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Quenched averaged correlation functions of the random magnets

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Abstract

It is shown that the ratios of the quenched averaged three and four-point correlation functions of the local energy density operator to the connected ones in the random-bond Ising model approach asymptotically to some *universal* functions. We derive the explicit expressions of these universal functions. Moreover it is shown that the individual logarithmic operators have not any contribution to the connected correlation functions of the disordered Ising model.

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1. Introduction

Random systems represent the spatial inhomogenuity where scale invariance is only preserved on average but not for specific disorder realization. The understanding of the role played by quenched impurities of the nature of phase transition is one of the significant subjects in statistical physics and has attracted a great deal of attention [1]. According to the Harris criterion [2], quenched randomness is a relevant perturbation at the second-order critical point for systems of dimension d, when its specific heat exponent α, of the pure system is positive. Concerning the effect of randomness on the correlation functions, it is known that the presence of randomness induces a logarithmic factor to the correlation functions of pure system [3]. Theoretical treatment of the quenched disordered systems is a non-trivial task in view of the fact that, one has to average the logarithm of the partition function over various realization of the disorder in the statistical ensemble and therefore find physical results. There are two standard methods to perform this averaging,

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the supersymmetry (SUSY) approach, and the well-known replica approach. Recently using the replica approach it has been shown by Cardy [4], that the logarithmic factor multiplying power law behavior are to be expected in the scaling behavior near non-mean field critical points. It is shown also that the results are valid for systems with short-range interactions and in an arbitrary number of dimensions. He concludes that in the limit of $n \to 0$ of replicas the theory possess of a set of fields which are degenerate (they have the same scaling dimensions) and finds a pair of fields which form a Jordan cell structure for dilatation operator and derives logarithmic operator in such disordered systems. Cardy proves that the quenched disordered theory with $Z = 1$ can be described by logarithmic conformal field theory as well. The logarithmic conformal field theories (LCFT) are extensions of conventional conformal field theories, which have emerged in recent years in a number of interesting physical problems of condensed matter physics [5–12] string theory [13–25], and nonlinear dynamical systems [26–29]. The LCFT are characterized by the fact that their dilatation operator L_0 are not diagonalized and admit a Jordan cell structure. The non-trivial mixing between these operators leads to logarithmic singularities in their correlation functions. It has been shown [30] that the correlator of two fields in such field theories, has a logarithmic singularity.

$$\left\langle \psi(r)\psi(r')\right\rangle \sim |r-r'|^{-2\Delta_\psi} \log|r-r'| + \cdots. \tag{1}$$

In this direction we show that the quenched averaged connected correlation functions of local energy density field can be written in terms of ordinary scaling operators which can be constructed by the difference of energy operators in two different replicas. We write the connected 3- and 4-point correlation functions of energy density explicitly in terms of such ordinary operators. Furthermore we prove that the logarithmic operators have no contribution in the quenched averaged connected correlation functions of the local density operator. However, these operators play a considerable role on the disconnected ones and produce some logarithmic factors in the correlation functions. We calculate the various types of quenched averaged 3- and 4-point correlation functions of the local energy density and show that the ratios of these correlation functions to the connected ones have the specific universal asymptotic and write down these universal functions explicitly.

We consider a quenched random ferromagnet, for instance an Ising model, with random-bond disorder. Let us describe this disordered system in the continuum limit by the following Hamiltonian,

$$H = H^0 + \int J(r)E(r)\,\mathrm{d}^d r, \tag{2}$$

where H^0 is the Hamiltonian of the renormalization group at fixed point describing the pure Ising model. The field $J(r)$ is a quenched random variable coupled to the local energy density $E(r)$. When the coupling $J(r)$ is independent of x and not random, the above Hamiltonian describes the behavior of the statistical model near it's critical point. For simplicity we assume that the random variable $J(r)$ is a gaussian variable which is characterized by its two moments $\langle J(r)\rangle = 0$ and $\langle J(r)J(r')\rangle = g\delta(r-r')$. The standard procedure of averaging over disorder is to introduce replicas, i.e., n identical copies of the same model for which

$$Z^n = \operatorname{Tr}\exp\left\{-\sum_{a=1}^{n} H_a^0 - \int \mathrm{d}^d r\, J(r) \sum_{a=1}^{n} E_a(r)\right\} \tag{3}$$

averaging over the disorder gives rise to the following effective replical Hamiltonian:

$$H_R = \sum_{a=1}^{n} H_a^0 - g \int \sum_{a\neq b} E_a(r) E_b(r)\, \mathrm{d}^d r, \tag{4}$$

we keep only the non-diagonal terms, since using the operator algebra of the pure system one can absorb the diagonal terms into H_a^0. The replicas are now coupled via the disorder. The scaling dimension of coupling g is $y_g = d - 2\Delta_E$ and is relevant at the pure fixed point if $y_g > 0$. For small y_g it is possible to use standard perturbation theory and find the possible random fixed point. It is noted by Cardy that the n operator E_a are degenerate at the pure fixed point and one can decompose them into irreducible representation of permutation group S_n. It has been shown that the combination $E_t = \sum_{a=1}^{n} E_a$ is a singlet (symmetric under the permutation of the replica group) and $\tilde{E}_a = E_a - \frac{1}{n}\sum_{b=1}^{n} E_b$ transforms according to an $(n-1)$-dimensional representation of S_n. The fields $\tilde{E}_a$ satisfy the condition $\sum_{a=1}^{n} \tilde{E}_a = 0$. The important observation is that the fields E_t and $\tilde{E}_a$ have the proper scaling dimensions close to $n \to 0$ as $\Delta_{E_t} = \Delta_{E_a}^{(0)} + \frac{1}{2}(1-n) y_g + O(y_g^2)$ and $\Delta_{\tilde{E}} = \Delta_E^{(0)} + \frac{1}{2} y_g + O(y_g^2)$, respectively. It is clear that the singlet field E_t becomes degenerate with the $(n-1)$ operators $\tilde{E}_a$. This is true to all orders [4]. However, they do not form the basis of the Jordan cell for the dilatation operator. To find the logarithmic pair according to [4] we define the correlation function of $\langle E_t(0) E_t(r)\rangle = A_1$ and $\langle \tilde{E}_a(0)\tilde{E}_a(r)\rangle = B_1$ and find the following relations for A_1 and B_1.

$$\begin{aligned} A_1 &= n(a - (n-1)b) \equiv n A(n) r^{-2\Delta_E(n)}, \\ B_1 &= \left(1 - \frac{1}{n}\right)(a-b) \equiv \left(1 - \frac{1}{n}\right) B(n) r^{-2\tilde{\Delta}_E(n)}, \end{aligned} \tag{5}$$

where $a = \langle E_i(0) E_i(r)\rangle$ and $b = \langle E_i(0) E_j(r)\rangle$ with $i \neq j$. The above equations enable us to write the quenched averaged connected two-point correlation functions of energy density operator in terms of a and b in the limit of $n \to 0$ as: $\overline{\langle E(0)E(r)\rangle_c} = a - b$ which is equal to $B(0) r^{-2\Delta_E}$ and it has a pure scaling behavior. However, the correlation functions a and b have the logarithmic singularities and behave as:

$$\begin{aligned} \langle E_1(0)E_1(r)\rangle &= \left(A'(0) - B'(0) + B(0) - B(0)\frac{y_g}{2}\ln r\right) r^{-2\Delta_E}, \\ \langle E_1(0)E_2(r)\rangle &= \left(A'(0) - B'(0) - A(0)\frac{y_g}{2}\ln r\right) r^{-2\Delta_E}, \end{aligned} \tag{6}$$

where $A(0) = B(0)$. The prime sign in the Eq. (6) means differentiating with respect to n. This means that in the limit $n \to 0$ the fields E_t and E_a form a basis of Jordan cell, i.e., their two point correlation functions behave as: $\langle E_t(0)E_t(r)\rangle = 0$, $\langle E_t(0)E_a(r)\rangle = a_1 r^{-2\Delta_E}$ and $\langle E_a(0)E_b(r)\rangle = (-2a_1 \ln r + D_{a,b}) r^{-2\Delta}$, where a_1 and $D_{a,b}$ are some constants. As noted by Cardy the ratio of quenched averaged two-point correlators of the energy density operator to the connected one has a universal r-dependence as:

$$\frac{\overline{\langle E(0)E(r)\rangle}}{\overline{\langle E(0)E(r)\rangle_c}} \sim \frac{\overline{\langle E(0)\rangle\langle E(r)\rangle}}{\overline{\langle E(0)E(r)\rangle_c}} \sim \ln r. \tag{7}$$

To understand the structure of Jordan cell, we note that in 2D one can define the operator L_0 as

$$L_0 = \begin{pmatrix} \Delta_E & 0 \\ 1 & \Delta_E \end{pmatrix}, \tag{8}$$

so that $L_0 E_t = \Delta_E E_t$ and $L_0 E_a = \Delta_E E_a + E_t$ in the limit of $n \to 0$. Using this representation for L_0 one can show that the field E_t with its logarithmic partner E_a have the standard logarithmic correlation functions [31], (see the correlation above the Eq. (8)). We note that in 2D we have dealt with two-dimensional conformal field theory, relying heavily on the underlying Virasoro algebra. For an extension to D dimensions one has to modify the representation of the Virasoro algebra to higher dimensions [32]. We consider a doublet of fields (Jordan cell) $\Phi = \binom{E_t}{E_a}$ and note that under D-dimensional conformal transformation $\boldsymbol{x} \to \boldsymbol{x}'$, we have, $\Phi(\boldsymbol{x}) \to \boldsymbol{\Phi}'(\boldsymbol{x}') = G^T \boldsymbol{\Phi}(\boldsymbol{x})$ where T is a two dimensional matrix which has Jordan form and $G = \|\frac{\partial x'}{\partial x}\|$ is the Jacobian. For our particular case T has the following Jordan form:

$$T = \begin{pmatrix} -\dfrac{2\Delta_E}{D} & 0 \\ 1 & -\dfrac{2\Delta_E}{D} \end{pmatrix}, \tag{9}$$

and one can show that the two fields E_t and E_a, transform as:

$$\begin{aligned} E_t(\boldsymbol{x}') &= G^{-\frac{2\Delta_E}{D}} E_t(\boldsymbol{x}), \\ E_a(\boldsymbol{x}') &= G^{-\frac{2\Delta_E}{D}} \big(\ln(G) E_t(x) + E_a(x)\big). \end{aligned} \tag{10}$$

This expresses that the top-field E_t always transforms as an ordinary scaling operator. It can be verified that the correlation functions of fields E_t and E_a have the standard D-dimensional logarithmic conformal field theory structure [31,32]. Using the above results, it is evident that the dimension of field-difference $E_a - E_b$ with $a \neq b$ is Δ_E and it transforms as an ordinary operator under the scaling transformation. The interesting observation is that the connected averaged correlation functions depends on the difference fields $E_a - E_b$ only and therefore they behave as the ordinary correlation functions. For instance, in the following we write the connected quenched averaged 2-, 3- and 4-point functions of local energy density in terms of the field-difference operators explicitly

$$\overline{\langle E(1)E(2)\rangle}_c = \frac{1}{2}\big\langle (E_a - E_b)_{(1)}(E_a - E_b)_{(2)}\big\rangle, \tag{11}$$

$$\overline{\langle E(1)E(2)E(3)\rangle}_c = \big\langle (E_a - E_b)_{(1)}(E_a - E_c)_{(2)}(E_a - E_b)_{(3)}\big\rangle, \tag{12}$$

$$\begin{aligned} &\overline{\langle E(1)E(2)E(3)E(4)\rangle}_c \\ &\quad = \big\langle (E_a - E_b)_{(1)}(E_a - E_c)_{(2)}(E_a - E_d)_{(3)}(E_a - E_b)_{(4)}\big\rangle \\ &\qquad - \frac{1}{2}\big\langle (E_a - E_b)_{(1)}(E_c - E_d)_{(2)}(E_c - E_d)_{(3)}(E_a - E_b)_{(4)}\big\rangle \end{aligned}$$

$$-\frac{1}{4}\big\langle (E_a - E_b)_{(1)}(E_c - E_d)_{(2)}(E_a - E_b)_{(3)}(E_c - E_d)_{(4)}\big\rangle$$
$$-\frac{1}{4}\big\langle (E_a - E_b)_{(1)}(E_a - E_b)_{(2)}(E_c - E_d)_{(3)}(E_c - E_d)_{(4)}\big\rangle,$$

where the last equation has only 15 independent terms.

To confirm this prediction also one can directly show that the quenched averaged connected correlation functions have a pure scaling behaviour which is determined by ordinary scaling operators and the logarithmic operators E_a do not change its behavior. This can be verified directly for the quenched averaged connected 3-point correlation function of energy density.

We are interested in deriving exactly the various 3-point quenched averaged functions as $\overline{\langle E(1)E(2)E(3)\rangle}$, $\overline{\langle E(1)E(2)\rangle\langle E(3)\rangle}$ and $\overline{\langle E(1)\rangle\langle E(2)\rangle\langle E(3)\rangle}$, which can be written in terms of the replica correlation functions $\langle E_1(1)E_1(2)E_1(3)\rangle = a$ $\langle E_1(1)E_1(2)E_2(3)\rangle = b$ and $\langle E_1(1)E_2(2)E_3(3)\rangle = c$, respectively. One can derive the correlation functions a, b and c by means of 3-point functions of E_t and $\tilde{E}_a$ as follows:

$$\big\langle E_t(1)E_t(2)E_t(3)\big\rangle = na + 3n(n-1)b, \quad n(n-1)(n-2)c \equiv nA_1, \tag{13}$$

$$\begin{aligned}\big\langle \tilde{E}_a(1)\tilde{E}_a(2)E_t(3)\big\rangle &= n_1 a + \left(n_1^2(n-1) - 4n_1^2 + \frac{1}{n^2}(n-1)^2 + \frac{2}{n^2}(n-1)(n-2)\right)b \\ &\quad + \left(-\frac{2}{n}n_1(n-1)(n-2) + \frac{1}{n^2}(n-2)^2(n-1)\right)c \\ &\equiv \left(1-\frac{1}{n}\right)B_1,\end{aligned} \tag{14}$$

and finally,

$$\begin{aligned}\big\langle \tilde{E}_a(1)\tilde{E}_a(2)\tilde{E}_a(3)\big\rangle &= \left(n_1^2 - \frac{n-1}{n^3}\right)a \\ &\quad + \left(-3n_1^2\frac{n-1}{n} - \frac{3}{n^3}(n-1)(n-2) + \frac{3}{n^2}n_1(n-1)\right)b \\ &\quad + \left(\frac{3}{n^2}n_1(n-1)(n-2) - \frac{1}{n^3}(n-1)(n-2)(n-3)\right)c \\ &\equiv \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)C_1,\end{aligned} \tag{15}$$

where $n_1 = (1 - 1/n)$ and A_1, B_1 and C_1 are pure scaling functions of variables $r_{i,j}$. To derive the above equations we use the replica symmetry and symmetry of the various types of 3-point correlation functions under interchanging of positions. We note that replica symmetry leads to have $\langle \tilde{E}_a(1)E_t(2)E_t(3)\rangle = 0$ and, therefore, dose not give any new relationship between a, b and c. Using the above equations, it can be found that the correlation functions a, b and c are as follows:

$$a = \frac{3nB_1 - 3nC_1 + n^2C_1 + A_1 - 3B_1 + 2C_1}{n^2},$$

$$b = \frac{nB_1 - nC_1 + A_1 - 3B_1 + 2C_1}{n^2},$$
$$c = \frac{A_1 - 3B_1 + 2C_1}{n^2}. \tag{16}$$

Using the above equations we can show that the connected quenched averaged 3-point function behaves as:

$$\overline{\langle E(1)E(2)E(3)\rangle} = 2c + a - 3b = C_1, \tag{17}$$

which is a scaling function and confirms the observation that the logarithmic operators (individually) have no role in the connected quenched averaged correlation functions. In addition one can derive the correlation functions $\langle E_i(1)E_j(2)E_k(3)\rangle$ for given $i,\ j$ and k in the limit of $n \to 0$ and show that they have the following form:

$$\langle E_i(1)E_j(2)E_k(3)\rangle = \big[\alpha_{ijk} - \beta_{ijk}D_1 + \gamma_{ijk}\big(4D_2 - D_1^2\big)\big] f(1,2,3), \tag{18}$$

where

$$f(1,2,3) = (r_{12}r_{13}r_{23})^{-2\Delta_E},$$
$$D_1 = \ln(r_{12}r_{13}r_{23}), \qquad D_2 = \ln r_{23}\ln r_{13} + \ln r_{13}\ln r_{12} + \ln r_{23}\ln r_{12}.$$

It can also be shown that the ratio of various symmetrized 3-point functions to the connected one behaves asymptotically as a *universal* function

$$\frac{1}{3}\big(4D_2 - D_1^2\big). \tag{19}$$

We generalize the above calculations to derive the various type of 4-point correlation functions and show that the ratio of the various disconnected to the connected one have the following universal asymptotic:

$$\sim \frac{1}{36}\big[O_1^3 - 6O_2 - 3O_3 - 12O_4 - 18O_5\big], \tag{20}$$

where $O_1 = \ln(r_{12}r_{13}r_{14}r_{23}r_{24}r_{34}),\ O_2 = (\ln r_{ij}\ln r_{kl}^2 + \cdots)$ with $i \neq j \neq k \neq l,\ O_3 = (\ln r_{ij}\ln r_{ik}^2 + \cdots)$ with $i \neq j \neq k,\ O_4 = (\ln r_{ij}\ln r_{kl}\ln r_{lj} + \cdots)$ with $i \neq j \neq k \neq l$, and finally $O_5 = (\ln r_{ij}\ln r_{ik}\ln r_{il} + \cdots)$ with $i \neq j \neq k \neq l$.

In summary, in this paper we have studied the correlation functions of disordered random magnets [33] and obtain the various types of 3- and 4-point quenched averaged correlation functions. One can check directly that these different types of the 3- and 4-point correlation functions have the general property of a logarithmic conformal field theory that the logarithmic partner can be regarded as the formal derivative of the ordinary fields (top field) with respect to their conformal weight [31]. In this case, one can consider the E_a fields as the derivative of E_t with respect to n. We emphasise that the derivative with respect to scaling weight can be written in terms of the derivative with respect to n. These properties enable us to calculate any N-point correlation function containing the logarithmic field E_a in terms of the correlation functions of the top-fields. The general expression of the correlation functions of the LCFT's are given in Ref. [31] and here we determine the unknown constants in the logarithmic correlation functions in terms of details of the random-bond Ising model. It is noted that the formal derivations with respect to scaling dimensions can not predict the unknown constants in the quenched averaged correlation functions of the local energy density operators. The constant depends on the detail of the statistical model. We have shown that the individual logarithmic operators E_a do not have any contribution to the quenched averaged connected correlation functions of the energy density. We also obtain that the connected correlation functions can be written in terms of the difference fields which transform as an ordinary scaling operator. However, they will play a crucial role to the disconnected averaged correlation functions. Also we find that the ratio of the various types of 3- and 4-point quenched averaged correlation functions to the connected ones have a universal asymptotic behavior and give their explicit form. These predictions can also be investigated numerically. Our analysis are valid in all dimensions as long as the dimension is below the upper critical dimensions. To derive the above results we have used the replica symmetry. Any attempt towards the breaking of this symmetry will change completely the above picture and produces more than one logarithmic fields in the block and produces higher order logarithmic singularities [31].

These results can be easily generalized to other problem such as polymer statistics, percolation and random phase sine-Gordon model, etc.

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