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Kolmogorov spectrum of renewable wind and solar power fluctuations

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Abstract. With increasing the contribution of renewable energies in power production, the task of reducing dynamic instability in power grids must also be addressed from the generation side, because the power delivered from such sources is spatiotemporally stochastic in nature. Here we characterize the stochastic properties of the wind and solar energy sources by studying their spectrum and multifractal exponents. The computed power spectrum from high frequency time series of solar irradiance and wind power reveals a power-law behaviour with an exponent $\sim 5/3$ (Kolmogorov exponent) for the frequency domain $0.001\,\mathrm{Hz} < f < 0.05\,\mathrm{Hz}$, which means that the power grid is being fed by turbulent-like sources. Our results bring important evidence on the stochastic and turbulent-like behaviour of renewable power production from wind and solar energies, which can cause instability in power grids. Our statistical analysis also provides important information that must be used as a guideline for an optimal design of power grids that operate under intermittent renewable sources of power.

1 Introduction

Wind power and solar irradiance have a stochastic nature, which is often ignored in stability analyses of power grids [1,2], or, if taken into account, commonly described in terms of uncorrelated Gaussian processes [3–6]. Their uncontrollable fluctuations in time are thought to be the major problem for keeping up the stability of power grids. Therefore understanding the nature of such fluctuations is extremely important. The power generation from wind turbines and photovoltaics exhibit non-Gaussian probability distribution functions [1,2]. The wind in the atmospheric boundary layer is distinctly turbulent and nonstationary [7–15]. As a consequence, the wind speed varies rather randomly on many different time scales that range from long-term variations

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Table 1. Data description.

dataset	rated power P_r	# data	frequency
wind farm	∼25 MW	$\sim 10^7$	1 Hz
wind turbine	∼2 MW	$\sim 10^7$	1 Hz
solar irradiance, Germany	–	$\sim 10^7$	1 Hz
solar irradiance, Hawaii	–	$\sim 10^7$	1 Hz

or seasonal trends (months) to very short ones (minutes down to less than a second). The latter are commonly considered to correspond to small (micro)-scale turbulence [16]. Such small-scale fluctuations are superimposed onto the mean velocity varying on larger scales [9,17]. Regarding the solar irradiance, much less is so far known about the statistical properties, multifractality (which corresponds to an unexpected high probability of *correlated* large irradiance fluctuations), volatility clustering, jump rates between sunny and cloudy states, etc. [18–23].

For the most efficient grid integration of wind and solar energies, the physics of the wind and solar power fluctuations and their stochastic behaviours must be understood in details. Recent investigations show that, such fluctuations exhibit multiple types of variability and nonlinearity on time scales of seconds. This includes intermittency, i.e. large correlated amplitude fluctuations, repeatedly transiting between steady states and rapid gusts for wind, and flickering, i.e. on-off type of fluctuations for solar [2].

Here we will study the power-output spectrum of the wind turbines and solar irradiance as well as their multifractal exponents. We find a *turbulent-like* exponent for the spectrum of solar irradiance and wind power fluctuations. Our analysis indicates that these renewable sources have strong intermittent behaviour and then compute the multi-fractal exponents of their fluctuations. The results here are derived from data measured on operating wind turbines and continuous solar irradiance measurements with sampling rate 1 Hz, in Germany and in the United States. In Table 1, we provide the data description for our present data sets. It contains the number of data points collected, as well as the duration and sampling frequency of the measurements. The wind farm studied has 12 wind turbines spread over an area of roughly $4\,\mathrm{km}^2$. The United States' National Renewable Energy Laboratory (NREL) performed a one-year measurement campaign at Kalaeloa Airport (21.312° N, −158.084° W), Hawaii, USA, from March 2010 until March 2011 using 19 LI-COR LI-200 pyranometers to measure effective solar radiation on horizontal and inclined surfaces with a 1 Hz temporal resolution. Two of the instruments were titled with a 45 degree orientation, while the other 17 were horizontally mounted and scattered across an area of about $750 \times 750\,\mathrm{m}^2$. From this publicly available data, the subset of effective horizontal solar radiation was selected, processed and checked to yield about 10^7 synchronised values of 1 Hz temporal resolution measured by the aforementioned 17 pyranometers on 378 days.

2 Spectrum of wind power and solar irradiance

Let us first study the spectrum of wind power and solar irradiance fluctuations. The time series are wind velocity, scaled power output of wind turbine $P(t)/P_r$, where P_r is its rated power, solar irradiance $G(t)$ and clear sky index $Z(t) = G(t)/G_{clearsky}$ (or detrended irradiance time series) [2]. One expects that the measured irradiance has a natural trend, because of its dependence on the sun's position in the sky and the longitude and latitude of the observation location. Therefore clear sky irradiance $G_{clearsky}$ is also a function of time. The typical fluctuations of clear sky index are depicted in Fig. 1.

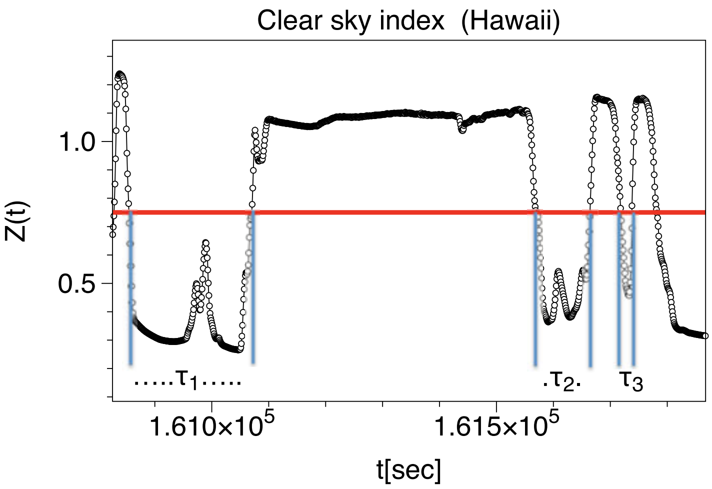

Fig. 1. Stochastic dynamics of clear sky index $Z(t)$. Waiting times for the cloudy-state, for instance with $Z \leq 0.75$ are denoted by $\tau_1, \tau_2, \dots$

Wind turbine and solar cells convert wind velocity and solar irradiance fluctuations into power fluctuations. Apparently, the conversion dynamics take place at short time scale of seconds for wind power and almost simultaneously for solar energy. The wind speed and power output of the single wind turbine have a similar spectral (and correlation) behavior at low frequencies $f < 0.1$ Hz, and obey the 5/3 Kolmogorov-law of turbulence i.e., $S(f) \sim f^{-5/3}$ [1,16], as shown in Fig. 2a. Due to the finiteness of the reaction time scale of wind turbines, the fast wind fluctuations are partly filtered, and slower wind fluctuations are adiabatically converted into power output fluctuations. The filtering will be more in high frequencies for the cumulative power fluctuations of wind park. Therefore one expects that the wind power has weaker high frequency fluctuations [1,24].

We note that the scaling region for the atmospheric wind spectrum is larger with respect to the wind power, due to the fact that these data are constrained by the turbine inertia at high frequencies and by the maximum output of the turbines at low frequencies. Also for frequencies $f \geq 0.1$ Hz, corresponding to the length scale of the order of 100 m, the wind fluctuations are smoothed out spatially by the rotor area (roughly 100 m diameter). Therefore a deviation from an ideal Kolmogorov spectrum (taken by a much smaller sensor) appears quite natural. Also in the range of seconds to minutes the control system (pitch in seconds, yaw in minute) is active and may serve as a filter too. It is worth pointing out that the power electronic from generator to convertor can be controlled in the range of *msec* and thus they could be potentially used to smooth out power fluctuations. In our data (with sampling rate 1 Hz), we do not see such filtering.

In contrast to the wind turbine, solar irradiance can transform to electric power in solar cells within time scale of μ-sec and therefore there is no considerable instrumental filtering in the resolution of 1 Hz irradiance sampling. The current in solar cell is proportional to the irradiance, which means that any fluctuations in the irradiance will transfer almost simultaneously to current fluctuations in solar cell. This is the reason that any discontinuity in solar irradiance, such as jumps in the intensity, will appear immediately in the output of solar cells.

In Figs. 2a and 2c, we plot the spectral density of irradiance fluctuations $G(t)$ for the data set measured in Germany and Hawaii and for single sensor as well as

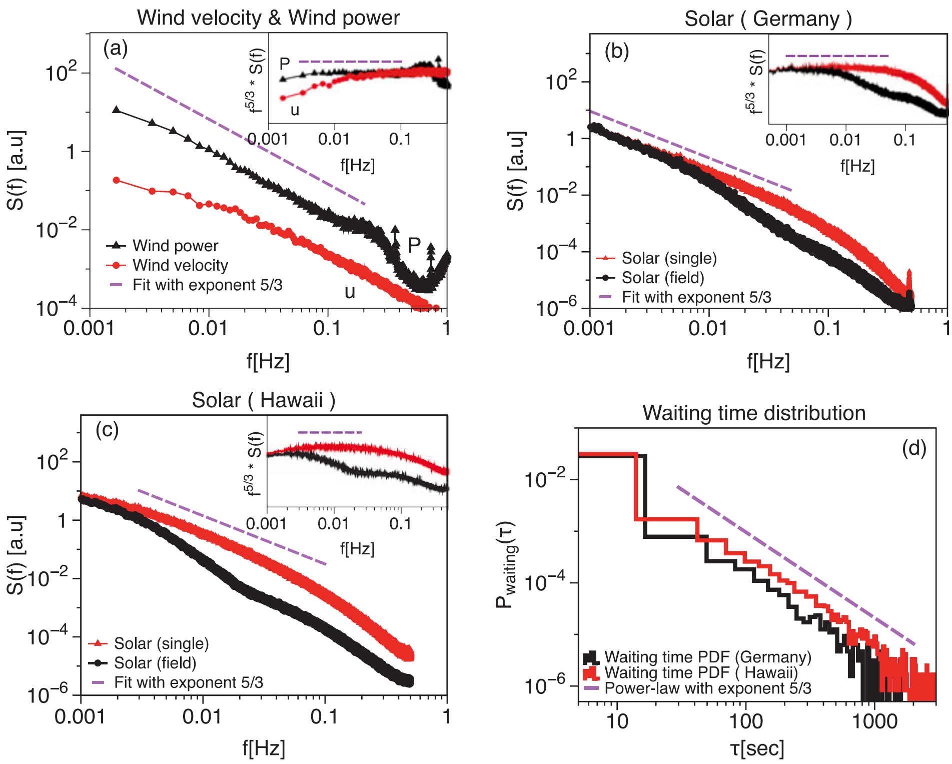

Fig. 2. a) Power spectra of wind velocity, wind power and irradiance fluctuation in log-log scale. The Kolmogorov exponent 5/3 are represented by dashed lines. (a) The power spectrum for wind velocity and wind power, from [1]. (b) and (c) The spectrum of solar irradiance measured in Germany and Hawaii. In the inset of (a) and (b), log-log plot of the compensated energy spectra $f^{5/3}S(f)$ versus frequency f are plotted. The turbulence type spectrum 5/3-law present in irradiance fluctuations is seen for single sensor, for frequencies $0.001 < f < 0.05$ Hz. (c) Power spectra of irradiance fluctuation for single (red) and averaged over 17 sensors (black) in log-log scale measured in Hawaii. (d) Waiting time distribution functions of cloudy-sky state. It shows that for time scales $10\,\mathrm{s} < \tau < 2000\,\mathrm{s}$, it has a power law behaviour with exponent $\sim 5/3$.

total irradiance in the field (with sample rate of 1 Hz and for more than 1 year of continues measurements of irradiance). The Figs. 2b and 2c show that the turbulent-type spectrum are present in irradiance fluctuations of the single sensor, within more than one order of magnitude, with exponents 1.65 ± 0.01 and 1.57 ± 0.01 (with error estimated at the 95% confidence level) for frequencies $0.001 < f < 0.05$ Hz and $0.003 < f < 0.03$ Hz in Germany and Hawaii, respectively. The spectrum of the whole field shows some filtering for high frequency region similar to the spectrum of total power of wind farm. Similar power-law behaviour is found for clear sky index $Z(t)$ (not presented here) and with the same exponent. We note that the spectrum is influenced by the cloud variability. The cloud situations in Germany and Hawaii have different type of variability. In Germany it has strong variability and large fluctuations expected in short time scales. However in Hawaii there is very low cloud variability and therefore the amplitude for high frequencies will be smaller.

2.1 Origin of the Kolmogorov type spectrum for wind power and solar irradiance

As we observed above, the spectral exponents for the power output of wind turbine and solar irradiance fluctuations have the same values and they obey both the Kolmogorov type spectrum, which means that in practice the power grids are fed with turbulent-like stochastic sources.

However, there are different origins to observe the exponent $5/3 \simeq 1.6$ in wind power and solar irradiance fluctuations. Wind velocity has fluid turbulence characteristic and it has the Kolmogorov type spectrum, i.e. $S(f) \sim f^{-5/3}$ in the inertial range (scaling region), where $S_u(f) = |u(f)|^2$ and $u(f)$ is the Fourier transform of wind velocity time series [16]. The wind power $P(t)$ is proportional to $u(t)^3$. We checked the ensemble averaged spectrum of $u(t)$ and $u^3(t)$ have the same power law exponent for frequencies $0.001 < f < 0.05\,\mathrm{Hz}$, of course with different amplitudes. This means that spectrum of $u^3(t)$, i.e. $S_{u^3}(f)$ behaves as $\sim f^{-\alpha}$ with $\alpha = 5/3$.

Observing the 5/3 exponent in solar irradiance has a root in the cloud size distributions and on-off nature of irradiance fluctuations. Analysing the cloud horizontal size distributions for the scale range $0.1 - 8000\,\mathrm{km}$ are shown to be well-represented using a single power-law relationship with an exponent of $\sim 1.66 \pm 0.04$ (with error estimated at the 95% confidence level) from 0.1 to 1000 km, i.e in four orders of magnitude [25]. The length scales in cloud sizes distribution can be transformed to time scales using the average cloud velocities [26]. Therefore different cloud sizes are equivalent to existence of different time scale fluctuation of solar irradiance. Intuitively, the spectrum of solar irradiance measures the contribution of each time scale (frequency) in the time series and it should be related to the exponent of cloud sizes distribution.

We can determine the cloud size distributions from the waiting time scale distribution of clear sky index for *cloudy-states* durations. In Fig. 1d, we plot the waiting time distribution functions of cloudy state (for single sensor as depicted in Fig. 1) and observe that for time scales $10\,\mathrm{s} < \tau < 2000\,\mathrm{s}$, it behaves as a power law with same exponent ~ 1.6 in Germany and Hawaii. We defined the cloudy-state when the clear sky Z has the values < 0.75. However the estimated exponent is almost constant with respect to some small variations of this condition. The spectrum of time series with on-off states (similar to clear sky index) can be written in terms of waiting time distributions $P_{waiting}(\tau)$ [27–29]. Scaling arguments show that for the waiting time distribution $P_{waiting}(\tau) \sim \tau^{-\eta}$, with exponent $\eta \simeq 1.6$, its spectrum behaves as $S(f) \sim f^{-\eta}$. We note that the observed exponent in cloud size distributions or waiting time distributions are empirical findings and their physical origin in not known yet [25].

3 Multifractal exponents of wind power and solar irradiance

We note that linear techniques, such as spectral and correlation analysis, can uncover only linear structures of given time series. However due to the nonlinear relation of wind velocity and its power (i.e. $P(t) \sim u^3(t)$) the power output for one single wind turbine is more intermittent than wind velocity itself. In what follows we study the non-linear properties and intermittent behaviour of these time series by studying their multifractal exponents. For these time series, intermittent behaviour can be quantified through increments statistics in some time lag τ, i.e. $X_\tau = X(t+\tau) - X(t)$, where $X(t)$ is the wind power, solar irradiance or clear sky index time series. Extreme values of increments are seen as gusts and strong jumps in the increments of wind power and solar irradiance, respectively.

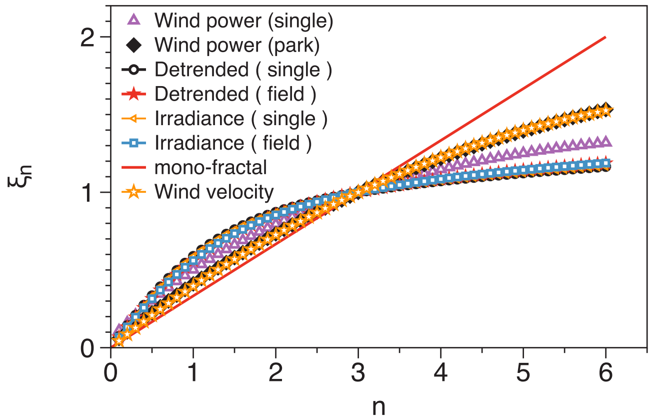

Fig. 3. Multifractal exponents of solar irradiance, clear sky index (detrended), wind power and wind velocity fluctuations. Multifractality has been checked by the scaling behavior of the moments $S_n(\tau)$ in terms of the third moment, i.e. extended self similarity method [31]. We observed that all of the moments (up to n = 6) of solar irradiance and wind power time series behave as $S_3^{\zeta_n}$, within the scaling region $10 - 1000\,\mathrm{s}$. Multifractality (non-linearity) is related to the scaling exponents ζ_n being a nonlinear function of n. For a mono-fractal (linear) process ζ_n is proportional to n.

Let $\{x(t)\}$ be a given time series and consider their statistics over a certain time scale τ, which is defined as:

$$\Delta x(\tau) = x(t+\tau) - x(t).$$

Let us denote $S(n, \tau)$, the order n absolute moment of $x(t)$;

$$S(n,\tau) = \langle |\Delta x(\tau)|^n \rangle. \tag{1}$$

We will say that the process is scale invariant, if the scale behavior of the absolute moment $S(n, \tau)$ (known as structure function) has a power law behavior in a certain range of τ [30]. Let us call ξ_n the exponent of power low, i.e.,

$$S(n,\tau) \simeq C_n \tau^{\xi_n} \tag{2}$$

where C_n is a prefactor. The process is called *monofractal* if ξ_n is a linear function of n and *multifractal* if ξ_n is nonlinear with respect to n. Multifractality has been introduced in the context of fully developed turbulence in order to describe the spatial fluctuations of the fluid velocity at very high Reynolds number [16].

An estimation of exponents ξ_n for given time series can be questioned if the time series do not, as is often the case, exhibit power-law behaviour over a broad interval. In such cases, instead of rejecting outright the existence of scale invariance, one must first explore the possibility of the data being scale invariant via the concept of extended self similarity (ESS) [31]. The ESS is a powerful tool for checking multifractal properties of data, and has been used extensively in research on turbulent flows. The basic idea is that, when the structure functions $S(n)$ are plotted against a structure function of a specific order, say $S(3)$, an extended scaling regime is found according to [8,16,31] and this defines the exponents ξ_n as $S(n) \sim S(3)^{\xi_n}$. Clearly, meaningful results are restricted to the regime where $S(3)$ is monotonic. For any Gaussian (or monofractal) process the exponents ξ follow a linear equation such as e.g.,

$$\xi_n = \frac{1}{3} n. \tag{3}$$

Therefore, any meaningful deviation from the simple scaling relation, (3), should be interpreted as deviation from monofractality or linearity. The estimated scaling exponents ξ_n from ESS, are displayed in Fig. 3, for single wind turbine, wind park, single irradiance measurements, irradiance of field, clear sky index (detrended) and finally for the measured wind velocity. In this respect, the solar data (irradiance and clear sky index) are strongly intermittent, and in turn, the wind power (single) and wind power (field) are less intermittent [1,2,7].

As a first step, it is important to recognise here the intermittent nature of such energy sources in order to properly adapt their integration in power grids. This is especially useful as large synchronous generators are progressively replaced by such volatile power plants. In particular, the nature and amplitude of possible fluctuations must be known to design and dimension appropriate solutions such as storage or even better, new optimised control systems to suppress the extreme fluctuations.

In summary, we show that the renewable power from wind and solar has a turbulent-like spectrum and have non-linear characteristics such as having the mutifractal exponents. The highly intermittent and turbulent-like power generation will become of central importance to the future electric grids that will be driven by an increasing share of renewable sources.

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