure P12.45**

# Example:

**Q|C** When a gymnast performing on the rings executes the *iron cross*, he maintains the position at rest shown in Figure P12.53a. In this maneuver, the gymnast's feet (not shown) are off the floor. The primary muscles involved in supporting this position are the latissimus dorsi ("lats") and the pectoralis major ("pecs"). One of the rings exerts an upward force $\vec{F}_h$ on a hand as shown in Figure P12.53b.

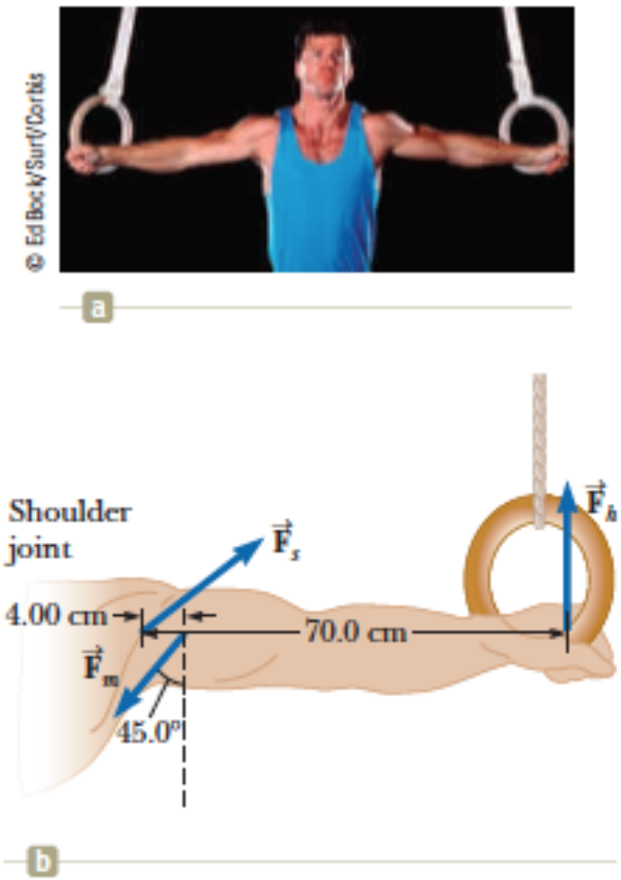



**Figure P12.53**

# Example:

**S** A uniform beam of mass $m$ is inclined at an angle $\theta$ to the horizontal. Its upper end (point $P$) produces a 90° bend in a very rough rope tied to a wall, and its lower end rests on a rough floor (Fig. P12.51). Let $\mu_s$ represent the coefficient of static friction between beam and floor. Assume $\mu_s$ is less than the cotangent of $\theta$. (a) Find an expression for the maximum mass $M$ that can be suspended from the top before the beam slips. Determine (b) the magnitude of the reaction force at the floor and (c) the magnitude of the force exerted by the beam on the rope at $P$ in terms of $m$, $M$, and $\mu_s$.

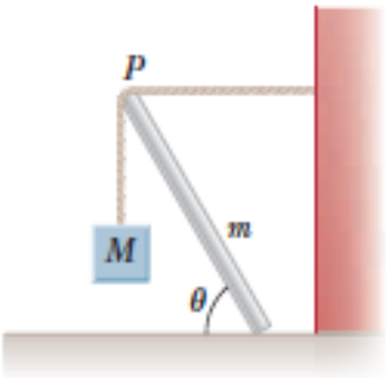



**Figure P12.51**

# Example:

••21 ILW The system in Fig. 12-38 is in equilibrium. A concrete block of mass 225 kg hangs from the end of the uniform strut of mass 45.0 kg. A cable runs from the ground, over the top of the strut, and down to the block, holding the block in place. For angles $\phi = 30.0°$ and $\theta = 45.0°$, find (a) the tension $T$ in the cable and the (b) horizontal and (c) vertical components of the force on the strut from the hinge.

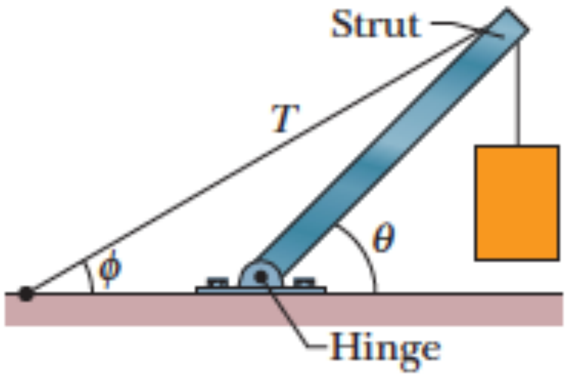



**Figure 12-38** Problem 21.

# Example:

••28 GO In Fig. 12-45, suppose the length $L$ of the uniform bar is 3.00 m and its weight is 200 N. Also, let the block's weight $W = 300$ N and the angle $\theta = 30.0°$. The wire can withstand a maximum tension of 500 N. (a) What is the maximum possible distance $x$ before the wire breaks? With the block placed at this maximum $x$, what are the (b) horizontal and (c) vertical components of the force on the bar from the hinge at $A$?

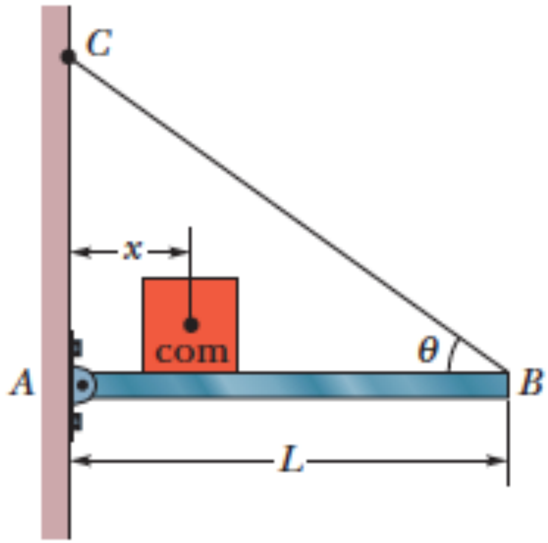
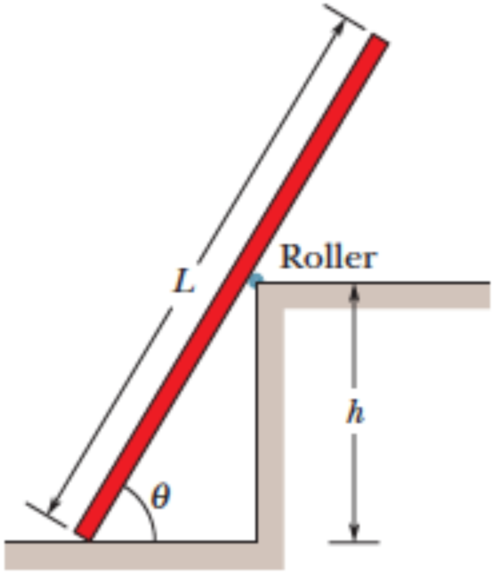



**Figure 12-45**
Problems 28 and 34.

# Example:

••37 GO In Fig. 12-51, a uniform plank, with a length $L$ of 6.10 m and a weight of 445 N, rests on the ground and against a frictionless roller at the top of a wall of height $h = 3.05$ m. The plank remains in equilibrium for any value of $\theta \geq 70°$ but slips if $\theta < 70°$. Find the coefficient of static friction between the plank and the ground.



# Example:

**65** In Fig. 12-73, a uniform beam with a weight of 60 N and a length of 3.2 m is hinged at its lower end, and a horizontal force $\vec{F}$ of magnitude 50 N acts at its upper end. The beam is held vertical by a cable that makes angle $\theta = 25°$ with the ground and is attached to the beam at height $h = 2.0$ m. What are (a) the tension in the cable and (b) the force on the beam from the hinge in unit-vector notation?

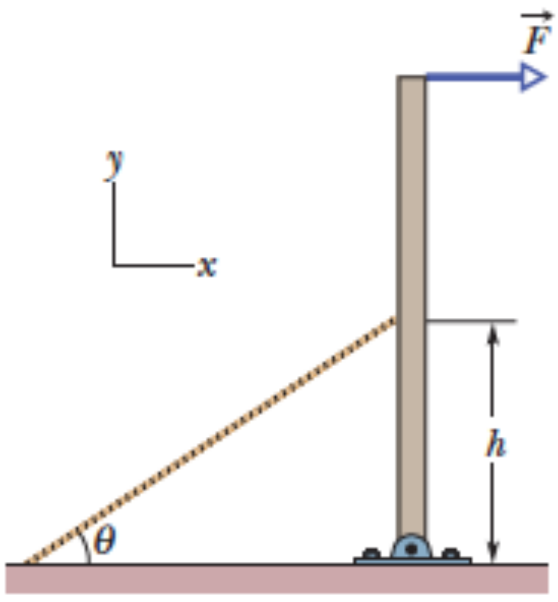
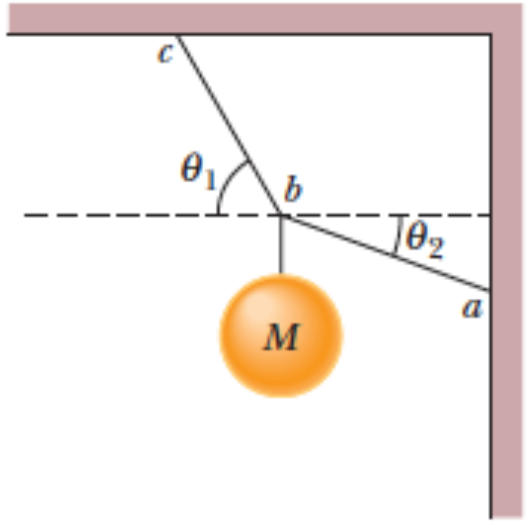



**Figure 12-73** Problem 65.

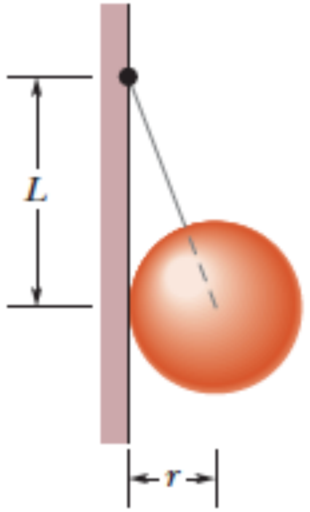
# Example:

**72** The system in Fig. 12-77 is in equilibrium. The angles are $\theta_1 = 60°$ and $\theta_2 = 20°$, and the ball has mass $M = 2.0$ kg. What is the tension in (a) string *ab* and (b) string *bc*?



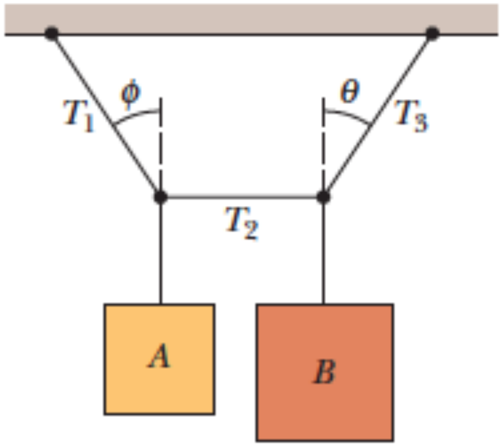
# Example:

•3 SSM WWW In Fig. 12-26, a uniform sphere of mass $m = 0.85$ kg and radius $r = 4.2$ cm is held in place by a massless rope attached to a frictionless wall a distance $L = 8.0$ cm above the center of the sphere. Find (a) the tension in the rope and (b) the force on the sphere from the wall.



# Example:

•10 GO The system in Fig. 12-28 is in equilibrium, with the string in the center exactly horizontal. Block *A* weighs 40 N, block *B* weighs 50 N, and angle $\phi$ is 35°. Find (a) tension $T_1$, (b) tension $T_2$, (c) tension $T_3$, and (d) angle $\theta$.



# Example:

•17 In Fig. 12-34, a uniform beam of weight 500 N and length 3.0 m is suspended horizontally. On the left it is hinged to a wall; on the right it is supported by a cable bolted to the wall at distance $D$ above the beam. The least tension that will snap the cable is 1200 N. (a) What value of $D$ corresponds to that tension? (b) To prevent the cable from snapping, should $D$ be increased or decreased from that value?

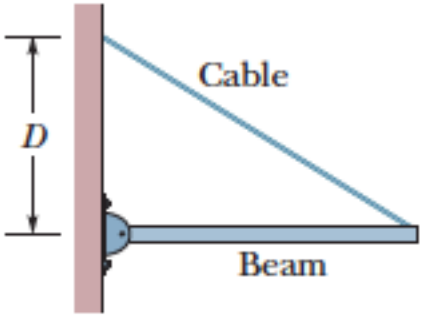



**Figure 12-34** Problem 17.

# Example:

••22 GO In Fig. 12-39, a 55 kg rock climber is in a lie-back climb along a fissure, with hands pulling on one side of the fissure and feet pressed against the opposite side. The fissure has width $w = 0.20$ m, and the center of mass of the climber is a horizontal distance $d = 0.40$ m from the fissure. The coefficient of static friction between hands and rock is $\mu_1 = 0.40$, and between boots and rock it is $\mu_2 = 1.2$. (a) What is the least horizontal pull by the hands and push by the feet that will keep the climber stable? (b) For the horizontal pull of (a), what must be the vertical distance $h$ between hands and feet? If the climber encounters wet rock, so that $\mu_1$ and $\mu_2$ are reduced, what happens to (c) the answer to (a) and (d) the answer to (b)?

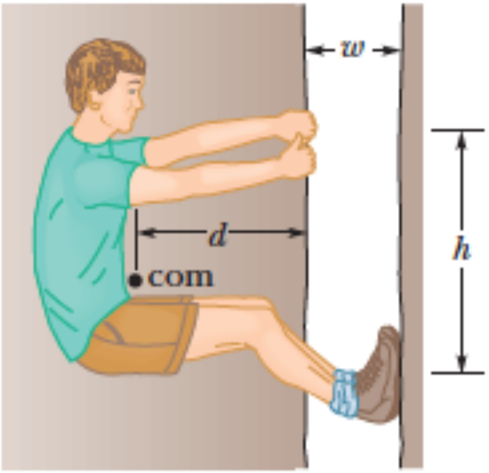
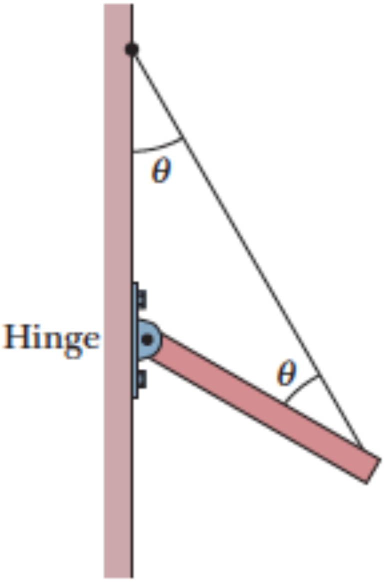



**Figure 12-39** Problem 22.

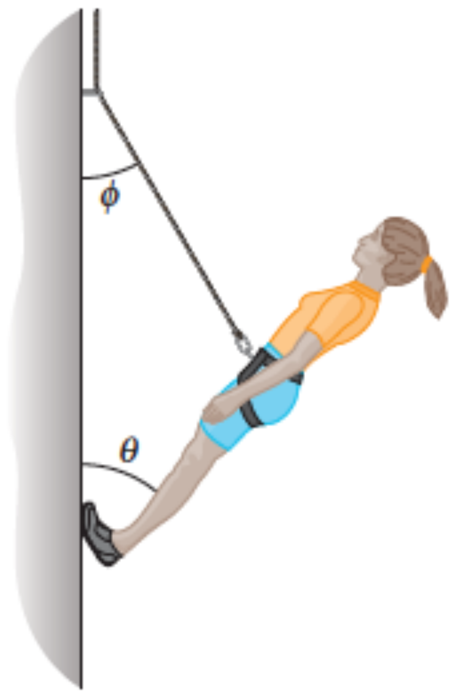
# Example:

••23 In Fig. 12-40, one end of a uniform beam of weight 222 N is hinged to a wall; the other end is supported by a wire that makes angles $\theta = 30.0°$ with both wall and beam. Find (a) the tension in the wire and the (b) horizontal and (c) vertical components of the force of the hinge on the beam.



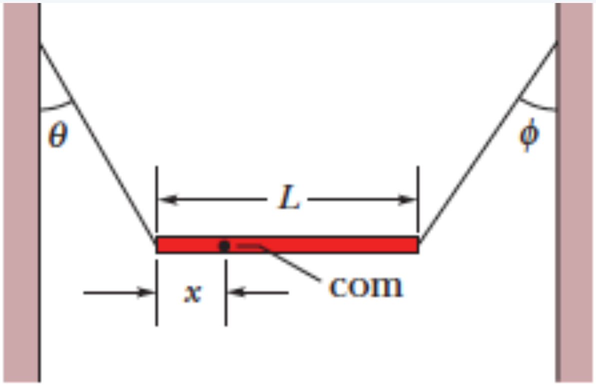
# Example:

••24 GO In Fig. 12-41, a climber with a weight of 533.8 N is held by a belay rope connected to her climbing harness and belay device; the force of the rope on her has a line of action through her center of mass. The indicated angles are $\theta = 40.0°$ and $\phi = 30.0°$. If her feet are on the verge of sliding on the vertical wall, what is the coefficient of static friction between her climbing shoes and the wall?



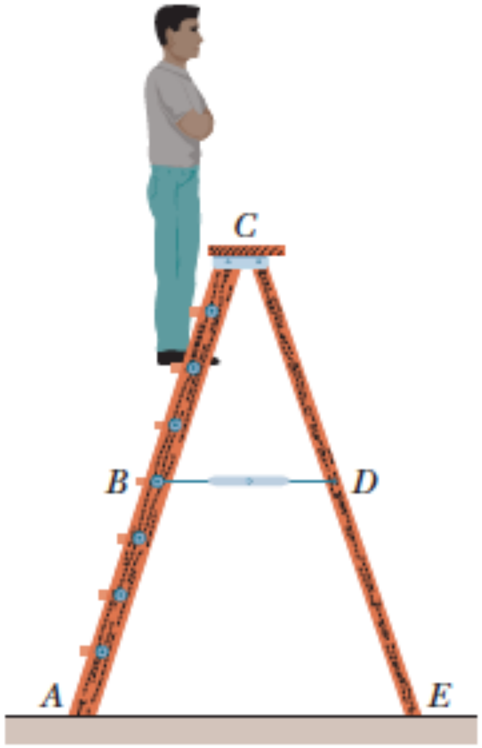
# Example:

••31 GO In Fig. 12-47, a nonuniform bar is suspended at rest in a horizontal position by two massless cords. One cord makes the angle $\theta = 36.9°$ with the vertical; the other makes the angle $\phi = 53.1°$ with the vertical. If the length $L$ of the bar is 6.10 m, compute the distance $x$ from the left end of the bar to its center of mass.



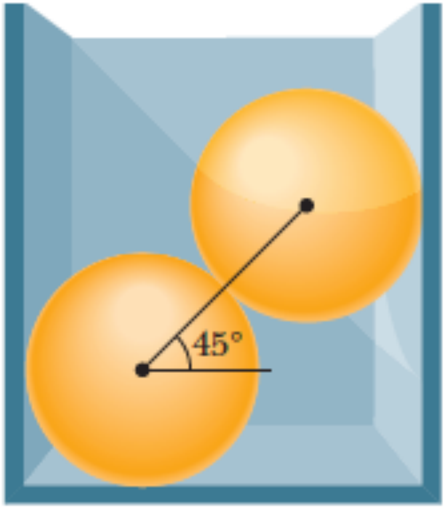
# Example:

•••39 For the stepladder shown in Fig. 12-53, sides *AC* and *CE* are each 2.44 m long and hinged at *C*. Bar *BD* is a tie-rod 0.762 m long, halfway up. A man weighing 854 N climbs 1.80 m along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find (a) the tension in the tie-rod and the magnitudes of the forces on the ladder from the floor at (b) *A* and (c) *E*. (*Hint:* Isolate parts of the ladder in applying the equilibrium conditions.)



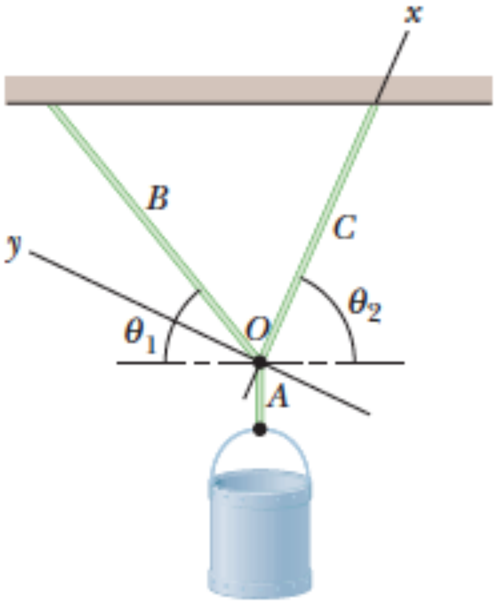
# Example:

**64** In Fig. 12-72, two identical, uniform, and frictionless spheres, each of mass $m$, rest in a rigid rectangular container. A line connecting their centers is at 45° to the horizontal. Find the magnitudes of the forces on the spheres from (a) the bottom of the container, (b) the left side of the container, (c) the right side of the container, and (d) each other. (*Hint:* The force of one sphere on the other is directed along the center–center line.)



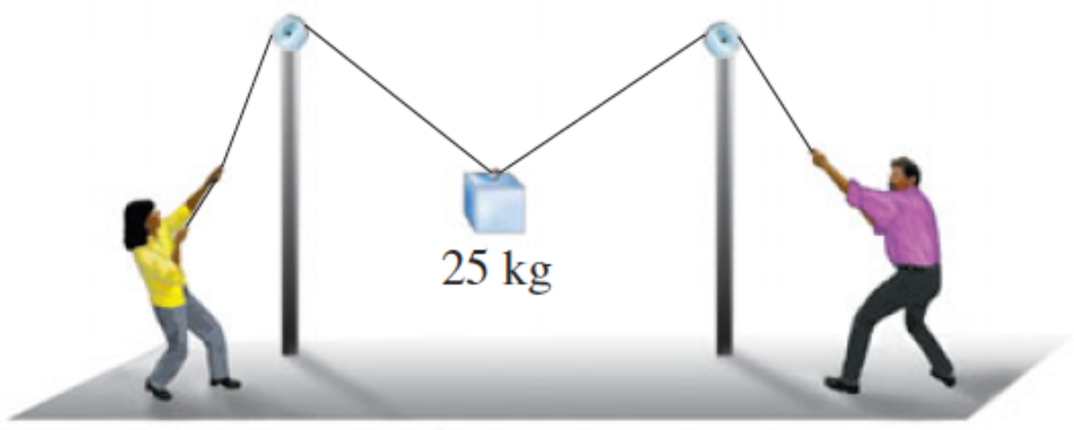
# Example:

**59** SSM In Fig. 12-68, an 817 kg construction bucket is suspended by a cable *A* that is attached at *O* to two other cables *B* and *C*, making angles $\theta_1 = 51.0°$ and $\theta_2 = 66.0°$ with the horizontal. Find the tensions in (a) cable *A*, (b) cable *B*, and (c) cable *C*. (*Hint:* To avoid solving two equations in two unknowns, position the axes as shown in the figure.)



# Example:

**79.** A 25-kg object is being lifted by pulling on the ends of a 1.15-mm-diameter nylon cord that goes over two 3.00-m-high poles that are 4.0 m apart, as shown in Fig. 12–90. How high above the floor will the object be when the cord breaks?



# Example:

**74.** A 15.0-kg ball is supported from the ceiling by rope A. Rope B pulls downward and to the side on the ball. If the angle of A to the vertical is 22° and if B makes an angle of 53° to the vertical (Fig. 12–88), find the tensions in ropes A and B.

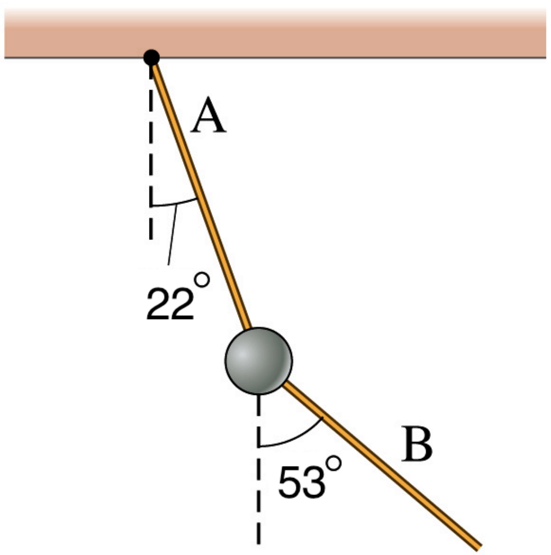



**FIGURE 12–88**
Problem 74.

# Example:

**92.** A 23-kg sphere rests between two smooth planes as shown in Fig. 12–106. Determine the magnitude of the force acting on the sphere exerted by each plane.

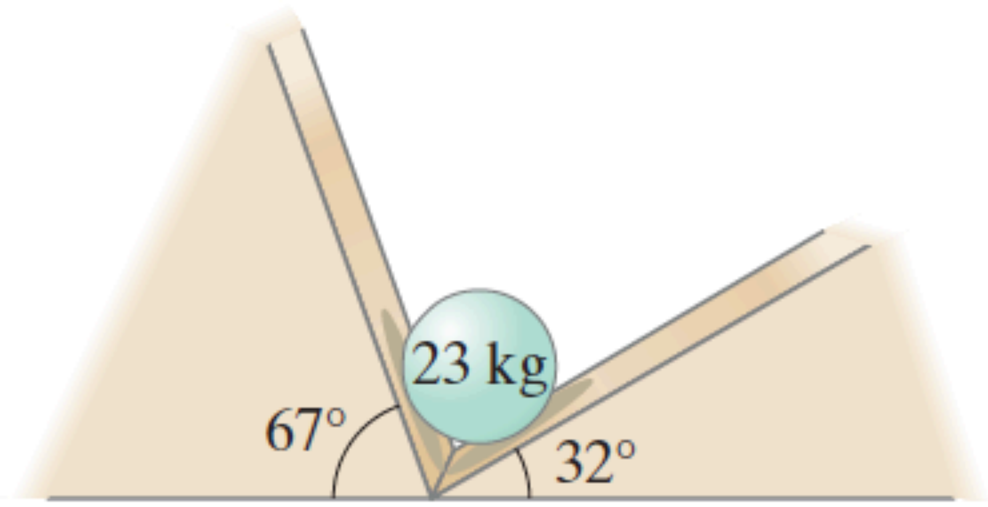



**FIGURE 12–106** Problem 92.

# Example:

**72.** A man doing push-ups pauses in the position shown in Fig. 12–86. His mass $m = 68$ kg. Determine the normal force exerted by the floor (*a*) on each hand; (*b*) on each foot.

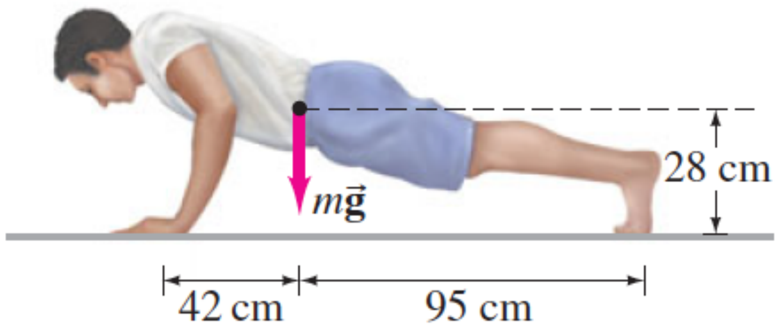



**FIGURE 12–86**
Problem 72.

# Example:

**32.** (III) A uniform ladder of mass $m$ and length $\ell$ leans at an angle $\theta$ against a frictionless wall, Fig. 12–67. If the coefficient of static friction between the ladder and the ground is $\mu_s$, determine a formula for the minimum angle at which the ladder will not slip.

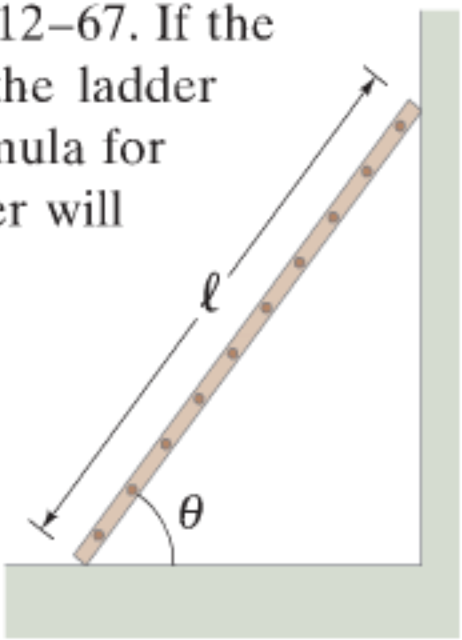



**FIGURE 12–67**
Problem 32.

# Example:

**23.** (II) Two wires run from the top of a pole 2.6 m tall that supports a volleyball net. The two wires are anchored to the ground 2.0 m apart, and each is 2.0 m from the pole (Fig. 12–62). The tension in each wire is 115 N. What is the tension in the net, assumed horizontal and attached at the top of the pole?

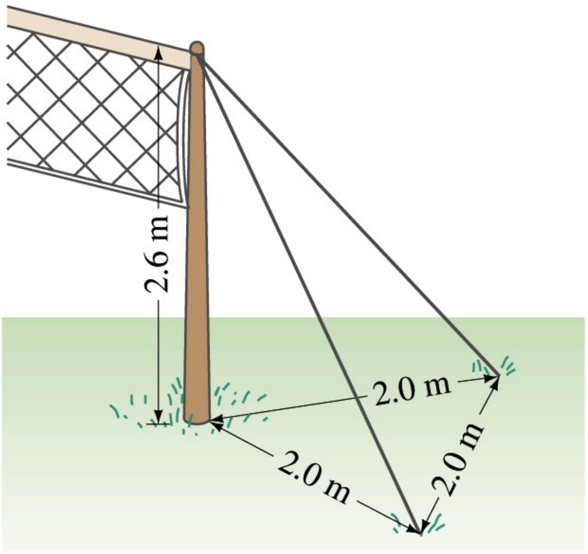



**FIGURE 12–62**
Problem 23.

# Example:

67. **Q|C** Figure P12.67 shows a vertical force applied tangentially to a uniform cylinder of weight $F_g$. The coefficient of static friction between the cylinder and all surfaces is 0.500. The force $\vec{\mathbf{P}}$ is increased in magnitude until the cylinder begins to rotate. In terms of $F_g$, find the maximum force magnitude $P$ that can be applied without causing the cylinder to rotate. *Suggestion:* Show that both friction forces will be at their maximum values when the cylinder is on the verge of slipping.

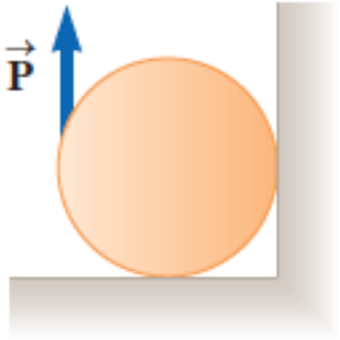
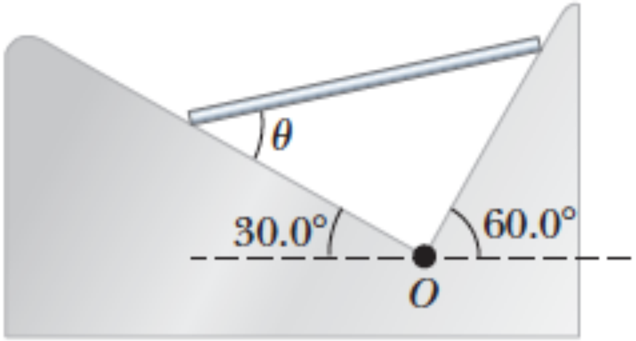



**Figure P12.67**

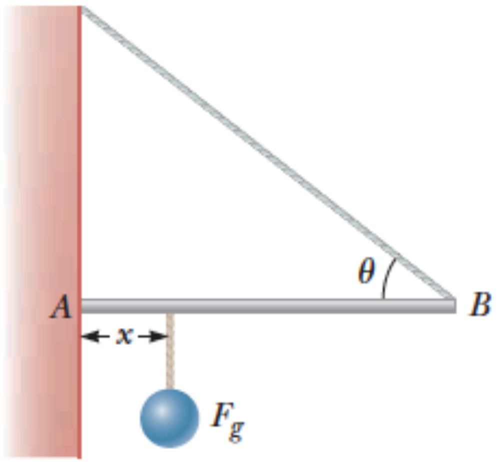
# Example:

**64.** A uniform rod of weight $F_g$ and length $L$ is supported at its ends by a frictionless trough as shown in Figure P12.64. (a) Show that the center of gravity of the rod must be vertically over point $O$ when the rod is in equilibrium. (b) Determine the equilibrium value of the angle $\theta$. (c) Is the equilibrium of the rod stable or unstable?



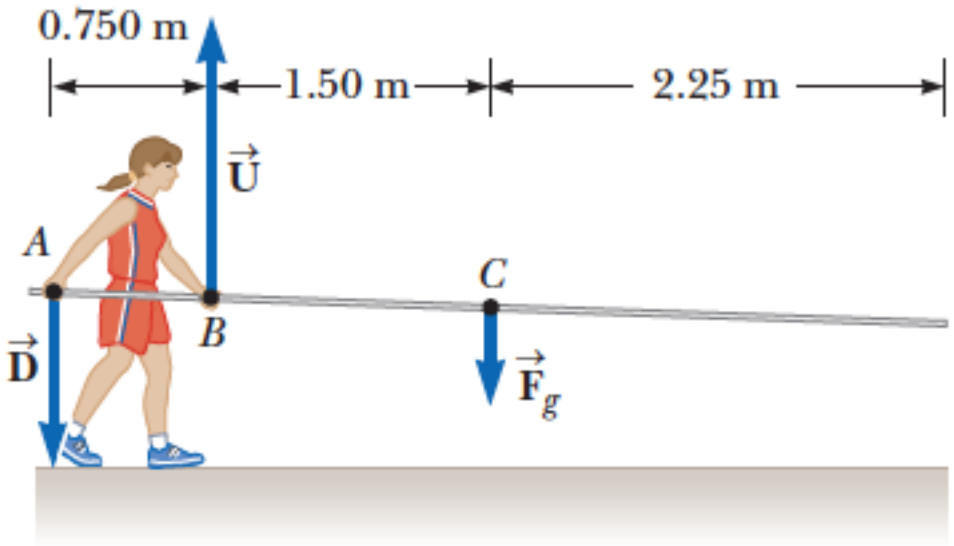
# Example:

**23.** One end of a uniform 4.00-m-long rod of weight $F_g$ is supported by a cable at an angle of $\theta = 37°$ with the rod. The other end rests against the wall, where it is held by friction as shown in Figure P12.23. The coefficient of static friction between the wall and the rod is $\mu_s = 0.500$. Determine the minimum distance $x$ from point $A$ at which an additional object, also with the same weight $F_g$, can be hung without causing the rod to slip at point $A$.



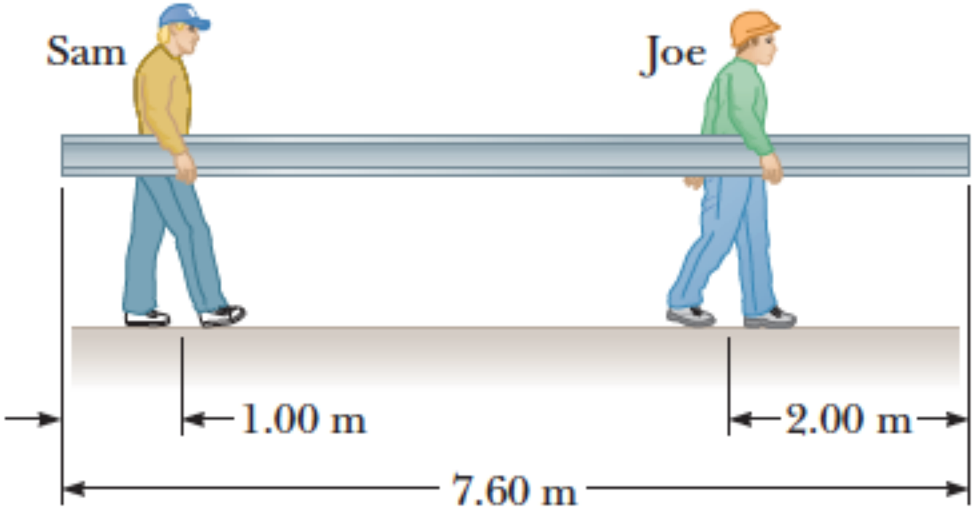
# Example:

12. A vaulter holds a 29.4-N pole in equilibrium by exerting an upward force $\vec{\mathbf{U}}$ with her leading hand and a downward force $\vec{\mathbf{D}}$ with her trailing hand as shown in Figure P12.12. Point *C* is the center of gravity of the pole. What are the magnitudes of (a) $\vec{\mathbf{U}}$ and (b) $\vec{\mathbf{D}}$?



# Example:

11. A uniform beam of length 7.60 m and weight $4.50 \times 10^2$ N is carried by two workers, Sam and Joe, as shown in Figure P12.11. Determine the force that each person exerts on the beam.



# Example:

85. A wheel of mass $M$ has radius $R$. It is standing vertically on the floor, and we want to exert a horizontal force $F$ at its axle so that it will climb a step against which it rests (Fig. 8–60). The step has height $h$, where $h < R$. What minimum force $F$ is needed?

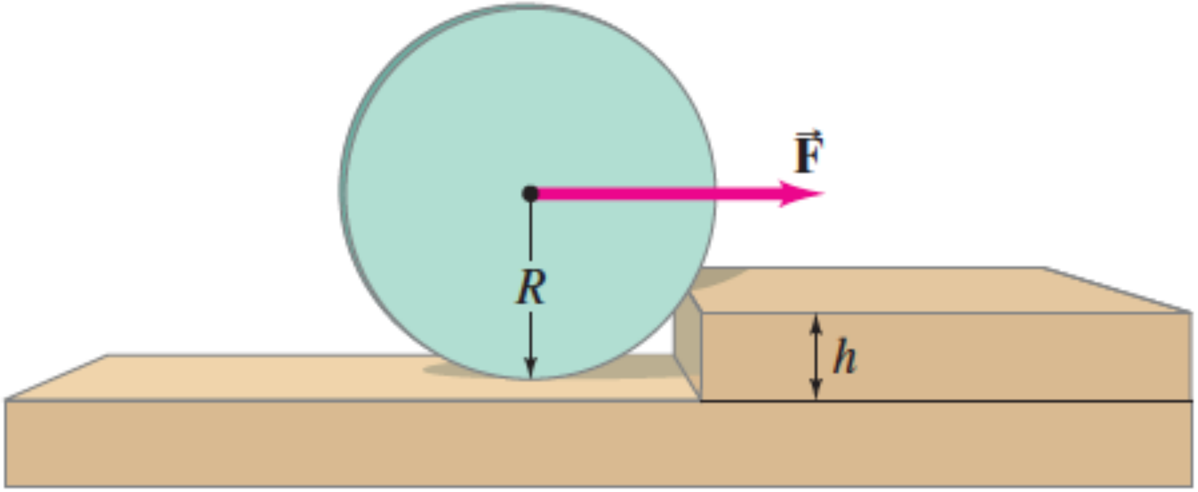



**FIGURE 8–60** Problem 85.

https://www.numerade.com/questions/a-23-kg-sphere-rests-between-two-smooth-planes-as-shown-in-fig-87-determine-the-magnitude-of-the-for/

27. (III) Consider a ladder with a painter climbing up it (Fig. 12–71). The mass of the uniform ladder is 12.0 kg, and the mass of the painter is 55.0 kg. If the ladder begins to slip at its base when the painter's feet are 70% of the way up the length of the ladder, what is the coefficient of static friction between the ladder and the floor? Assume the wall is frictionless.

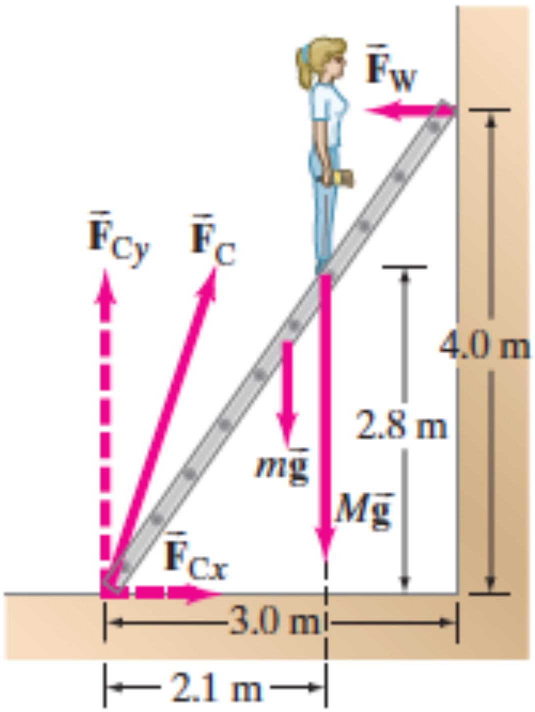



**FIGURE 12–71** Problem 27.