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Scale dependence of the directional relationships between coupled time series

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Abstract. Using the cross-correlation of the wavelet transformation, we propose a general method of studying the scale dependence of the direction of coupling for coupled time series. The method is first demonstrated by applying it to coupled van der Pol forced oscillators and coupled nonlinear stochastic equations. We then apply the method to the analysis of the log-return time series of the stock values of the IBM and General Electric (GE) companies. Our analysis indicates that, on average, IBM stocks react earlier to possible common sector price movements than those of GE.

Keywords: nonlinear dynamics, stochastic processes

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Contents

1. Introduction

Large interacting systems are abundant in Nature and society and have been studied for a long time [1]. They vary anywhere from hydrology, ecology, and biological systems to traffic flow, stock markets, and spatial distributions of unemployed people. The key to understanding such a wide variety of systems and phenomena is a simple fact: an effect is preceded by a cause. Hence, one must develop the ability for distinguishing the cause—the *driver*—from the effect—the *recipient*—which would then make it possible to estimate the direction along which information flows, or the direction of causal influence. If the system is such that one cannot interfere in it, then the fact that the cause precedes the effect implies that it contains information about the effect's or the recipient's future that is not contained in its past, whereas the opposite is not true. Such a simple observation was in fact the basis for the Granger causality [2], one of the best known methods of determining the direction of causal influence in the analysis of time series. Although the Granger causality was originally developed for econometric analysis, it has since been applied to a wide variety of problems in physics, geoscience and social sciences, and biology [3]–[6].

There are already many methods for uncovering causal influences between two processes. They include, in addition to the Granger causality [2, 7], information-theoretic characteristics [8]–[11], state-space [12]–[14] and double-wavelet [15, 16] analyses, and modelling of phase dynamics [17]–[19]. The last approach is based on concepts from the nonlinear theory of oscillations, and is perhaps one of the most sensitive methods currently available for analysing nonlinear systems that are endowed with a relatively stable oscillation period [20, 21]. Tokuda *et al* [22] and Timme [23] proposed estimators of couplings in ensembles of oscillators based on phase dynamics modelling, which formulate and solve the problem for deterministic processes. In the former case the estimators are computed on the basis of a single time series, but the coupling architecture must be simple and known *a priori*, while at the same time the generalization of the method to systems with a large number of quantities to be estimated is very difficult. The approach

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of Timme [23] requires close individual frequencies of the oscillators, in order to be able to manipulate them. The coupling strengths are determined from a set of time series that correspond to different, strictly synchronized regimes [24, 25]. Other approaches include those that use the multidimensional Langevin dynamics [26] to investigate which time series drives the other, the permutation information approach [27] and the phase-slope index method [28].

In this paper we introduce a robust method for studying the timescale dependence of the correlation and directional relationships between two time series. The method, which is based on using the wavelet transformation (WT) of the time series, can detect the driving source at any given timescale or frequency. The main advantage of the method is that it detects the directionality in each timescale, whereas other methods provide information only for the entire time series. The method is demonstrated by first analysing a pair of coupled van der Pol (VDP) forced oscillators and coupled nonlinear stochastic equations. We then apply the method to high frequency log-return time series of the IBM and General Electric (GE) companies, representing the product of the stock values and their volumes. In this case the method provides a simple explanation as to why the stocks react to a common sector price movements with different delays.

The rest of the paper is organized as follows. In section 2 the proposed method is described in detail. Its demonstration for coupled nonlinear oscillators, for which we already know the result, is presented in section 3. Next, coupled nonlinear stochastic equations are analysed in section 4. We then apply the method to the coupling of time series of two companies, namely, IBM and GE, which also demonstrates why one stock may react to a common factor faster or slower than the other one. The paper is summarized in section 6.

2. The method

As the first step we compute the WTs $\mathcal{W}_X(a,t)$ of the two time series, say A and B. The WT of a time series X is defined by

$$\mathcal{W}_X(a,t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} X(u)\Psi^*\left(\frac{u-t}{a}\right) \mathrm{d}u. \tag{1}$$

Here, Ψ is the mother wavelet for the wavelets that should be soliton-like with zero mean, and $*$ indicates a complex conjugate. We use the Morlet function, $\Psi(u) = \pi^{-1/4}\exp(2\mathrm{i}\pi f_0 u)\exp(-u^2/2)$ as the mother wavelet, where f_0 is a constant. $\mathcal{W}_X(a,t)$ is usually referred to as the wavelet detail coefficient in which a is a timescale parameter. The coefficients $\mathcal{W}_X(a,t)$ are complex-valued and have amplitude $W_X = |\mathcal{W}_X(a,t)|$ and a scale–time dependent phase $\phi(a,t)$. The probability distribution function (PDF) of the WT coefficients, $P(\mathcal{W}_X(a,t))$, depends on the amplitude W_X and phase ϕ, i.e. $P(\mathcal{W}_X(a,t),\mathcal{W}_X^*(a,t))\,\mathrm{d}\mathcal{W}_X\,\mathrm{d}\mathcal{W}_X^* = P(W_X,\phi;a,t)\,\mathrm{d}W_X\,\mathrm{d}\phi$. Integration over the phase ϕ yields the PDF for the amplitudes W_X, and the same holds for the PDF of the phase. The explicit form of the PDF for W_X with Gaussian-distributed real and imaginary parts of $\mathcal{W}_X(a,t)$ is given in [29].

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Next, we define the correlation functions of $\mathcal{W}_X(a,t)$ for two time series A and B as

$$\begin{aligned}&\langle \mathcal{W}_{\rm A}(a=1/f,t-\tau)\mathcal{W}^*_{\rm B}(a=1/f,t)\rangle\\ &\langle W_{\rm A}(a=1/f,t-\tau)W_{\rm B}(a=1/f,t)\exp(\phi_{\rm A}(a,t-\tau)-\phi_{\rm B}(a,t))\rangle\end{aligned} \tag{2}$$

where $\langle\cdots\rangle$ denotes the ensemble averaging. This is the two-point correlation function of four fields, $W_{\rm A}$, $W_{\rm B}$, $\phi_{\rm A}$ and $\phi_{\rm B}$, and it is, in general, difficult to delineate how the correlation function depends on the four fields. Therefore, we define correlation functions for the WT amplitudes and the averaged phase differences by

$$C_1(\tau)=\langle W_{\rm A}(a=1/f,t-\tau)W_{\rm B}(a=1/f,t)\rangle \tag{3}$$

and

$$C_2(\tau)=\langle|\phi_{\rm A}(a,t-\tau)-\phi_{\rm B}(a,t)|\rangle \tag{4}$$

The PDF in averaging $C_1(\tau)$ is determined from the joint PDF of $\mathcal{W}_X(a,t)$s by integrating over ϕ_1 and ϕ_2, which gives us the joint PDF for W_Xs, i.e. $P(W_{\rm A},W_{\rm B};a,t)$. The correlation function $C_1(\tau)$ measures the intensity of the correlation of two time series at scale a, which may have a maximum in some lag τ. The statistical averaging of the phase difference, i.e. $C_2(\tau)$, provides information about the time lag over which the two series are synchronized. If the phase shift is constant, then the systems are synchronized. The in-phase state then corresponds to 0 phase shift, and anti-phase to phase shift π.

We say that series A has correlation with series B over timescale a (or frequency $f=1/a$) if the calculated cross-correlation coefficients of the absolute values of the WTs of A and B series over the timescale a have a well-defined maximum at some finite lag τ. For positive τ the time series A is ahead of the time series B and vice versa. In general, the time lag τ is a function of the timescale a. Here, the values of the cross-correlation function for a given τ are determined with a 1σ confidence level.

3. Application to coupled nonlinear oscillators

To demonstrate the utility of the method, let us analyse two coupled forced nonlinear oscillators, a subject of much recent interest [30]–[32], due to their importance to the modelling of a variety of phenomena and processes. Of these, the classical self-sustained VDP oscillator is a paradigm for oscillating limit cycles or relaxation oscillations. Thus, we first demonstrate the method's utility by investigating directionality in a pair of coupled VDP forced oscillators, for which we know the result. The coupled VDP forced oscillators are given by

$$\begin{aligned}&\ddot{x}_1-\mu(1-x_1^2)\dot{x}_1+\omega_1x_1+\epsilon_1x_2=\eta_1(t),\\ &\ddot{x}_2-\mu(1-x_2^2)\dot{x}_2+\omega_2x_2+\epsilon_2x_1(t-t_0)=\eta_2(t).\end{aligned} \tag{5}$$

At first, we set $\omega_1=1$, $\omega_2=10$, $\mu=1$, $t_0=0$ and consider mutually independent noises, $\langle\eta_i(t)\eta_j(t')\rangle=\sigma^2\delta_{i,j}\delta(t-t')$, with the standard deviations $\sigma_j^2=0.3$ for $j=1,2$. All the parameters are fixed, except the coupling ϵ_2 from oscillator 1 to 2; we also set $\epsilon_1=0$. Thus, any change detected by the proposed method would be related to the changes made in the actual coupling between the two systems. The time series $x_{1,2}(t)$ were generated for three values of ϵ_2, namely, 0.0, 0.1, and 10. We choose the time step of the integration of equation (5) to be $\Delta t=0.001$.

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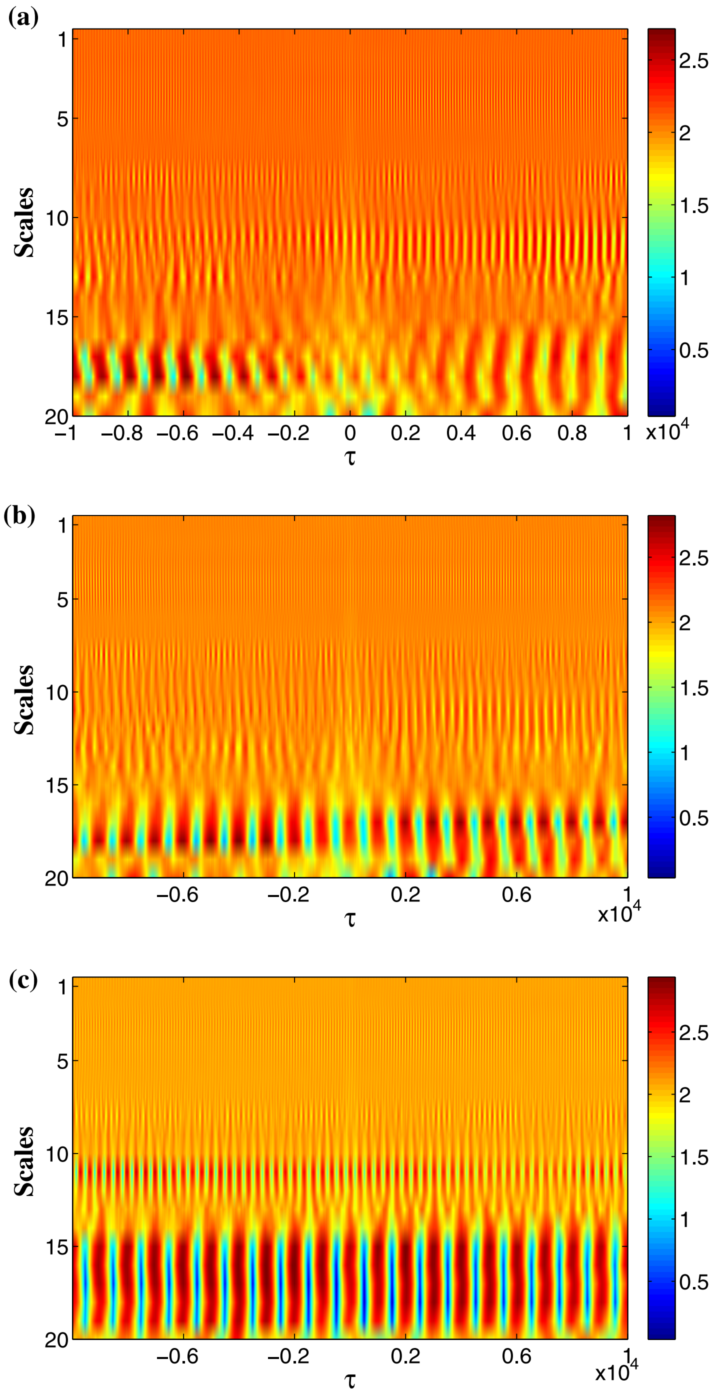

Figure 1. (a) Averaged phase difference of the WTs of the two VDP oscillators for (a) $\epsilon_2 = 0$; (b) $\epsilon_2 = 0.1$ and (c) $\epsilon_2 = 10$. The series contains 10^4 data points.

Figures 1(a)–(c), present the averaged phase difference WTs of the two time series for $\epsilon_2 = 0$, 0.1, and 10. Figures 2(a) and 2(b) present the amplitudes of the cross-correlation coefficients of the two WTs for $\epsilon_2 = 0$ and 10. It appears that for the uncoupled VDP oscillators ($\epsilon_2 = 0$) there is no order in the cross-correlations of the amplitudes $C_1(\tau)$, and that the averaged phase difference $C_2(\tau)$, with each scale behaving independently of the

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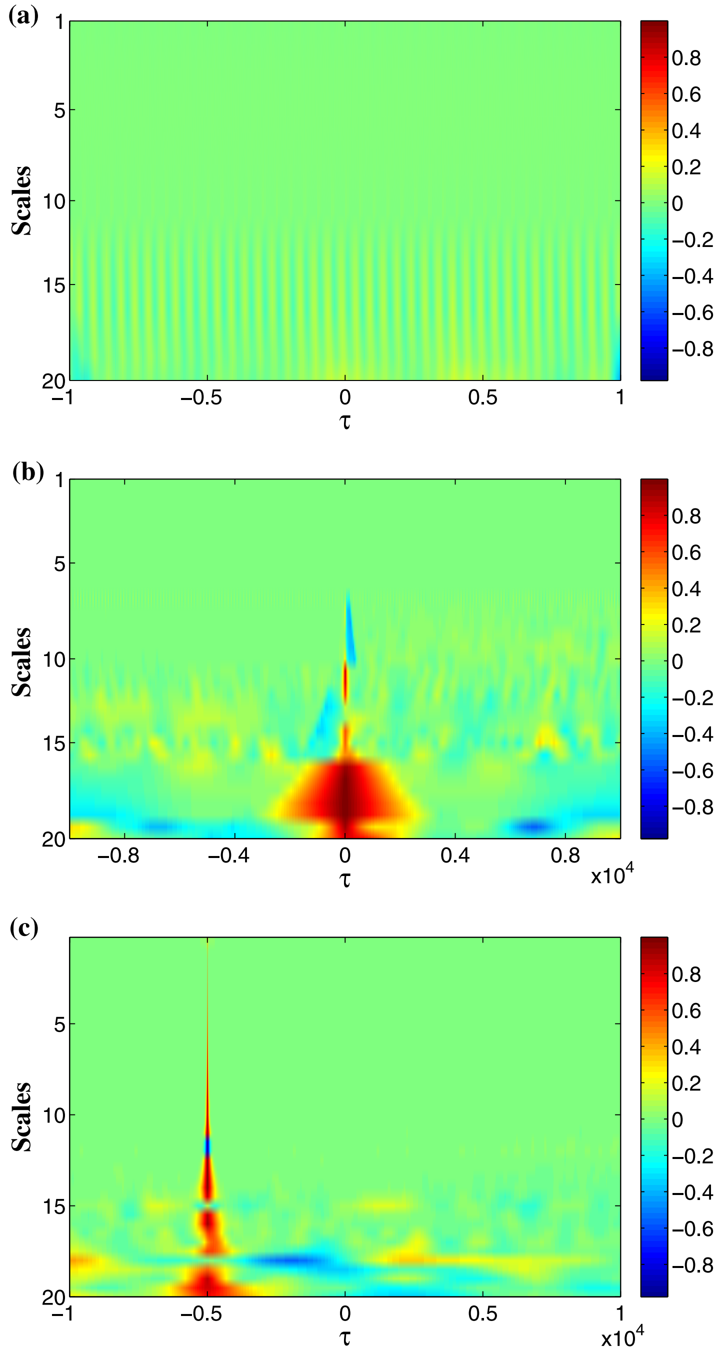

Figure 2. (a) The cross-correlation coefficients of the wavelet amplitudes of the VDP oscillators for the couplings (a) $\epsilon_2 = 0.0$ and (b) $\epsilon_2 = 10$. (c) The cross-correlation coefficients of the wavelet amplitudes of the VDP oscillators for the coupling $\epsilon_2 = 10$ and $t_0 = -5000$ data points.

others, is exactly similar to the cross-correlations of the amplitudes and phase differences of two independent white noises.

For the coupled case ($\epsilon_2 = 0.1, 10$), however, the averaged phase difference is between 0 (in phase) and π (anti-phase), and its behaviours over different scales are not independent.

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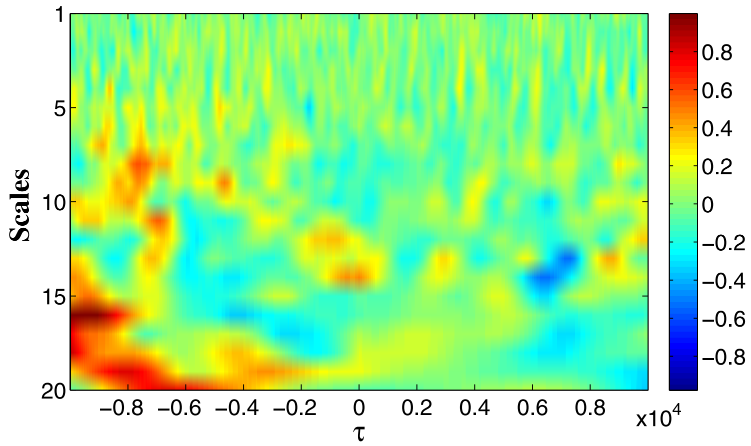

Figure 3. The cross-correlation coefficients of the wavelet amplitudes of the coupled nonlinear stochastic equations for $\epsilon = 10$ with $t_0 = -8000$ data points.

We note that due to the existence of random forcing $\eta_i(t)$, the time series $x_{1,2}(t)$ have a wide spectrum with peaks at the main frequencies ω_1 and ω_2.

In figures 2(b) and (c) the red and blue areas possess, respectively, positive and negative correlation coefficients. The amplitude of the cross-correlation coefficients indicates the coupling between the two time series, whereas the sign of the location of the maximum of the correlation coefficients determines which oscillator is ahead or follows the other. As shown in figure 2(b), the peaks in the cross-correlation coefficients are located in lag $\tau = 0$.

Next, consider the case in which all the parameters are fixed, except the time delay t_0. Thus, any change detected by the proposed method would be related to the changes made in the delay time of the two systems. The time series $x_{1,2}(t)$ were generated for $\epsilon_2 = 10$. As shown in figure 2(c) the peak of the cross-correlation has shifted to $\tau = t_0$ (we chose t_0 to be -5000 data points). Therefore, as expected, the method enables us to detect the time delay of the coupling of two oscillators.

4. Application to coupled nonlinear stochastic equations

As the second demonstration of the method we consider the coupled nonlinear stochastic equations, given by [26]

$$\mathrm{d}y_1 = -y_1\,\mathrm{d}t + \sqrt{\mathrm{d}t}\, f_1(t); \qquad \mathrm{d}y_2 = -y_2\,\mathrm{d}t - \epsilon y_1(t - t_0) y_2(t)\,\mathrm{d}t + \sqrt{\mathrm{d}t} f_2(t), \tag{6}$$

with $\langle f_i(t)\rangle = 0$, $\langle f_i(t)^2\rangle = 0.3$, and $\langle f_1(t) f_2(t')\rangle = 0$. Once again, all the parameters are fixed, except the coupling ϵ from the process 1 to 2. Thus, any change detected by the proposed method would be related to the changes made in the actual coupling between the two processes. The time series $y_{1,2}(t)$ were generated for two values of ϵ, namely, 0 and 10, with $\Delta t = 0.001$.

We find results for the phase difference of two time series that are similar to those for the VDP oscillators. Figure 3 presents the amplitudes of the cross-correlation coefficients of the WTs of the two time series for $\epsilon_2 = 10$ and $t_0 = -8000$ data points. We find that the results for the system of coupled nonlinear stochastic equations are similar to those

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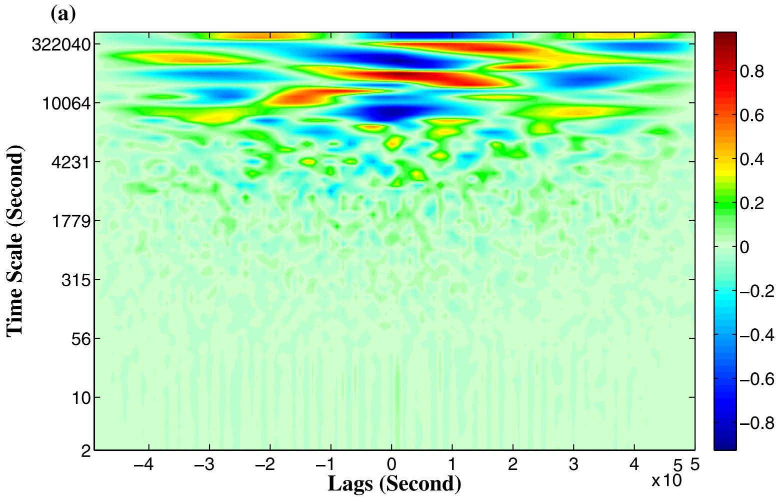

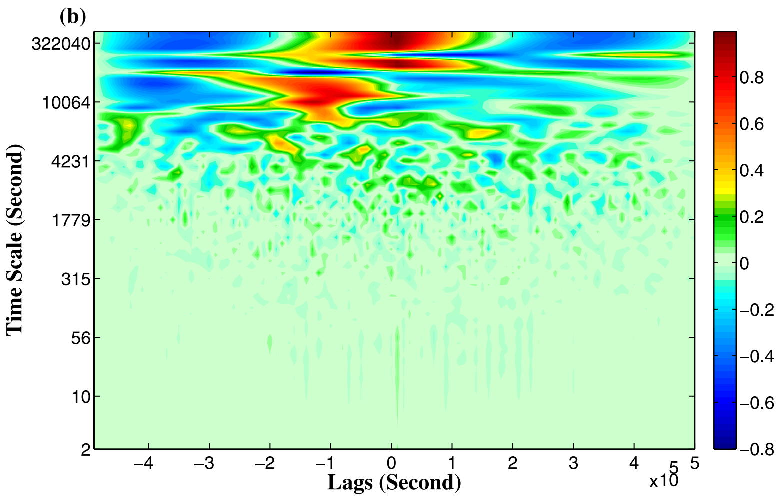

Figure 4. The cross-correlation of the wavelet amplitudes of the IBM and GE series over the timescale 2–322 340 s during (a) August 1995, and (b) December 1997. Blue and red relate, respectively, to the time and timescales for which the IBM or GE series is ahead of the other one. The colour bars show the strength of the amplitude of the wavelets' correlation. The first peak on the left side of zero with the largest amplitude for each timescale determines which series is ahead of the other one in responding to a common cause (taking into account the estimated errors of the cross-correlation coefficients).

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for the VDP forced oscillators. Indeed, a non-zero t_0 results in a shift in the position of the peaks of amplitude correlation functions from $\tau = 0$ to t_0.

5. Application to financial time series

Next, we use the method to study the log-return time series of IBM and GE, which have zero mean and are normalized (unit variance). The log-returns are defined by $A(t) = \ln(x_{t+1}/x_t)$, where x_t is the product of the price and volume of stock of A at time t. The data analysed are for 1995–2000 with high frequency sampling over 8000 h;

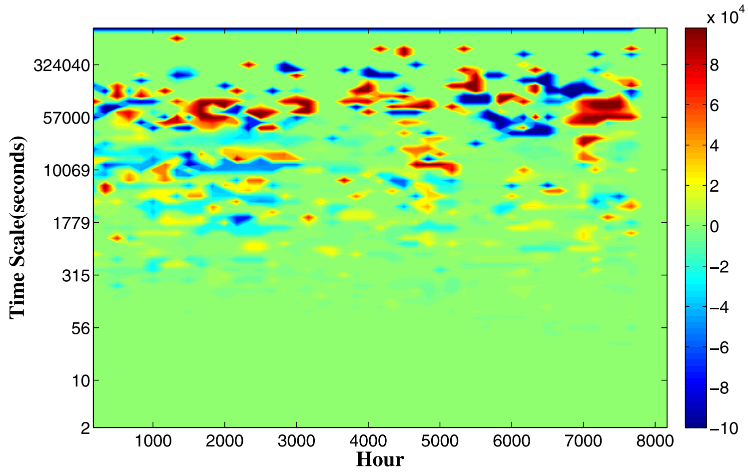

Figure 5. The location τ of the largest amplitude of the cross-correlations of the WTs for each timescale. Blue and red indicate, respectively, the times and frequencies at which the GE's or IBM's series is ahead. The colour bars are in units of seconds.

on average we have one data point for every 23 s. Between every two data points we use a linear fit to connect them. The reason is that the data belong to different clicking times and, thus, need some pre-processing before one is able to consider them as the two time series at a specific time t.

Figures 4(a) and (b) present the cross-correlations coefficients of the amplitudes of the WTs of the two time series for about one month, that contain data over about 10^6 s, during August 1995 and December 1997. The timescales are from 2 to 322 340 s. Blue and red relate, respectively, to the time and timescales for which the IBM or GE stocks are ahead of the others, i.e. respond faster to a change in the market. The averaged coefficients in each row indicate that the GE stocks are ahead of IBM's in August 1995, whereas it is the IBM stocks that are ahead of GE's in December 1997.

In figure 5 we moved along the data from 1995 to 2000, with a window size of about one month, to detect the location τ of the largest amplitude of the cross-correlations of the WTs for each timescale. Then, we investigated the averaged time lags from 1995 to 2000 for various timescales. If the averaging is done in the horizontal direction of figure 5, the results are those shown in figure 6, which indicate that the time lag τ is positive, particularly over the timescales 10^4–10^5 s, implying that it is the IBM stocks that are ahead of GE's during 1995–2000. In other words, the IBM stocks react to possible common sector price movements earlier than GE's.

The results were further checked by using the permutation information (PI) approach [27], which enables one to quantify the directionality of a coupling between interacting oscillators and to detect dynamical changes in complex time series. We obtained results similar to those described above. The statistical significance of the results was checked by generating 10^3 realizations of surrogate data from the original ones. For each realization we applied an independent and different surrogate technique, specifically designed to destroy only the cross-correlation, and preserve the structure of the data intact [27].

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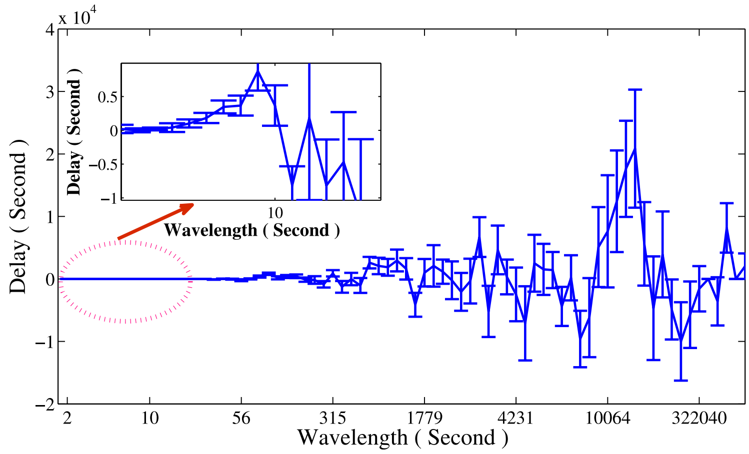

Figure 6. Averaged time lags from 1995–2000 for various timescales (frequencies). The averaging was done in the horizontal direction of figure 4. The area that has positive averaged time lag indicates that it is the IBM series that is ahead of the GE's.

6. Summary

Using the cross-correlation of the amplitudes of the WTs of two coupled time series, we proposed a general method for studying the direction of the coupling for the two series. The main advantage of our method is that it enables one to determine the *timescale* dependence of the directionality for the two series. The method was demonstrated through its application to coupled nonlinear oscillators and coupled nonlinear stochastic equations. It was also used to analyse the log-return time series of IBM and GE.

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