General Physics I

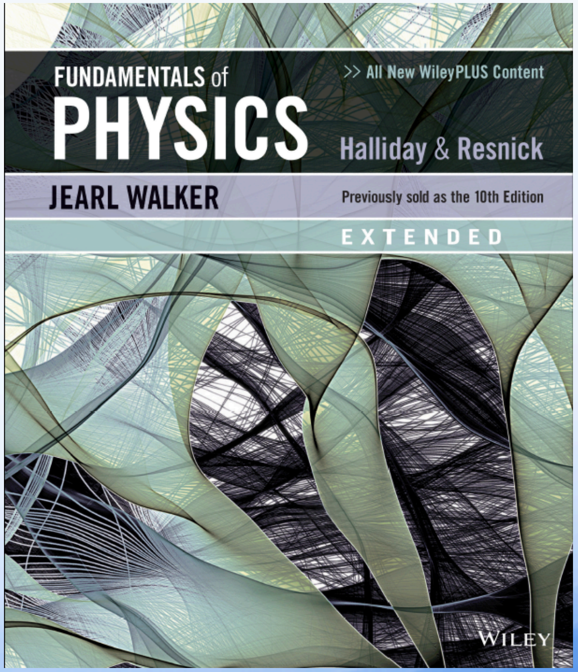
chapter 1

Sharif University of Technology
Mehr 1401 (2022-2023)

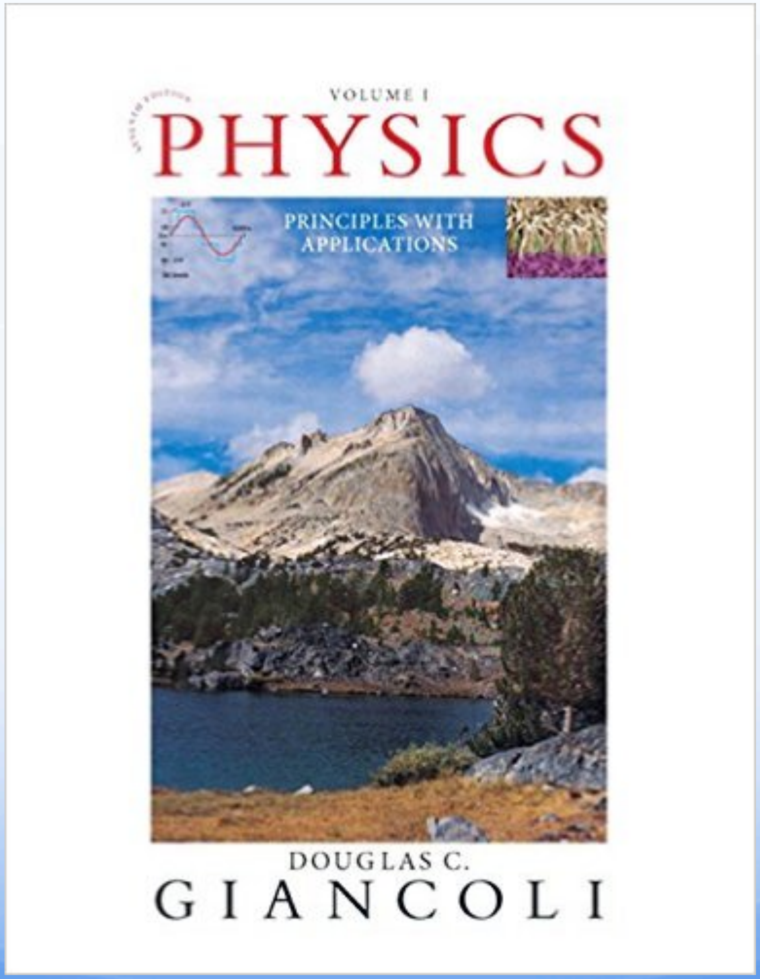
M. Reza Rahimi Tabar

Books

- 11^{th} Ed.

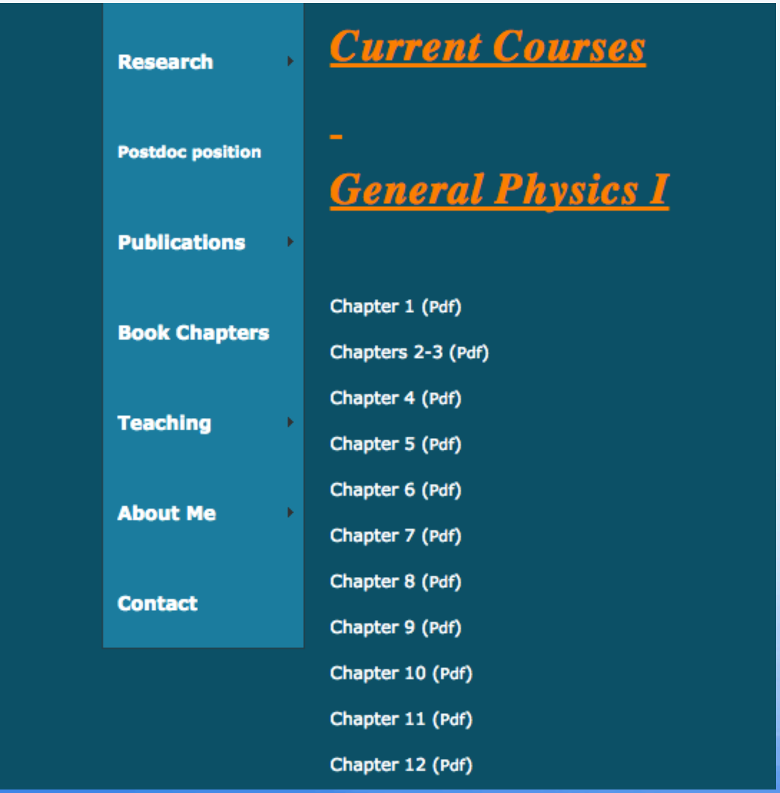


- Physics: Principles with Applications (7th Edition)



Lectures

- http://sharif.edu/~rahimitabar/current_courses.html



https://vc.sharif.edu/ch/rahimitabar

Chapters

- 1- Measurement
- 2- Motion Along a Straight Line
- 3- Vectors
- 4- Motion in Two and Three Dimensions
- 5- Force and Motion—I
- 6- Force and Motion—II
- 7- Kinetic Energy and Work
- 8- Potential Energy and Conservation of Energy
- 9- Center of Mass and Linear Momentum
- 10- Rotation
- 11- Rolling, Torque, and Angular Momentum
- 12- Equilibrium and Elasticity
- 13- Gravitation
- 14- Fluids
- 15- Oscillations
- 16- Waves—I
- 17. Waves—II

Mid-term 26 Aban 1401, 9h – End of Chap. 8 4 + 4 + 12 = 20

Physics

The most basic of all sciences!

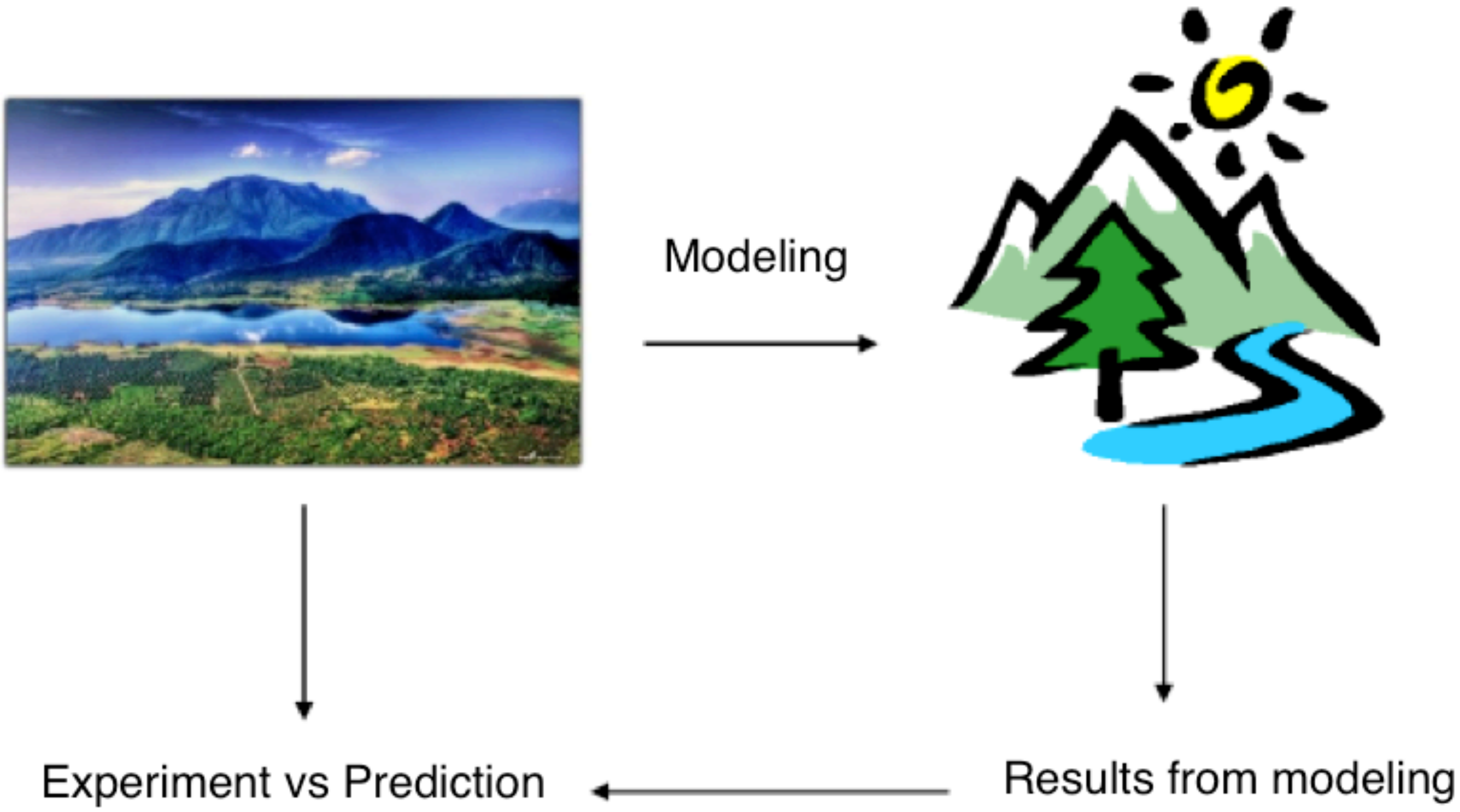
- Physics:

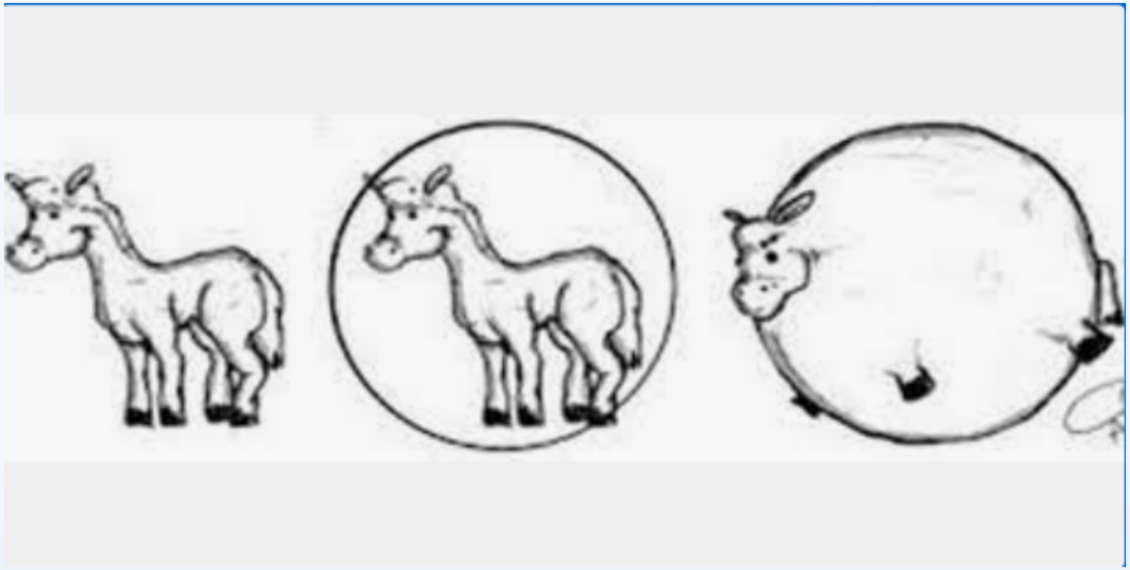
The "Parent" of all sciences!

- Physics =

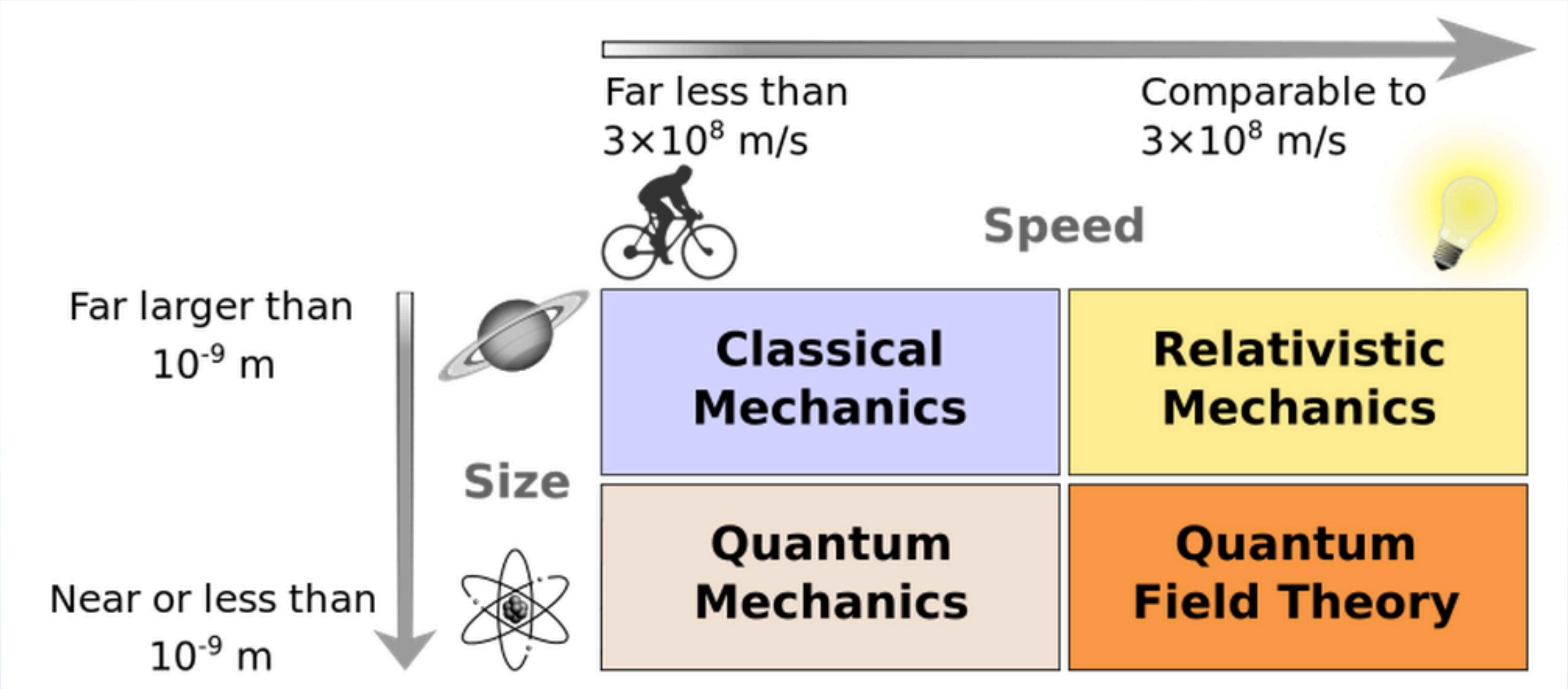
The study the structure and dynamics of matter and fields

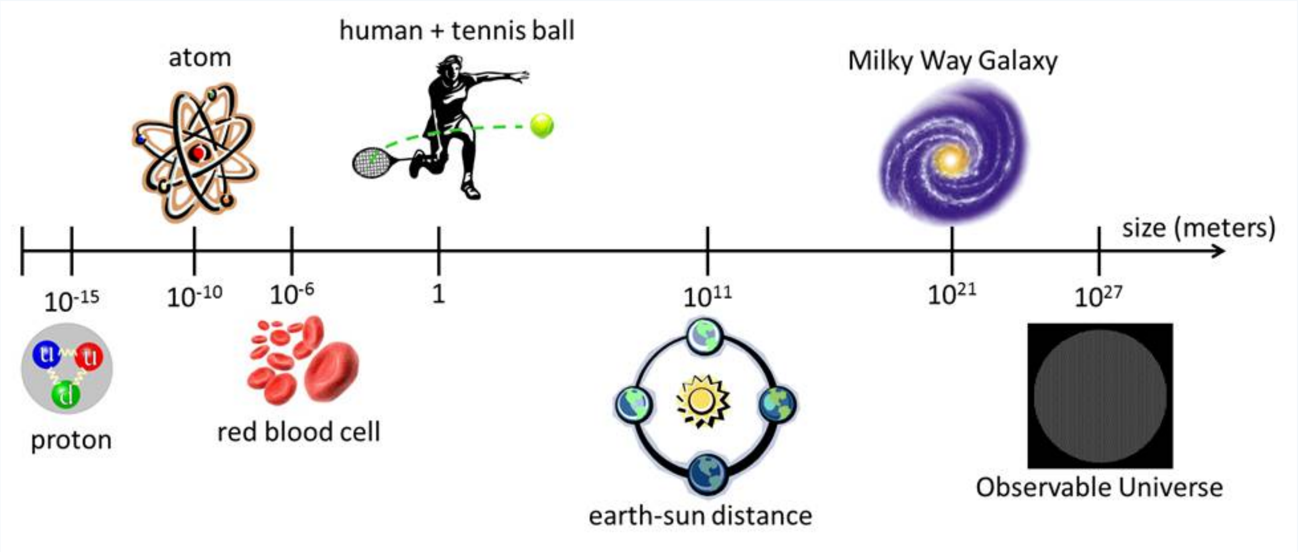
Physics



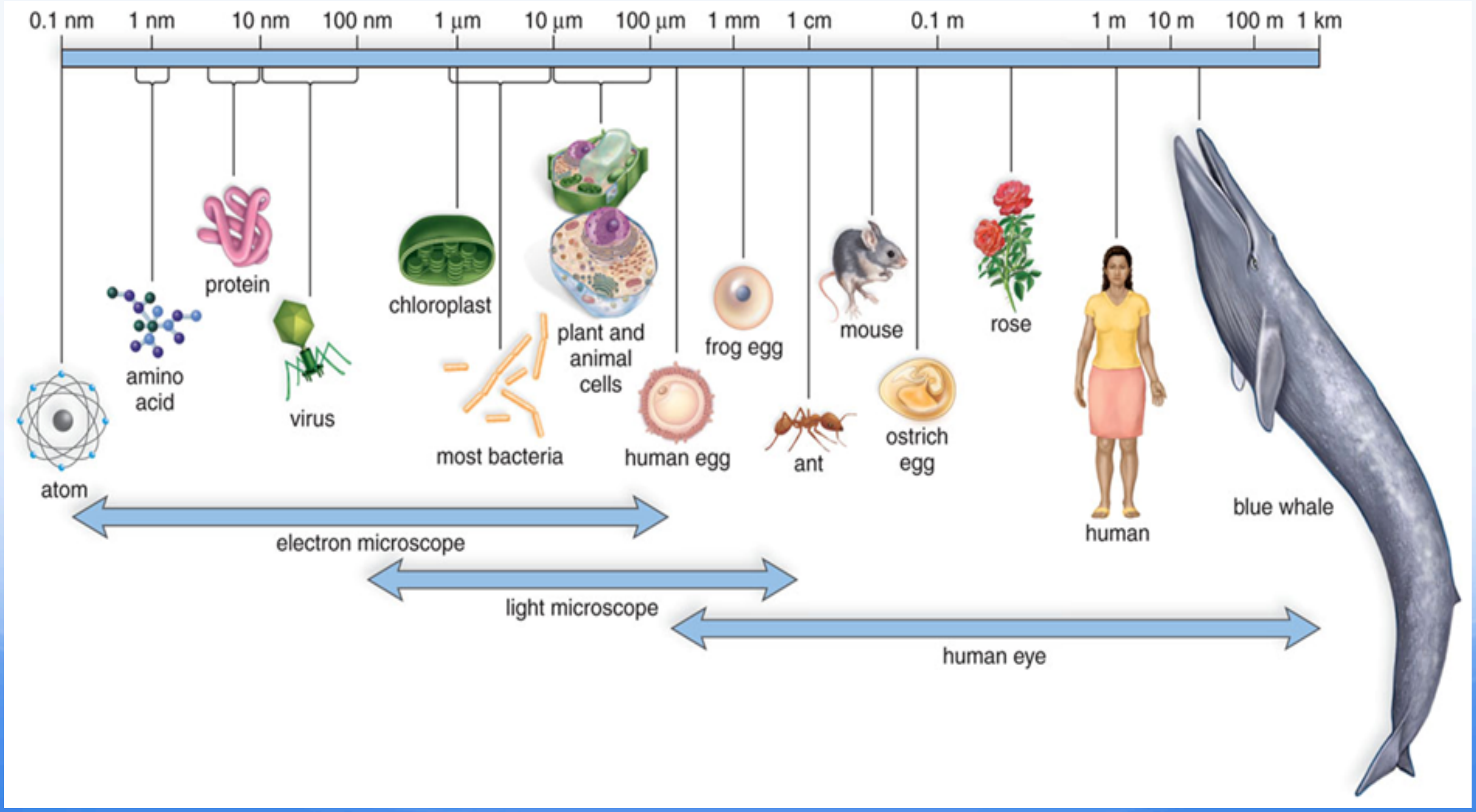


Length scale --- Velocity

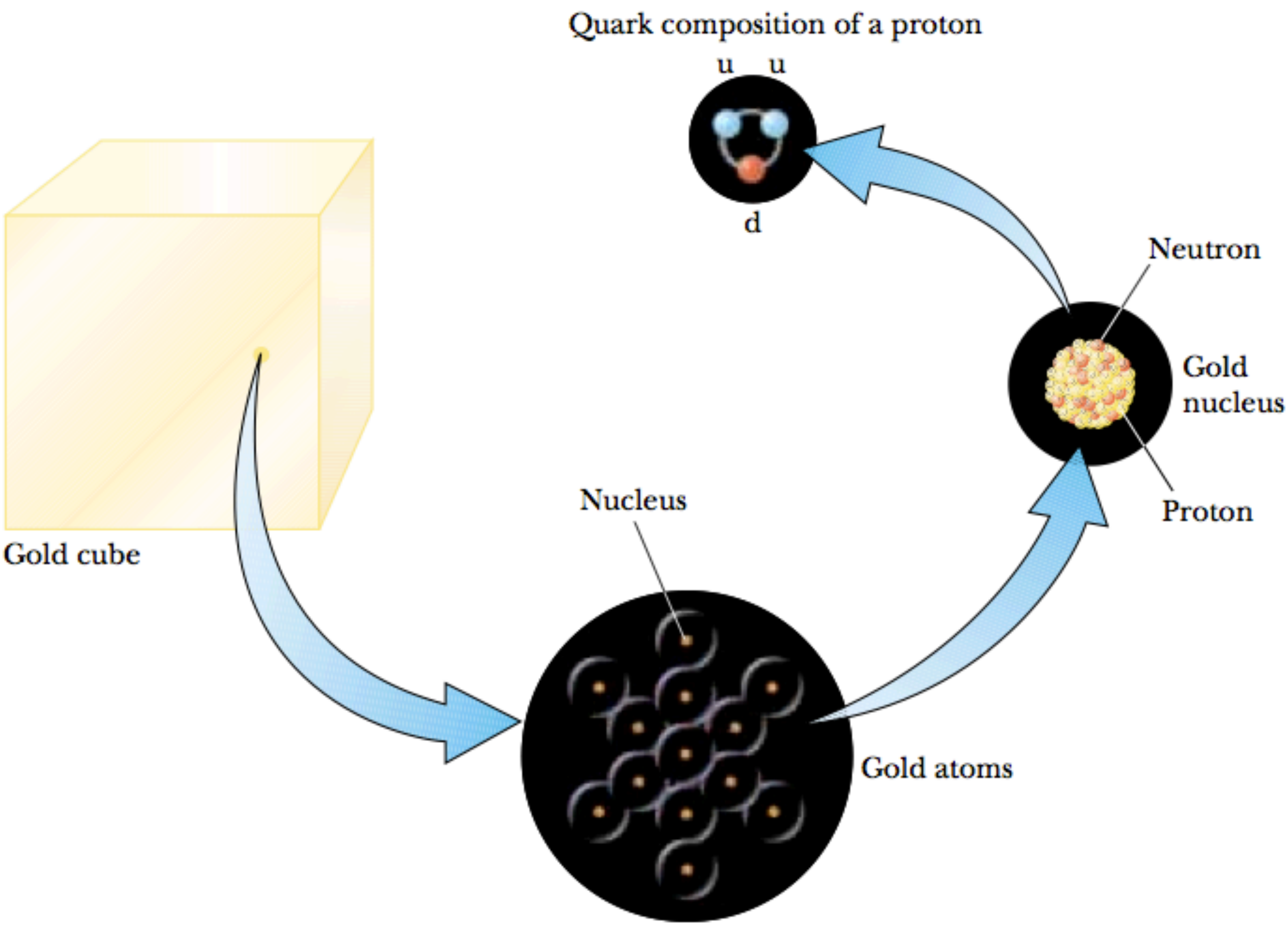




human + tennis ball
atom
Milky Way Galaxy
size (meters)
10^{-15}
10^{-10}
10^{-6}
1
10^{11}
10^{21}
10^{27}
proton
red blood cell
earth-sun distance
Observable Universe



0.1 nm
1 nm
10 nm
100 nm
1 µm
10 µm
100 µm
1 mm
1 cm
0.1 m
1 m
10 m
100 m
1 km
protein
chloroplast
amino acid
virus
plant and animal cells
frog egg
mouse
rose
most bacteria
human egg
ant
ostrich egg
atom
blue whale
human
electron microscope
light microscope
human eye


Quark composition of a proton
u
u
d
Neutron
Gold
nucleus
Proton
Nucleus
Gold cube
Gold atoms

CHAPTER 1

Measurement

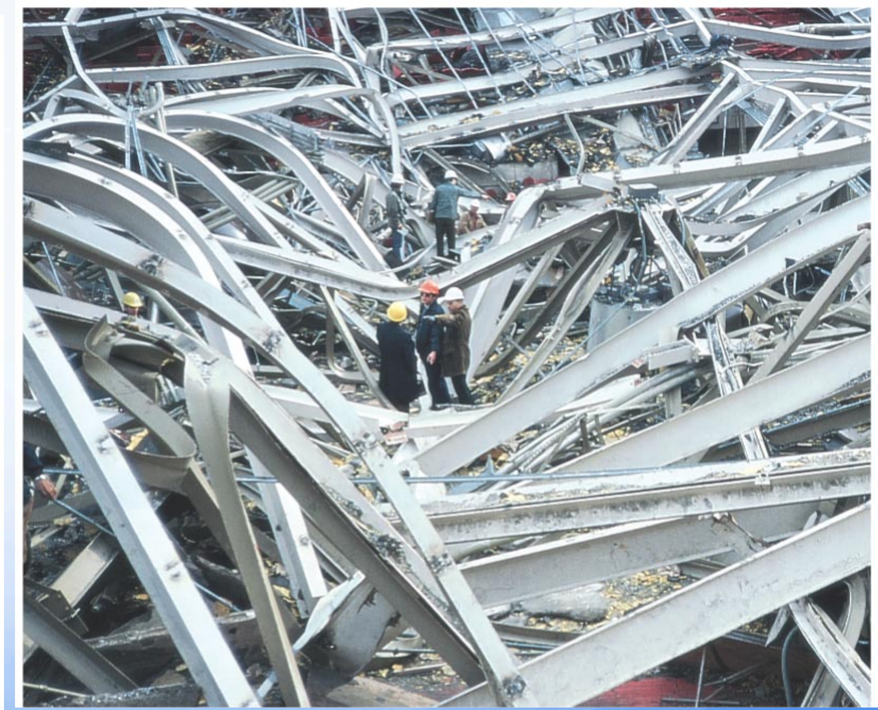
Learning Objectives:

- After reading this module, you should be able to . . .
- 1.01- Identify the base quantities in the SI system.
- 1.02- Name the most frequently used prefixes for SI units.
- 1.03- Change units (here for length, area, and volume) by using chain-link conversions.
- 1.04- Explain that the meter is defined in terms of the speed of light in vacuum.

1. Measurements

- Basis of testing theories in science
- Need to have consistent systems of units for the measurements
- Uncertainties are inherent
- Need rules for dealing with the uncertainties

Physics Principles are used in many practical applications, including construction. As the photo on the right clearly shows, communication of physics principles between Architects & Engineers is sometimes essential if disaster is to be avoided!!



The Nature of Science

- Physics is an EXPERIMENTAL science!

Experiments & Observations:

  - Are important first steps toward a scientific theory. It requires **imagination** to tell what is important, to develop a theory, & to test it in the laboratory.

Theories

  - Are created to explain experiments & observations. Can also make predictions

Experiments & Observations:

  - Can tell if predictions are accurate.

- But, no theory can be absolutely 100% verified!
  - But a theory can be proven false.

Theory

A Quantitative (Mathematical) Description of experimental observations.

- Not just WHAT is observed but WHY it is observed as it is and HOW it works the way it does.

Tests of Theories

1. Experimental Observations:

More experiments & more observation!!

2. Predictions

Made before observations & experiments.

Model, Theory, Law

- Model: An analogy of a physical phenomenon to something we are familiar with.
- Theory: More detailed than a model. Puts the model into mathematical language

Law

- A concise & general statement about how nature behaves. Must be verified by many, many experiments! Only a few laws.

How does a new theory get accepted?

- It's Predictions:

Agree better with data than those of an old theory

- It Explains:

A greater range of phenomena than old theory

Example

- Aristotle:

Believed that objects would return to rest once put in motion.

- Galileo:

Realized that an object put in motion would stay in motion until some force stopped it.

- Newton:

Developed his Laws of Motion to put Galileo's observations into mathematical language.

Measurement & Uncertainty; Significant Figures

No measurement is exact.

There is always some uncertainty due to limited instrument accuracy & difficulty reading results.

The photograph to the left illustrates this – it would be difficult to measure the width of this wood to better than a millimeter.

Measurement & Uncertainty

- Physics is an EXPERIMENTAL science!

  It finds mathematical relations between physical quantities. It also expresses those relations in math language.

  This gives rise to LAWS & THEORIES

- Experiments are NEVER 100% accurate.
  - They ALWAYS have UNCERTAINTY in the final result.

    ≡ Experimental Error.

  - It is common to state this precision (when known).

- Consider a simple measurement of the width of a board. Suppose the result is 23.2 cm.
- However, suppose we know that our measurement is only accurate to an estimated 0.1 cm.

$\Rightarrow$ The width is written as (23.2 ± 0.1) cm

$$\pm 0.1 \text{ cm} \equiv \text{Experimental Uncertainty}$$

- The Percent Uncertainty is then:

$$\pm (0.1/23.2) \times 100 \approx \pm 0.4\%$$

Significant Figures

("sig figs")

≡ The number of significant figures is the number of reliably known digits in a number.

- It is usually possible to tell The Number of Significant Figures by the way the number is written:

23.21 cm has 4 significant figures

0.062 cm has 2 significant figures

(initial zeroes don't count)

80 km is ambiguous:

It could have 1 or 2 significant figures.

If it has 3, it should be written 80.0 km.

Calculations Involving Several Numbers

<u>When Multiplying or Dividing Numbers</u>:

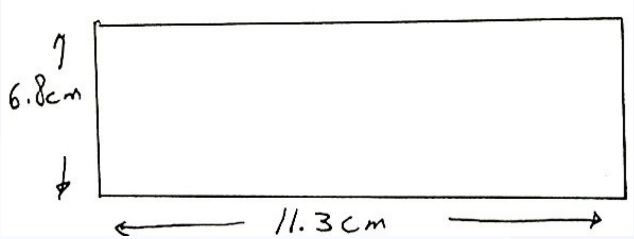
The number of significant digits in the result ≡

The same as the number used in the calculation which have the fewest significant digits.

<u>When Adding or Subtracting Numbers</u>:

The answer is no more accurate than the least accurate number used.

Example

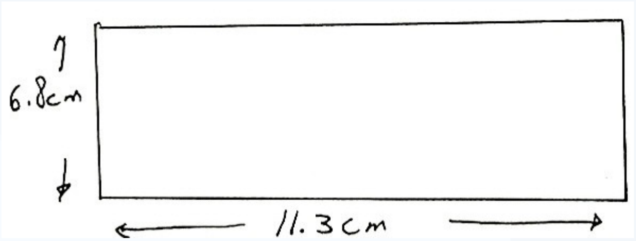


(Not to scale!)

- Calculate the area A of a board with dimensions

11.3 cm & 6.8 cm.

Example



(Not to scale!)

- Calculate the area A of a board with dimensions

11.3 cm & 6.8 cm.

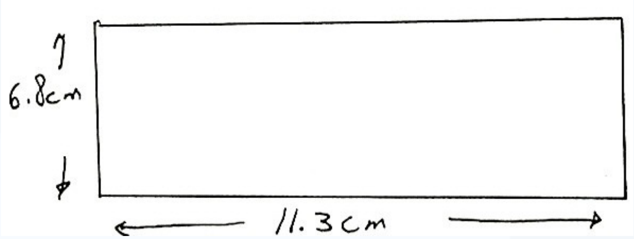
$$A = (11.3) \times (6.8) = 76.84\ \text{cm}^2$$

11.3 has 3 sig figs & 6.8 has 2 sig figs

$\Rightarrow$ A has too many sig figs!

Example

(Not to scale!)



- Calculate the area A of a board with dimensions

11.3 cm & 6.8 cm.

$$A = (11.3) \times (6.8) = 76.84 \text{ cm}^2$$

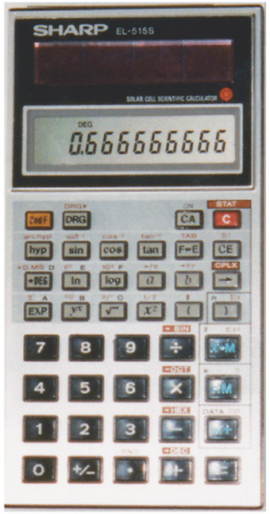
11.3 has 3 sig figs & 6.8 has 2 sig figs

$\Rightarrow$ A has too many sig figs!

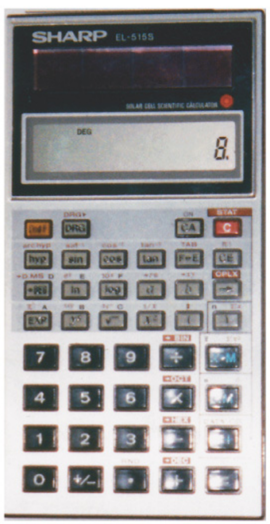
Proper number of sig figs in the answer = 2

$\Rightarrow$ Round off 76.84 & keep only 2 sig figs

$\Rightarrow$ A <u>Reliable Answer</u> for $A = 77 \text{ cm}^2$



(a)



(b)

Calculators will not give you the right number of significant figures; they usually give too many, but sometimes give too few (especially if there are trailing zeroes after a decimal point).

The top calculator shows the result of

2.0 / 3.0.

The bottom calculator shows the result of

2.5×3.2.

All digits on your calculator are NOT significant!!

Conceptual Example 1-2: Significant figures

- Using a protractor, you measure an angle of 30°.

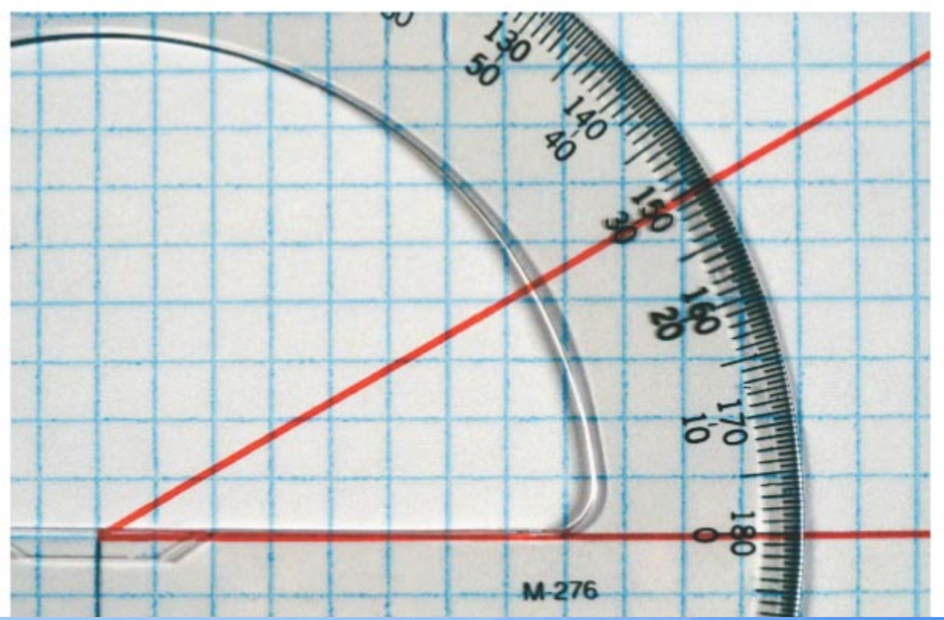
(a) How many significant figures should you quote in this measurement?

(b) Use a calculator to find the cosine of the angle you measured.

(a) Precision ~ 1° (not 0.1°).

So 2 sig figs & angle is 30° (not 30.0°).

(b) Calculator: cos(30°) = 0.866025403. But angle precision is 2 sig figs so answer should also be 2 sig figs. So cos(30°) = 0.87



Powers of 10 (Scientific Notation)

- It is common to express very large or very small numbers using power of 10 notation.
- Examples:

$$39{,}600 = 3.96 \times 10^{4}$$

(moved decimal 4 places to left)

$$0.0021 = 2.1 \times 10^{-3}$$

(moved decimal 3 places to right)

PLEASE USE SCIENTIFIC NOTATION!!

Units, Standards, SI System

- All measured physical quantities have units.
- Units are VITAL in physics!!
- In this course (and in most of the modern world, except the USA!) we will use (almost) exclusively the SI system of units.

SI = "Systéme International" (French)

More commonly called the "MKS system" (meter-kilogram-second) or more simply,

"The Metric System"

SI or MKS System

- Defined in terms of standards for length, mass, & time.
- Length unit: Meter (m) (kilometer = km = 1000 m)
  - Standard meter. Newest definition in terms of speed of light ≡ Length of path traveled by light in vacuum in (1/299,792,458) of a second!
- Time unit: Second (s)
  - Standard second. Newest definition ≡ time required for 9,192,631,770 oscillations of radiation emitted by cesium atoms!
- Mass unit: Kilogram (kg)
  - Standard kilogram ≡ Mass of a specific platinum-iridium alloy cylinder kept at Intl Bureau of Weights & Measures in France

Larger & smaller units defined from SI standards by powers of 10 & Greek prefixes

These are the standard SI prefixes for indicating powers of 10. Many (k, c, m, μ) are familiar; Y, Z, E, h, da, a, z, and y are rarely used.

TABLE 1–4
Metric (SI) Prefixes

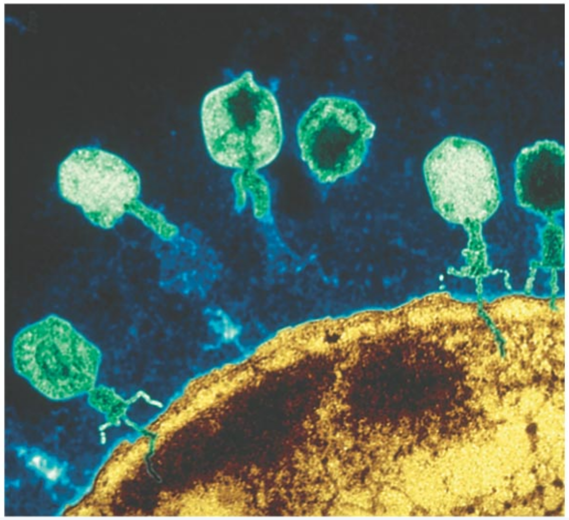
Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

Typical Lengths (approx.)

TABLE 1–1 Some Typical Lengths or Distances (order of magnitude)

Length (or Distance)	Meters (approximate)
Neutron or proton (diameter)	10^{-15} m
Atom (diameter)	10^{-10} m
Virus [see Fig. 1–5a]	10^{-7} m
Sheet of paper (thickness)	10^{-4} m
Finger width	10^{-2} m
Football field length	10^{2} m
Height of Mt. Everest [see Fig. 1–5b]	10^{4} m
Earth diameter	10^{7} m
Earth to Sun	10^{11} m
Earth to nearest star	10^{16} m
Earth to nearest galaxy	10^{22} m
Earth to farthest galaxy visible	10^{26} m



Typical Times (approx.)

TABLE 1–2 Some Typical Time Intervals

Time Interval	Seconds (approximate)
Lifetime of very unstable subatomic particle	10^{-23} s
Lifetime of radioactive elements	10^{-22} s to 10^{28} s
Lifetime of muon	10^{-6} s
Time between human heartbeats	10^{0} s (= 1 s)
One day	10^{5} s
One year	3×10^{7} s
Human life span	2×10^{9} s
Length of recorded history	10^{11} s
Humans on Earth	10^{14} s
Life on Earth	10^{17} s
Age of Universe	10^{18} s

Typical Masses (approx.)

TABLE 1–3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron →→→	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship →→→	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

We will work only in the SI system, where the basic units are kilograms, meters, & seconds.

TABLE 1–5 SI Base Quantities and Units

Quantity	Unit	Unit Abbre-viation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Other systems of units:

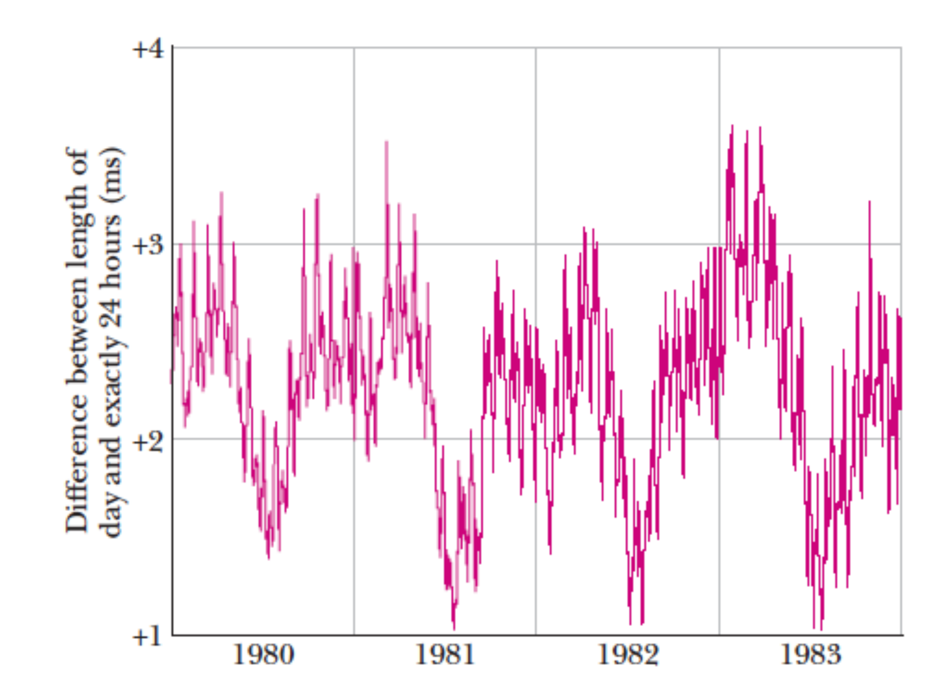
cgs: units are grams, centimeters, & seconds.

British (engineering) system (everyday US system): force instead of mass as one of its basic quantities, which are feet, pounds, & seconds.

Time

- Units
  - seconds, s in all systems

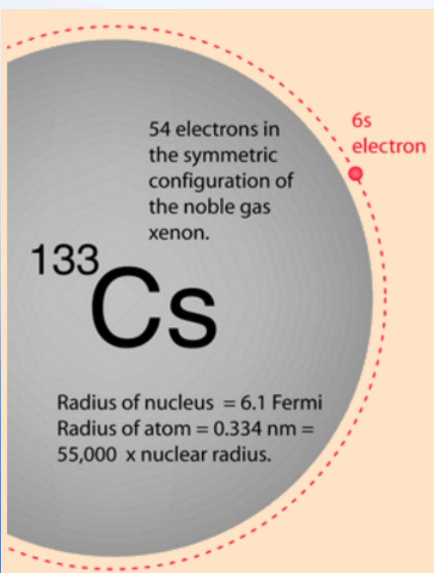

Steven Pitkin

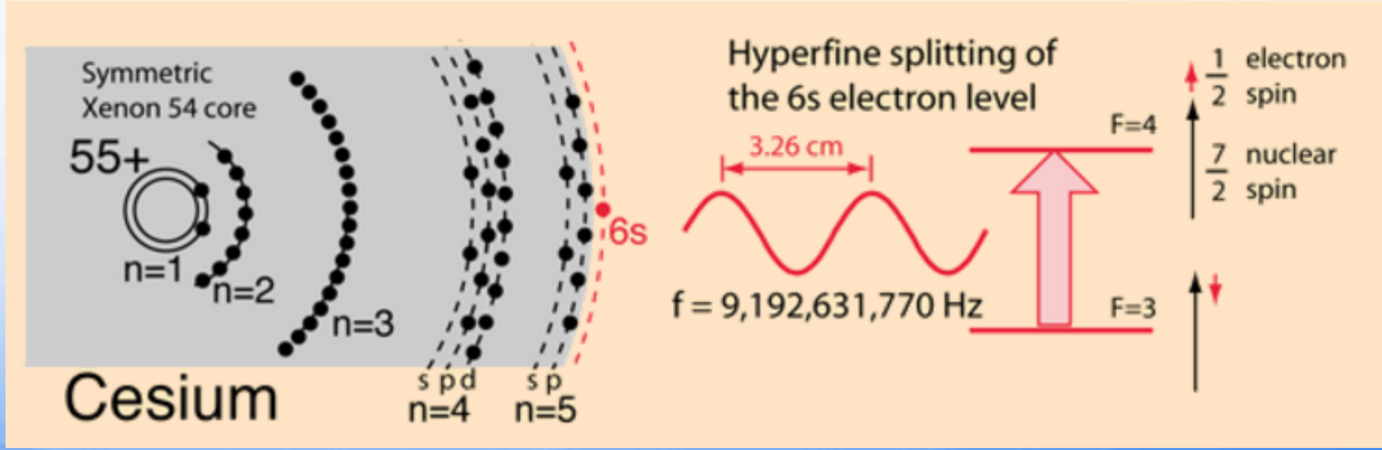


One Second

- Defined in terms of the oscillation of radiation from a cesium atom

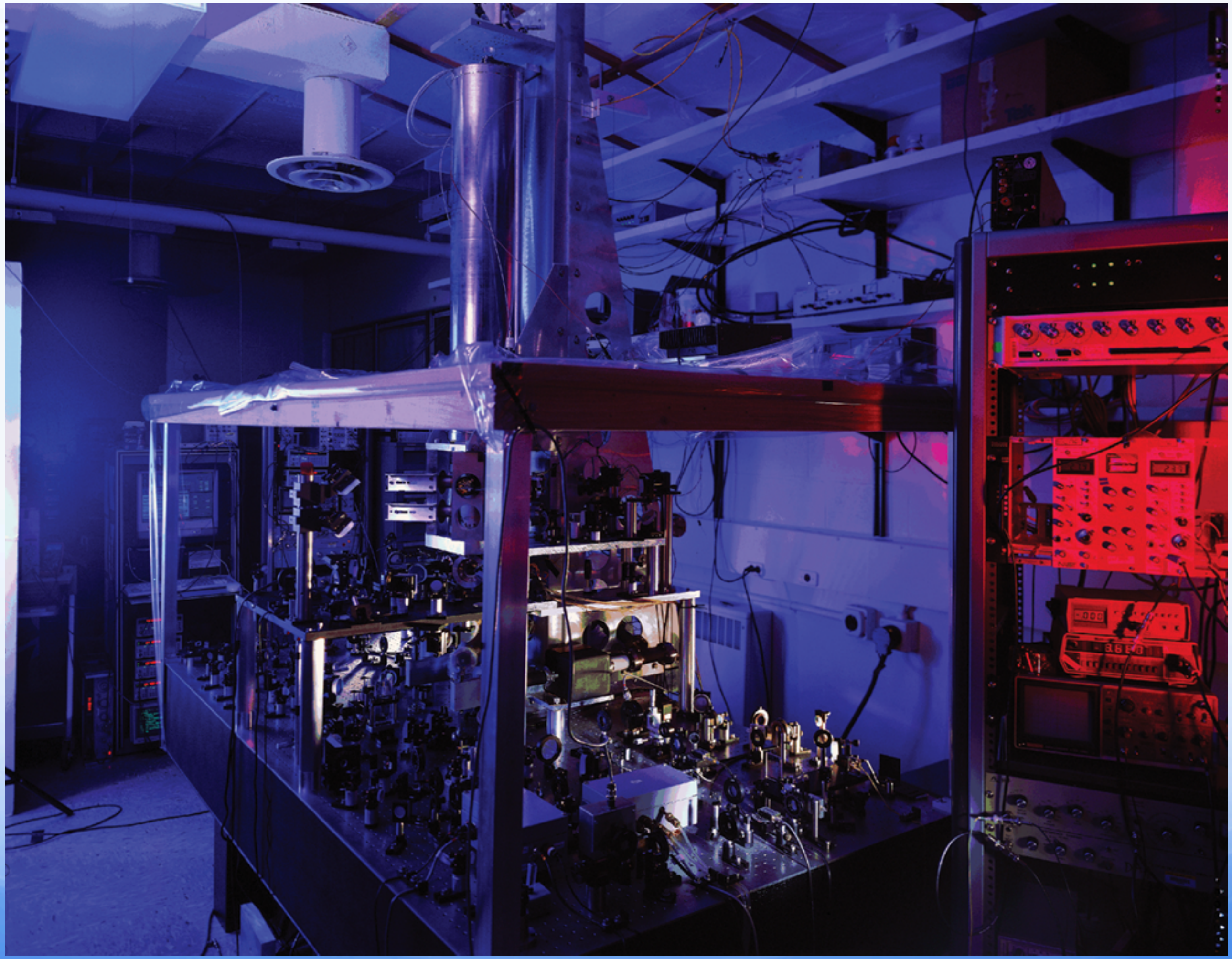
✓ the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom





with a precision of 1 second in 1.4 million years!

Atomic Clock



Basic & Derived Quantities

- Basic Quantity ≡ Must be defined in terms of a standard (meter, kilogram, second,….).

- Derived Quantity ≡ Defined in terms of combinations of basic quantities
  - Unit of speed (v = distance/time) = meter/second = m/s
  - Unit of density ($\rho = m/V$) = kg/m^3

Units and Equations

- In dealing with equations, remember that the units must be the same on both sides of an equation (otherwise, it is not an equation)!
- Example: You go 90 km/hr for 40 minutes. How far did you go?
  - Ch. 2 equation from Ch. 2: $x = vt$.
  - So, $v = 90$ km/hr, $t = 40$ min. To use this equation, first convert t to hours:

    $t = (⅔)$hr so, $x = (90 \text{ km/hr}) \times [(⅔)\text{hr}] = 60 \text{ km}$

  The hour unit (hr) has (literally) cancelled out in the numerator & denominator!

Converting Units

- As in the example, units in the numerator & the denominator can cancel out (as in algebra)
- Illustration: Convert 80 km/hr to m/s

Conversions: 1 km = 1000 m; 1hr = 3600 s

$\Rightarrow$ 80 km/hr =

(80 km/hr) (1000 m/km) (1hr/3600 s)

(Cancel units!)

80 km/hr $\cong$ 22 m/s (22.222...m/s)

- Useful conversions:

1 m/s $\cong$ 3.6 km/hr; 1 km/hr $\cong$ (1/3.6) m/s

✔ Order of Magnitude; Rapid Estimating

- Sometimes, we are interested in only an approximate value for a quantity. We are interested in obtaining rough or order of magnitude estimates.

- Order of magnitude estimates: Made by rounding off all numbers in a calculation to 1 sig fig, along with power of 10.
  - Can be accurate to within a factor of 10 (often better)

Example: Ali has 2 apples, Nima has 4 apples.
Their numbers of apples are "of the same order of magnitude"

Masses of Various Objects (Approximate Values)

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

TABLE 1–3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

Density

Densities of Various Substances

Substance	Density ρ (10^3 kg/m^3)
Platinum	21.45
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air at atmospheric pressure	0.0012

$$\rho = \frac{m}{V}$$

Mean density of Earth
5.52 g/cm^3

Arsenic 5.727 g/cm3

Germanium 5.323 g/cm3

TABLE 1–1 Some Typical Lengths or Distances (order of magnitude)

Length (or Distance)	Meters (approximate)
Neutron or proton (diameter)	10^{-15} m
Atom (diameter)	10^{-10} m
Virus [see Fig. 1–8a]	10^{-7} m
Sheet of paper (thickness)	10^{-4} m
Finger width	10^{-2} m
Football field length	10^{2} m
Height of Mt. Everest [see Fig. 1–8b]	10^{4} m
Earth diameter	10^{7} m
Earth to Sun	10^{11} m
Earth to nearest star	10^{16} m
Earth to nearest galaxy	10^{22} m
Earth to farthest galaxy visible	10^{26} m

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time interval for one revolution of the Earth about its axis)	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Sample Problem 1.01 Estimating order of magnitude, ball of string

- The world’s largest ball of string is about 2 m in radius. To the nearest order of magnitude, what is the total length L of the string in the ball?

Calculations: Let us assume the ball is spherical with radius $R = 2$ m. The string in the ball is not closely packed (there are uncountable gaps between adjacent sections of string). To allow for these gaps, let us somewhat overestimate

the cross-sectional area of the string by assuming the cross section is square, with an edge length $d = 4$ mm. Then, with a cross-sectional area of d^2 and a length L, the string occupies a total volume of

$$V = (\text{cross-sectional area})(\text{length}) = d^2 L.$$

This is approximately equal to the volume of the ball, given by $\frac{4}{3}\pi R^3$, which is about $4R^3$ because π is about 3. Thus, we have the following

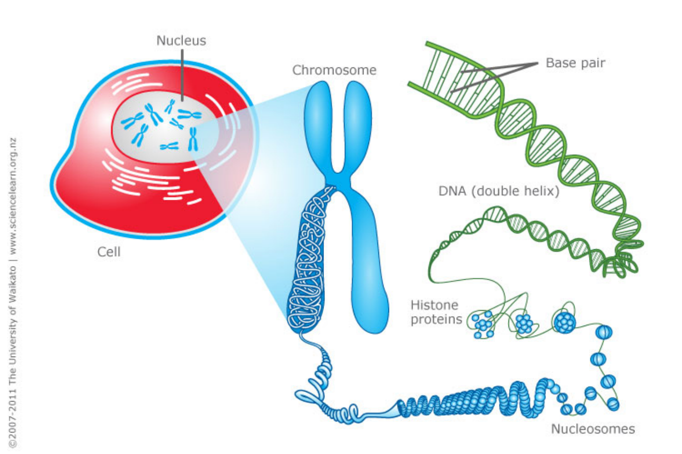
$$d^2 L = 4R^3,$$

or

$$L = \frac{4R^3}{d^2} = \frac{4(2\text{ m})^3}{(4 \times 10^{-3}\text{ m})^2}$$

$$= 2 \times 10^6\text{ m} \approx 10^6\text{ m} = 10^3\text{ km}. \quad \text{(Answer)}$$

(Note that you do not need a calculator for such a simplified calculation.) To the nearest order of magnitude, the ball contains about 1000 km of string!



Example

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\frac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{L}^n \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^m = \frac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \qquad \text{and} \qquad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in $\mathrm{m/s^2}$.

Example

Example 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on w[illegible] the person is exercising, sleeping, angry, serene, [illegible]nd so forth. To the nearest order of magnitude, we shall ch[illegible]ose 10 breaths per minute as our estimate of the average. [illegible]This is certainly closer to the true value than 1 breath per mi[illegible] or 100 breaths per minute.) The number of minutes in a year is approximately

$$1\ \text{yr}\left(\frac{400\ \text{days}}{1\ \text{yr}}\right)\left(\frac{25\ \text{h}}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 6 \times 10^5\ \text{min}$$

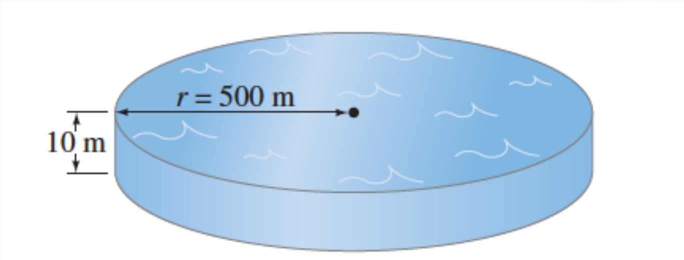
Notice how much simpler it is in the expression above to multiply 400×25 than it is to work with the more accurate 365×24. These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70\ \text{yr})(6 \times 10^5\ \text{min/yr}) = 4 \times 10^7$ min. At a rate of 10 breaths/min, an individual would take 4×10^8 breaths in a lifetime, or on the order of 10^9 breaths.

What If? **What if the average life span were estimated as 80 years instead of 70? Would this change our final estimate?**

Answer We could claim that $(80\ \text{yr})(6 \times 10^5\ \text{min/yr}) = 5 \times 10^7$ min, so that our final estimate should be 5×10^8 breaths. This is still on the order of 10^9 breaths, so an order-of-magnitude estimate would be unchanged. Furthermore, 80 years is 14% larger than 70 years, but we have overestimated the total time interval by using 400 days in a year instead of 365 and 25 hours in a day instead of 24. These two numbers together result in an overestimate of 14%, which cancels the effect of the increased life span!

Example

Example (volume of lake)



EXAMPLE 1–6 ESTIMATE **Volume of a lake.** Estimate how much water there is in a particular lake, Fig. 1–10a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

APPROACH No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 1–10b).

SOLUTION The volume V of a cylinder is the product of its height h times the area of its base: $V = h\pi r^2$, where r is the radius of the circular base.† The radius r is $\frac{1}{2}$ km $= 500$ m, so the volume is approximately

$$V = h\pi r^2 \approx (10\,\text{m}) \times (3) \times \left(5 \times 10^2\,\text{m}\right)^2 \approx 8 \times 10^6\,\text{m}^3 \approx 10^7\,\text{m}^3,$$

where π was rounded off to 3. So the volume is on the order of $10^7\,\text{m}^3$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $\left(10^7\,\text{m}^3\right)$ is probably better to quote than the $8 \times 10^6\,\text{m}^3$ figure.

NOTE To express our result in U.S. gallons, we see in the Table on the inside front cover that $1\,\text{liter} = 10^{-3}\,\text{m}^3 \approx \frac{1}{4}\,\text{gallon}$. Hence, the lake contains $\left(8 \times 10^6\,\text{m}^3\right)\left(1\,\text{gallon}/4 \times 10^{-3}\,\text{m}^3\right) \approx 2 \times 10^9\,\text{gallons}$ of water.

Example

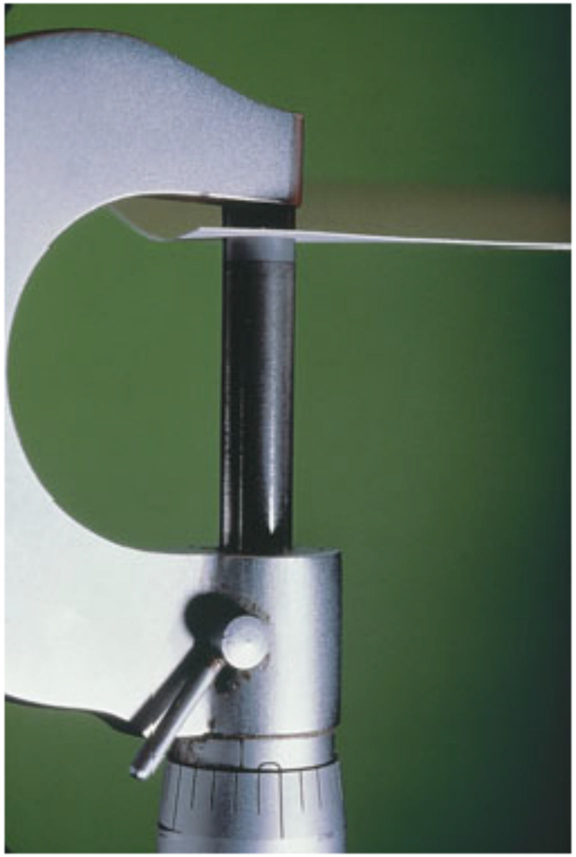


FIGURE 1–11 Example 1–7.
Micrometer used for measuring small thicknesses.

Example

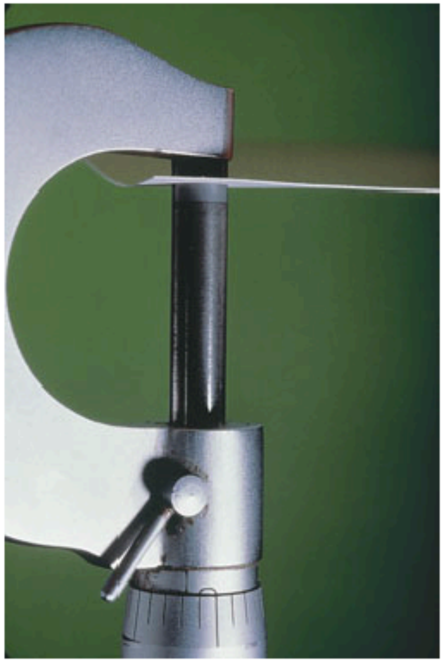


FIGURE 1–11 Example 1–7. Micrometer used for measuring small thicknesses.

EXAMPLE 1–7 ESTIMATE **Thickness of a sheet of paper.** Estimate the thickness of a page of this book.

APPROACH At first you might think that a special measuring device, a micrometer (Fig. 1–11), is needed to measure the thickness of one page since an ordinary ruler can not be read so finely. But we can use a trick or, to put it in physics terms, make use of a *symmetry*: we can make the reasonable assumption that all the pages of this book are equal in thickness.

SOLUTION We can use a ruler to measure hundreds of pages at once. If you measure the thickness of the first 500 pages of this book (page 1 to page 500), you might get something like 1.5 cm. Note that 500 numbered pages, counted front and back, is 250 separate pieces of paper. So one sheet must have a thickness of about

$$\frac{1.5\text{ cm}}{250\text{ sheets}} \approx 6 \times 10^{-3}\text{ cm} = 6 \times 10^{-2}\text{ mm},$$

or less than a tenth of a millimeter (0.1 mm).

Example

EXAMPLE 1–9 ESTIMATE Estimating the radius of Earth. Believe it or not, you can estimate the radius of the Earth without having to go into space (see the photograph on page 1). If you have ever been on the shore of a large lake, you may have noticed that you cannot see the beaches, piers, or rocks at water level across the lake on the opposite shore. The lake seems to bulge out between you and the opposite shore—a good clue that the Earth is round. Suppose you climb a stepladder and discover that when your eyes are 10 ft (3.0 m) above the water, you can just see the rocks at water level on the opposite shore. From a map, you estimate the distance to the opposite shore as $d \approx 6.1\ \text{km}$. Use Fig. 1–14 with $h = 3.0\ \text{m}$ to estimate the radius R of the Earth.

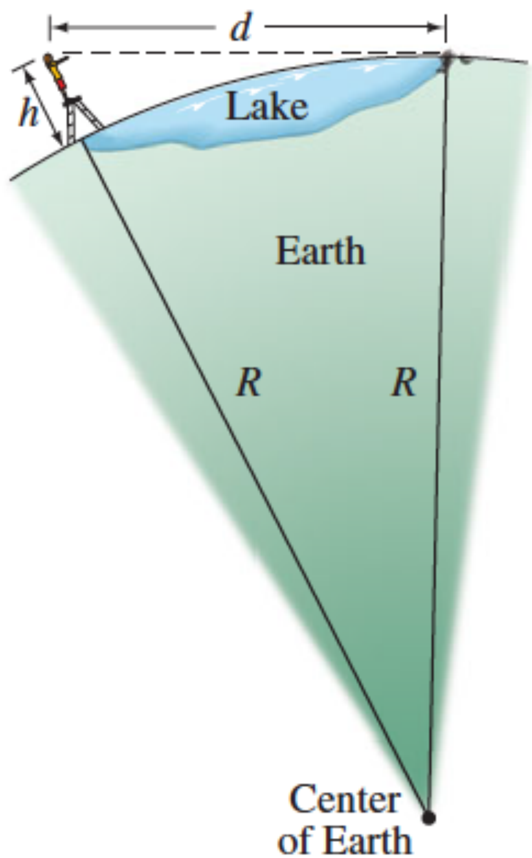


FIGURE 1–14 Example 1–9, but not to scale. You can just barely see rocks at water level on the opposite shore of a lake 6.1 km wide if you stand on a stepladder.

APPROACH We use simple geometry, including the theorem of Pythagoras,

$$c^2 = a^2 + b^2,$$

where c is the length of the hypotenuse of any right triangle, and a and b are the lengths of the other two sides.

SOLUTION For the right triangle of Fig. 1–14, the two sides are the radius of the Earth R and the distance $d = 6.1\text{ km} = 6100\text{ m}$. The hypotenuse is approximately the length $R + h$, where $h = 3.0\text{ m}$. By the Pythagorean theorem,

$$R^2 + d^2 \approx (R + h)^2$$
$$\approx R^2 + 2hR + h^2.$$

We solve algebraically for R, after cancelling R^2 on both sides:

$$R \approx \frac{d^2 - h^2}{2h} = \frac{(6100\text{ m})^2 - (3.0\text{ m})^2}{6.0\text{ m}}$$
$$= 6.2 \times 10^6\text{ m}$$
$$= 6200\text{ km}.$$

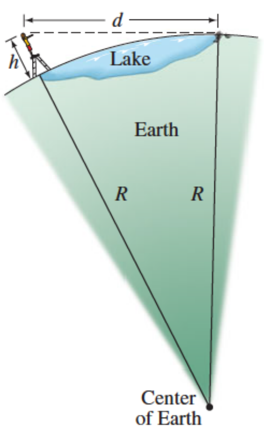


FIGURE 1–14 Example 1–9, but not to scale. You can just barely see rocks at water level on the opposite shore of a lake 6.1 km wide if you stand on a stepladder.

NOTE Precise measurements give 6380 km. But look at your achievement!

Example

29. (II) Estimate how long it would take one person to mow a football field using an ordinary home lawn mower (Fig. 1–15). (State your assumption, such as the mower moves with a 1-km/h speed, and has a 0.5-m width.)

FIGURE 1–15
Problem 29.

Example

*37. (II) The speed v of an object is given by the equation $v = At^3 - Bt$, where t refers to time. (*a*) What are the dimensions of A and B? (*b*) What are the SI units for the constants A and B?

Example

48. Hold a pencil in front of your eye at a position where its blunt end just blocks out the Moon (Fig. 1–19). Make appropriate measurements to estimate the diameter of the Moon, given that the Earth–Moon distance is 3.8×10^5 km.

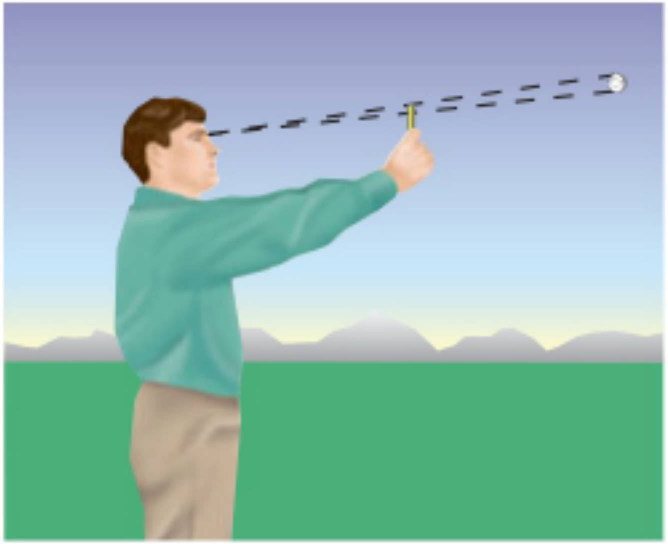


FIGURE 1–19
Problem 48. How big is the Moon?

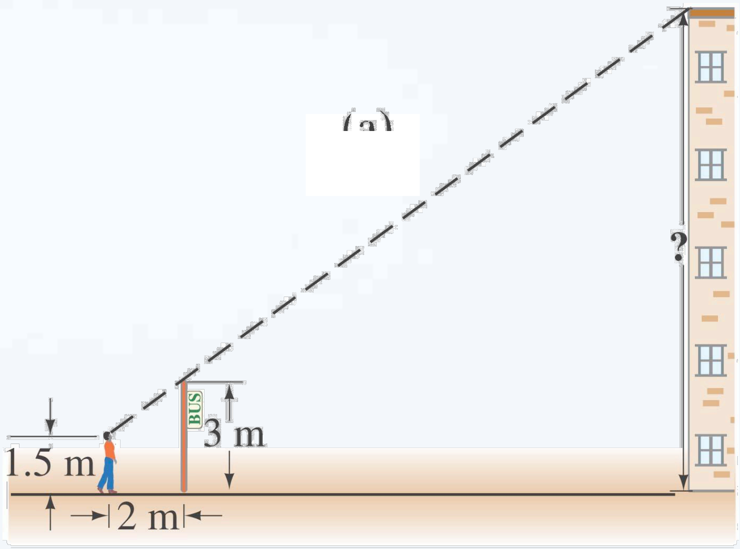
Example

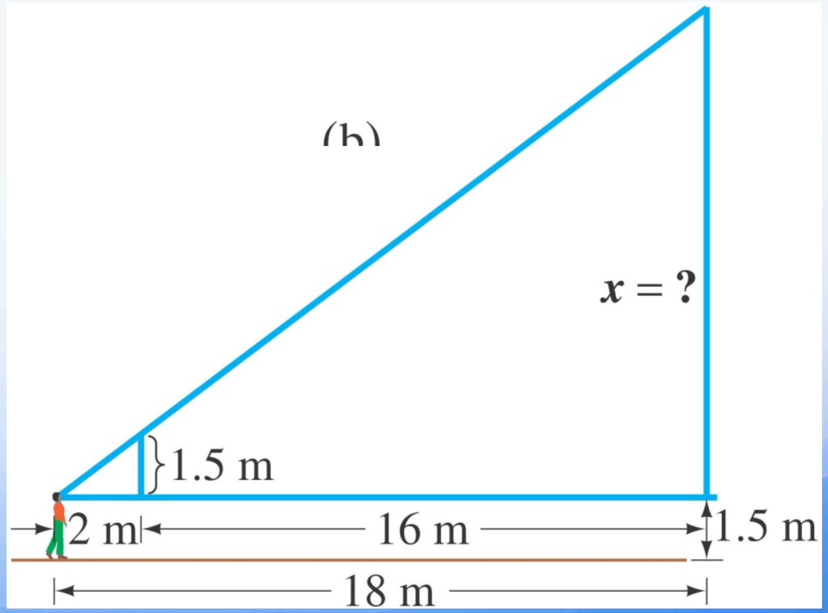
45. Estimate the number of jelly beans in the jar of Fig. 1–18.

FIGURE 1–18
Problem 45. Estimate the number of jelly beans in the jar.

Example 1-9: Height by triangulation.

Estimate the height of the building shown by "triangulation," with the help of a bus-stop pole and a friend. (See how useful the diagram is!)





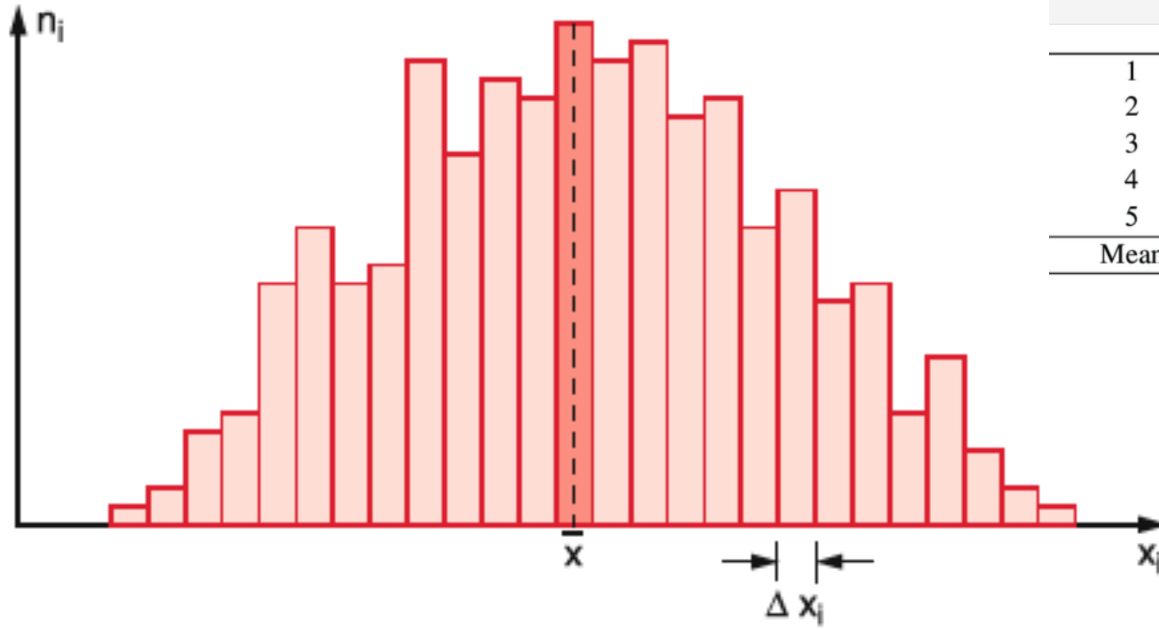
Accuracy and Precision; Measurement Uncertainties and Errors

- Systematic and statistical errors.

measuring equipment

Systematic and statistical errors.

1	234,41
2	234,39
3	234,79
4	234,04
5	234,42
6	234,81
7	234,24
8	234,57
9	234,54
10	234,29
11	234,63



	Free fall
1	9.52
2	10.00
3	9.76
4	9.33
5	9.60
Mean	9.77

Figure 1.33 Typical histogram of the statistical distribution of measured values x_i around the mean value $\bar{x}$

$$x_w = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} x_i \,.$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \,,$$

The quantity

$$\sigma = \sqrt{\langle e^2 \rangle} = \sqrt{\frac{\sum (x_\mathrm{w} - x_i)^2}{n}} \tag{1.8a}$$

is named **standard deviation** or *root mean square deviation*. It equals the square root of the squared arithmetic mean $\langle e^2 \rangle$

$$\langle e^2 \rangle = \frac{1}{n} \sum e_i^2 = \frac{1}{n} \sum_{i=1}^{n} (x_\mathrm{w} - x_i)^2 \tag{1.8b}$$

The smaller quantity

$$\begin{aligned} \sigma_\mathrm{m} = \sqrt{\overline{\varepsilon^2}} &= \sqrt{\frac{1}{n^2} \sum e_i^2} \\ &= \frac{1}{n} \sqrt{\sum_i (x_\mathrm{w} - x_i)^2} \end{aligned} \tag{1.8c}$$

is the **mean error of the arithmetic mean** $\overline{x}$.

$$\sigma_\mathrm{m} = \frac{\sigma}{\sqrt{n}} \, .$$

For the standard deviation of the individual results x_i we obtain the mean deviation of the arithmetic mean value

$$\sigma^2 = \frac{n}{n-1}s^2 \rightarrow \sigma = \sqrt{\frac{\sum(\bar{x}-x_i)^2}{n-1}}\,, \qquad (1.13)$$

which can be obtained from measurements and is therefore a known quantity.

For the mean deviation of the arithmetic mean (also called standard deviation of the arithmetic means) we get

$$\sigma_{\mathrm{m}}^2 = \frac{1}{n-1}s^2 \rightarrow \sigma_{\mathrm{m}} = \sqrt{\frac{\sum(\bar{x}-x_i)^2}{n(n-1)}}\,. \qquad (1.14)$$

Example

For 10 measurements of the period of a pendulum the following values have been obtained:
$T_1 = 1.04$ s; $T_2 = 1.01$ s; $T_3 = 1.03$ s; $T_4 = 0.99$ s; $T_5 = 0.98$ s; $T_6 = 1.00$ s; $T_7 = 1.01$ s; $T_8 = 0.97$ s; $T_9 = 0.99$ s; $T_{10} = 0.98$ s.

The arithmetic mean is $\overline{T} = 1.00$ s. The deviations $x_i = T_i - \overline{T}$ of the values T_i from the mean $\overline{T}$ are
$x_1 = 0.04$ s; $x_2 = 0.01$ s; $x_3 = 0.03$ s; $x_4 = -0.01$ s; $x_5 = -0.02$ s; $x_6 = 0.00$ s; $x_7 = 0.01$ s; $x_8 = -0.03$ s; $x_9 = -0.01$ s; $x_{10} = -0.02$ s. This gives

$$\Sigma(T_i - \langle T\rangle)^2 = \Sigma x_i^2 = 46 \cdot 10^{-4}\,\mathrm{s}^2 .$$

The standard deviation is then

$$\sigma = \sqrt{(46 \cdot 10^{-4}/9)} = 2.26 \cdot 10^{-2}\,\mathrm{s}$$

and the standard deviation of the arithmetic mean is

$$\sigma_\mathrm{m} = \sqrt{(46 \cdot 10^{-4}/90)} = 0.715 \cdot 10^{-2}\,\mathrm{s} .$$

$$x_\mathrm{w} = \overline{x} \pm \sigma_\mathrm{m} = \overline{x} \pm \sqrt{\frac{\sum (x_i - \overline{x})^2}{n(n-1)}} .$$

Significant Figures

- A significant figure is one that is reliably known
- All non-zero digits are significant
- Zeros are significant when
  - between other non-zero digits
  - after the decimal point and another significant figure
  - can be clarified by using scientific notation

$$17400 = 1.74 \times 10^4$$ 3 significant figures

$$17400. = 1.7400 \times 10^4$$ 5 significant figures

$$17400.0 = 1.74000 \times 10^4$$ 6 significant figures

Operations with Significant Figures

- Accuracy -- number of significant figures

  Example: meter stick: $\pm 0.1\,cm$

- When multiplying or dividing, round the result to the same accuracy as the least accurate measurement

  2 significant figures

  Example: rectangular plate: 4.5 cm by 7.3 cm

  area: 32.85 cm^2 33 cm^2

- When adding or subtracting, round the result to the smallest number of decimal places of any term in the sum

  Example: 135 m + 6.213 m = 141 m

Example

5.60 x 7.102 = 39.7712 ==> 39.8

3 s.f. 4.s.f. so this will have 3 s.f.

15 ÷ 3.155 = 4.754358 ==>4.8

2s.f. 4.s.f. so this will have 2 s.f.

A. Shariati, Significant Figures
http://www.gammajournal.ir/pdf/gamma_no_25_ar_02v01.pdf