

PAPER: CLASSICAL STATISTICAL MECHANICS, EQUILIBRIUM AND NON-EQUILIBRIUM

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PAPER: Classical statistical mechanics, equilibrium and non-equilibrium

Jump events in the human heartbeat interval fluctuations

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Abstract. Jumps are discontinuous variations in time series and one expects that the higher jump activity will cause higher uncertainty in the stochastic behavior of measured time series. Here we study jump events in beat-to-beat fluctuations in the heart rates of healthy subjects, as well as those with congestive heart failure (CHF). The analysis shows that the interbeat time series belong to the class of non-continuous stochastic processes. The estimated drift and diffusion coefficients and jump characteristics of healthy and CHF subjects reveal the distinguishability of two subjects.

Keywords: nonlinear dynamics, stochastic processes

³ Authors contributed equally to this work.

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1. Introduction

Physiological data and time series are generated by complex self-regulating systems and seem to be highly stochastic, represent nonstationary data, and fluctuate in an irregular and complex manner [1–14]. Among physiological time series, the study of the statistical properties of heartbeat interval sequences has attracted much recent attention. Extensive analysis of interbeat interval variability has been carried out, as it represents an important quantity for elucidating the possibly nonhomeostatic physiological variability. In the conventional approaches to analyzing such data, the fluctuations are usually ignored because it is assumed that there is no meaningful structure in the apparent noise, so these studies focus on *averaged* quantities. Nowadays it is known that the effect of such fast dynamics can often be treated as dynamical fluctuations which is substantial when one aims to study the given complex beat-to-beat time series [12]. Such stochastic time series has many aspects to be studied due to their intrinsic complexities. Linear and non-linear correlations, fractal and multi-fractality and continuity are just a few examples of these aspects that might be related to each other by mathematical theorems [13, 14].

One particular model to analyze such processes was proposed by Stanley and co-workers which is fractal Brownian motion (FBM). FBM is a non-stationary stochastic process which induces long-range correlations, the successive increments of which are, however, stationary and follow a Gaussian distribution [4]. The power spectrum of FBM is given by $S(f) \propto f^{-(2H+1)}$ where H is Hurst exponent and they could distinguish healthy and congestive heart failures (CHF) by the value of H . Healthy data has Hurst exponent $H < 1/2$ and CHF has $H > 1/2$, but for the values of H near to $1/2$ there are ambiguities, so one needs a more efficient method [4–12, 15]. Another approach have been used by Ivanov *et al* [7]. They were able distinguish the healthy and CHF ones with multifractal detrended fluctuation analysis [16]. Bogachev *et al*

[17, 18], studied the statistics of return intervals between large heartbeat intervals of their multiplicative random cascade model above a certain threshold in order to obtain the probability density function of return interval, and *a priori* information about the occurrence of the next large heartbeat interval. In addition, Witt *et al* [19] have proposed a peaks over the threshold model, where high values of the superposition of a fractional noise and a white noise model the temporal occurrence of premature ventricular contractions.

It is known that the Kramers–Moyal (KM) expansion for the probability density of given stochastic process can reduce to the Fokker–Planck equation if higher-order (>2) KM coefficients vanish [20]. There are, however, many physical experiments that indicate non-vanishing higher-order KM coefficients [12, 21]. Nowadays, the non-vanishing higher-order KM coefficients are related to jump events in given time series [14]. In practice jump events can participate in the observed non-Gaussian feature of increments (ramp up and ramp down) statistics of many time series [22]. This is the reason that most of the jump detection techniques are based on threshold values for the differential of time series, i.e. $y = dx/dt$ (it is standard to consider the events with $|y| > 6\sigma_y$ as jumps).

Recently, it has been demonstrated that a finite sampling interval not only influences the first- and second order KM coefficients but also causes non-vanishing higher-order ones [21]. Using information about these higher-order KM conditional moments, we utilize a novel criterion (as a necessary condition) to check whether, the underlying process has a continuous or discontinuous trajectory [21]. Also this novel approach is able to detect jump events in the time series.

This paper is organized as follows. In section 2, we describe Kramers–Moyal conditional moments of conventional Langevin equation with finite sampling interval dt . In section 3, we provide a brief review of a non-parametric approach to estimate drift, diffusion and jump characteristics of given time series. In section 4, we assert that the cardiac inter-beat intervals time series have jump contributions, and with estimating their stochastic characteristics, one can distinguish the healthy subjects from those with congestive heart failures. At the end, we summarize our studies in the conclusion part.

2. Continuous stochastic processes

Continuous stochastic processes are commonly modeled by conventional Langevin equation,

$$dx(t) = D^{(1)}(x, t)dt + \sqrt{2D^{(2)}(x, t)}dW(t). \quad (1)$$

Here $D^{(1)}(x, t)$ and $D^{(2)}(x, t)$ are the first and second Kramers–Moyal coefficients which are known as drift and diffusion coefficients, respectively and $W(t)$ is a scalar Wiener process. The time series generated by equation (1) is a continuous diffusion process [14]. The Kramers–Moyal coefficients are related to the conditional moments for infinitesimal dt as follow

$$\begin{aligned} \langle (x(t + dt) - x(t))^1 |_{x(t)=x} \rangle &= D^{(1)}(x, t)dt \\ \langle (x(t + dt) - x(t))^2 |_{x(t)=x} \rangle &= 2D^{(2)}(x, t)dt \\ \langle (x(t + dt) - x(t))^{2+s} |_{x(t)=x} \rangle &= 0 \end{aligned} \tag{2}$$

where $s > 0$. Here, the KM coefficients are given by $D^{(m)}(x, t) = M^{(m)}(x, t)/m! = \lim_{dt \rightarrow 0} \frac{1}{m!dt} K^{(m)}(x, t)$ with the conditional moments defined as

$$K^{(m)}(x, t) = \langle (x(t + \tau) - x(t))^m |_{x(t)=x} \rangle \tag{3}$$

The probability density function $p(x, t)$ of given Markov process satisfies a partial differential Kramers–Moyal equation, which is the infinite order in the state variable x and of first order in time t . According to the Pawula theorem, the Kramers–Moyal expansion could be truncated after the second term if the fourth-order coefficient $D^{(4)}(x, t)$ vanishes. In this case, the time series will be statistically continuous.

To derive the equation (2), we assumed that the time series is continuous, even though it is already sampled in discrete times due to the limited sampling frequency, which in turn will cause non-vanishing $D^{(4)}(x, t)$. So we should take care of whether non-vanishing $D^{(4)}(x, t)$ is related to non-continuous behavior in the time series or just is an artifact of discrete sampling. The finiteness of sampling rate force us to derive the correction terms in the equation (2). One finds the following finite sampling expansions of the KM conditional moments as [21];

$$\begin{aligned} K_d^{(2)}(x, dt) &= b^2 dt + \frac{1}{2} [2a(a + bb') + b^2(2a' + b^2 + bb'')] dt^2 + \mathcal{O}(dt)^3, \\ K_d^{(4)}(x, dt) &= 3b^4 dt^2 + \mathcal{O}(dt)^3, \\ K_d^{(6)}(x, dt) &= 15b^6 dt^3 + \mathcal{O}(dt)^4 \end{aligned} \tag{4}$$

where $D^{(1)}(x, t) = a(x, t)$ and $D^{(2)}(x, t) = b^2(x, t)/2$. Here we omit the x - and t -dependence of a and b to enhance readability. In general, checking the Pawula theorem for given empirical data is not an easy task. With the second- and fourth-order conditional moments for small dt one finds $K^{(4)}(x)/3(K^{(2)}(x))^2$, which confirms Wick’s theorem [20, 21] and follows from the fact that the short-time propagator of the Langevin dynamics (equation (1)) is a Gaussian distribution.

3. Non-parametric jump-diffusion modeling

According to the Pawula theorem, if a process has non-vanishing $D^{(4)}(x, t) \neq 0$ (or the criterion $K^{(4)}(x)/3(K^{(2)}(x))^2$ does not fulfil [21]), the higher order KM coefficients will not vanish. Therefore the continuous diffusion Langevin equation will not be appropriate to model such systems. For such time series jumps will play important role [14].

A typical jump-diffusion dynamics can be defined as [14];

$$dx(t) = D^{(1)}(x, t)dt + \sqrt{D^{(2)}(x, t)}d\omega(t) + \xi dJ(t) \tag{5}$$

where $J(t)$ is a time-homogeneous Poisson jump process. Also $\{W(t), t \geq 0\}$ is a scalar Wiener process, $D^{(1)}(x, t)$ the drift function and $D^{(2)}(x, t)$ the diffusion function. The jump has rate $\lambda(x)$, can be state dependent and size ξ which we assume it has zero mean gaussian distribution with variance $\sigma_\xi^2(x)$, which is known as jump amplitude.

For infinitesimal dt , it is shown recently [14] that;

$$\begin{aligned} K_j^{(1)}(x, dt) &= D^{(1)}(x, t)dt, \\ K_j^{(2)}(x, dt) &= [D^{(2)}(x, t) + \langle \xi^2 \rangle \lambda(x)] dt, \\ K_j^{(2m)}(x, dt) &= \langle \xi^{2m} \rangle \lambda(x)dt, \text{ for } 2m > 2, \end{aligned} \tag{6}$$

the subscript ‘j’ denotes jumpy process. Using the relation $\langle \xi^{2m} \rangle = \frac{(2m)!}{2^m(m!)^2} \langle \xi^2 \rangle^m$, for the Gaussian random variable ξ in equation (6), the jump rate $\lambda(x, t)$ and jump amplitude σ_ξ^2 can be written in terms of KM coefficients as:

$$\lambda(x, t) = \frac{M^{(4)}(x, t)}{3\sigma_\xi^4(x, t)} \tag{7}$$

$$\sigma_\xi^2(x, t) = \frac{M^{(6)}(x, t)}{5M^{(4)}(x, t)}. \tag{8}$$

Once the jump components $\sigma_\xi^2(x, t)$ and $\lambda(x, t)$ are identified, the second moment $M^{(2)}(x, t)$ identifies the diffusion function $D^{(2)}(x, t)$ and the first moment gives us the estimate for the drift function $D^{(1)}(x, t)$.

4. Result

4.1. Reconstruction of Ornstein–Uhlenbeck process with jumps

To demonstrate the ability of our approach, we estimate drift, diffusion, and jump characteristics from time series of well-known jump-diffusion processes (Ornstein–Uhlenbeck process in the presence of jump) with preset coefficients. We consider a linear drift $D^{(1)}(x, t) = -10x$, constant diffusion coefficient $D^{(2)}(x, t) = 1$, with jump rate $\lambda = 0.6$ and jump amplitude $\sigma_\xi^2 = 1$ in equation (5) and then using the equations (6)–(8), we reconstruct these functions with Gaussian kernel estimator. For all estimated functions and coefficients, we obtain a very good agreement with theory (figure 1). In figure 2 the state dependence of $K^{(4)}(x)$ and $3(K^{(2)}(x))^2$ for jump rates $\lambda = 0$ and $\lambda = 0.6$ are plotted. When $K^{(4)}(x)$ equals to $3(K^{(2)}(x))^2$, would Langevin equation proper for modeling. For jump-diffusion processes the fraction $K^{(4)}(x)/3(K^{(2)}(x))^2$ can be larger than unity [21].

4.2. Detection of non-diffusive behaviour in time series

In order to distinguish that if the underlying process is jumpy or diffusive, a novel criterion Q is driven as follows [21]:

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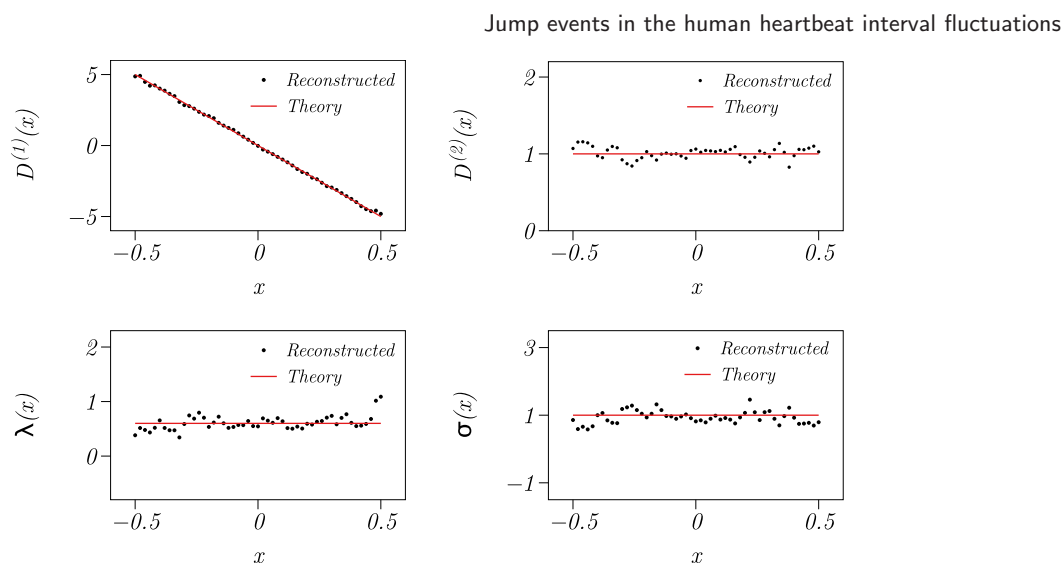


Figure 1. Reconstruction of Ornstein-Uhlenbeck process with jump. The numerical integration has been done with $dt = 10^{-3}$ with 10^7 data points for $\lambda = 0.6$, $\sigma_\xi^2 = 1$.

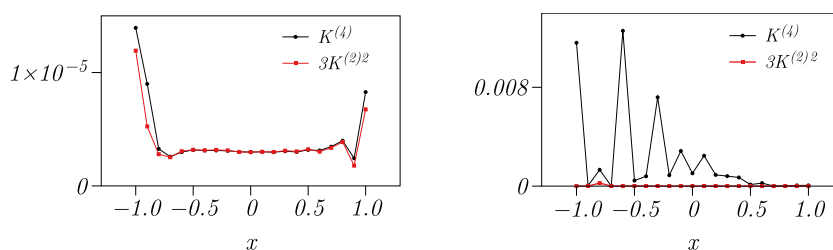


Figure 2. Checking the criterion $K^{(4)}(x) \simeq 3(K^{(2)}(x))^2$ for diffusive (left panel), for instance Ornstein-Uhlenbeck process without jump and jumpy (right panel) processes.

$$Q(x) = \frac{K_{\bullet}^{(6)}(x, dt)}{5K_{\bullet}^{(4)}(x, dt)} \approx \begin{cases} b^2(x)dt, & \text{diffusive } (\bullet = 'd') \\ \sigma_\xi^2(x), & \text{jumpy } (\bullet = 'j'). \end{cases} \quad (9)$$

Here the subscript \bullet is a placeholder for either ‘ d ’ or ‘ j ’. For a jumpy process, $Q(x)$ has the value $\sigma_\xi^2(x)$ which does not have dt -dependence. Also for a diffusive process, $Q(x)$ exhibits a vanishing behavior with decreasing time interval dt . We check the values of $Q(x)$ and its dependence on dt for Ornstein-Uhlenbeck process with and without jumps in (figure 3).

4.3. Non-diffusive behaviour in inter-beat intervals time series

Ghasemi *et al* construct Langevin dynamics for analyzing cardiac inter-beat intervals [23]. They distinguished the healthy subjects from those with CHF using estimated drift and diffusion coefficients. We take a closer look at heartbeat data which they used, and show that the data have non-diffusive characteristics and one needs to model it with jump-diffusion dynamics.

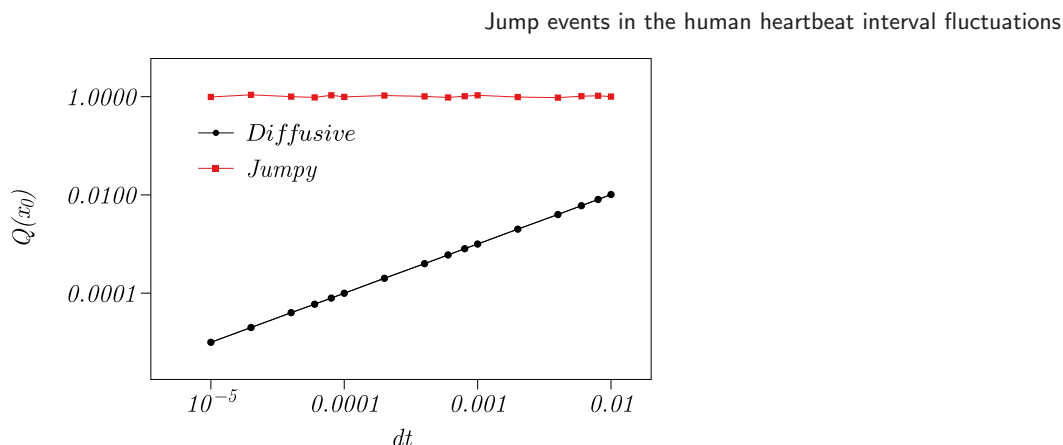


Figure 3. Dependence of $Q(x_0)$ on time interval dt exemplary time series of Ornstein–Uhlenbeck process with and without jumps with jump amplitude $\sigma_\xi^2 = 1$. We chose x_0 around the mean of the respective time series. Lines are for eye guidance only. We have chosen $dt \in \{10^{-5}, 2 \times 10^{-5}, 4 \times 10^{-5}, 6 \times 10^{-5}, \dots, 10^{-2}\}$, in integration of corresponding stochastic equations in Euler–Maruyama scheme.

The data we analyze here were part of previous studies [23]. The number of data points is of the order of 30 000, taken over a period of about six hours which have been started on the morning, and the subjects are awake and at rest. The database includes ten healthy subjects (seven females and three males with ages between 20 and 50, and an average age of 34.3 years), and 12 subjects with CHF (three females and nine males with ages between 22 and 71, and an average age of 60.8 years). The data is available at www.physionet.org/challenge/chaos/. Two typical inter-beat time series of healthy and CHF subjects are shown in figure 4.

First of all, to check whether inter-beat time series are diffusive, we calculate the value of $K^{(4)}(x)/3(K^{(2)}(x))^2$ for healthy and CHF subjects and the results are shown in figure 5 for zero mean time series. As it is clear, by considering the error bars, $K^{(4)}(x)$ is not equal to $3(K^{(2)}(x))^2$ which demonstrates that $D^{(4)}(x) \neq 0$, and therefore we can rule-out the diffusive nature of the studied time series. However this cannot say that whether the time series is jumpy or not. Therefore, we use Q -criterion as it was introduced in equation (9).

Before going into details, we briefly mention that in the case of empirically derived time series that were sampled with a fixed sampling interval Δ , equation (9) can be verified by scaling the time interval as $dt = \alpha\Delta$, where $\alpha = 1, 2, 3, \dots$ (i.e. by considering data points $\{x(0), x(\alpha\Delta), \dots\}$ only). Checking the dependence of $Q(x)$ on $dt = \alpha\Delta$ then indicates a possibly diffusive or jumpy behavior [21]. For coarse scales ($\alpha \gg 1$), we expect $Q(x)$ to take on non-vanishing values, given that data discretized at such scales appears as a succession of discontinuous jumps, even if the underlying trajectory is continuous. For small scales ($\alpha = \mathcal{O}(1)$) and diffusive processes, $Q(x)$ approaches zero since the Brownian-type (Wiener-type) behavior of the process produces a continuous trajectory [21]. Thus, the small-scale behavior of $Q(x)$ for $\alpha = \mathcal{O}(1)$ is an indicator for rapid changes or jumps in a given time series.

In figure 6, $Q(x)$ versus coarse scale α for healthy subjects and CHF ones are shown and we can clearly see the $Q(x)$ is not approaching zero when $\alpha \rightarrow 0$, which represents that time series is inherently jumpy. So, modeling inter-beat fluctuation of heart rate with jump-diffusion equation is authentic in this case.

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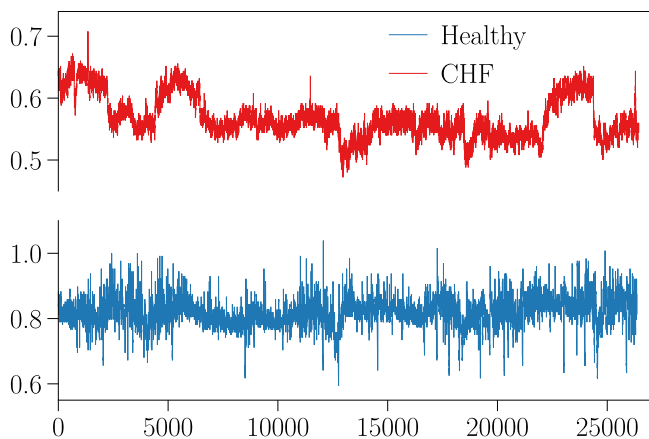


Figure 4. Segments of inter-beat time series of healthy and CHF subjects. The x -axis is the number of data points of beat-to-beat data.

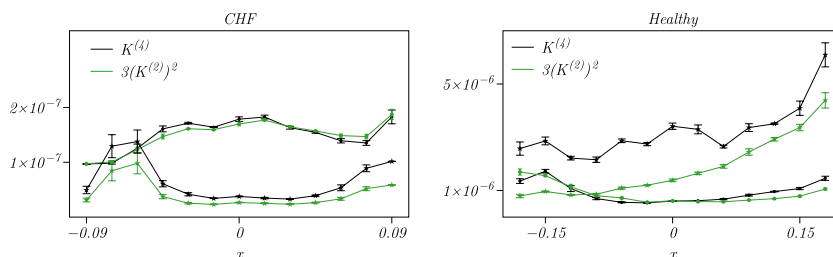


Figure 5. Checking the Pawula theorem with criterion $K^{(4)}(x) \simeq 3(K^{(2)}(x))^2$. The plots show that interbeat time series of (two) healthy and (two) CHF subjects are not belong to the class of continuous stochastic processes.

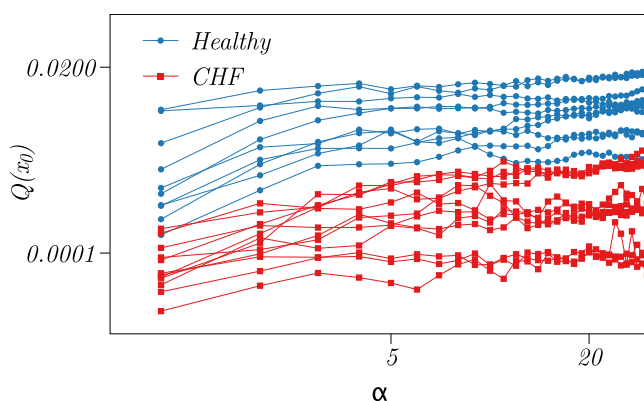


Figure 6. Dependent of $Q(x_0)$ on time interval α for inter-beat time series of healthy and CHF subjects. We chose x_0 around the mean of the respective time series. Lines are for eye guidance only. The zero limit of α show that $Q(x_0)$'s approaches to constants.

4.4. Jump-diffusion modeling of inter-beat intervals time series

Now using the relations equations (6)–(8), we estimate the drift and diffusion coefficients, as well as jump rate and jump amplitude, in a range $\pm 2\sigma$ for the interbeat time series,

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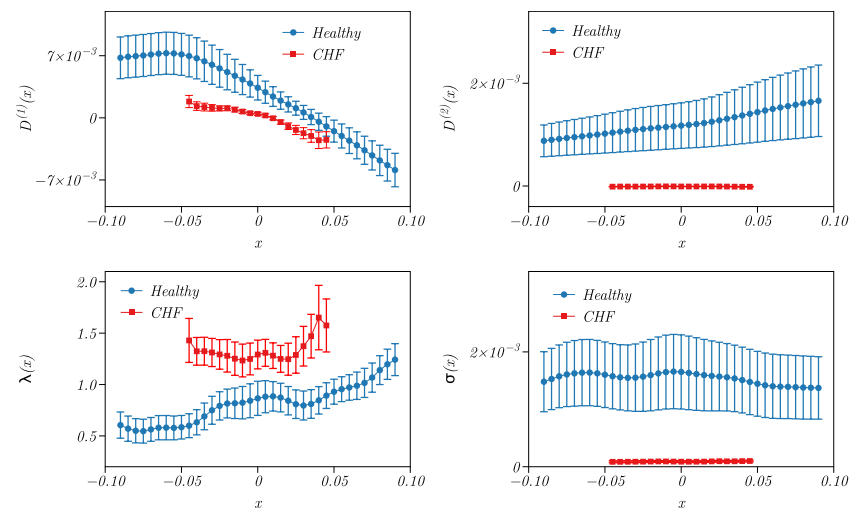


Figure 7. Estimated drift and diffusion coefficients as well as jump rate and jump amplitude for inter-beat time series of healthy and CHF subjects, reveal the distinguishability of two subjects.

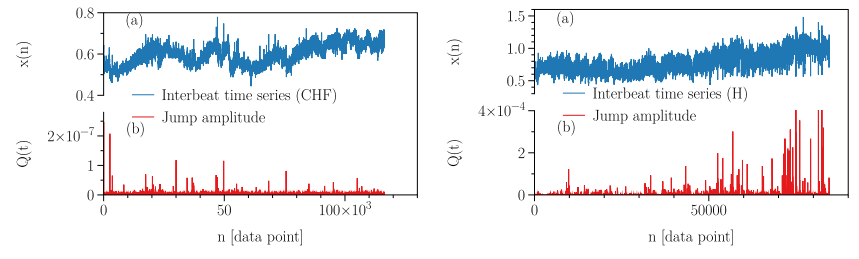


Figure 8. Time-resolved estimates of $Q(x(t)) \equiv Q(t)$ for the time series for the time series of a healthy (right) and a CHF subject (left).

and standard errors are shown in figure 7. As shown in the figure 7, the estimated drift coefficient $D^{(1)}(x)$, $D^{(2)}(x)$ and jump characterises, i.e. jump rate λ and jump amplitude σ_{ξ}^2 reveal the distinguishability of two subjects. The inverse of the slope of the linear portion of drift coefficient estimates the correlation time scale, so by comparing the drift coefficient between these two groups, the slope of the $D^{(1)}(x)$ for healthy group is greater than CHF for the subjects which means they have shorter correlation time scales. The higher values of jump amplitude for healthy subjects show that they have stronger jumps than the ones with congestive heart failure. The higher values of healthy ones' error bars show that we have a wide variety of healthy people, however, the heart disease causes their distribution to collapse in a narrow range.

5. Conclusion and discussion

Many empirical time series exhibit fluctuations that are interrupted by jumps in very short time between different states of a system. Examples include dynamics of stochastic resonance, fluctuations of wind and solar power systems, transitions in financial and

climate data, eye movements, or movement and foraging paths of animals, see [12, 21] and references therein. In this paper we provide evidence that inter-beat time series of healthy and CHF subjects are not continuous stochastic processes and then extract the functions and parameters in jump-diffusion modeling. The estimated quantities reveal the distinguishability of two subjects.

We note that the inter-beat fluctuation time series belong to the non-stationary processes [24], and their stochastic properties will depend on time. Using a kernel estimation of Kramers–Moyal coefficients, one can find the local stochastic properties of given time series [21]. In figure 8 time-resolved $Q(x(t))$ for ~ 24 h inter-beat time series of healthy and CHF are given, which show high variability of jump properties during different hours of day. The temporal evolution of $Q(x(t))$ provides strong evidence for an intermittent switching between diffusive and jumpy behavior [14, 21, 25].

We finally note that most of the previous works are model-based [17–19], while our proposed approach is a time-series-based approach and all the functions of the modeling can be found directly from data, therefore is applicable in analysing large class of complex time series. Modeling distribution of inter-beat time series with q-statistics will be one of the next steps in study of such time series [26–29].

References

- [1] Tabar M R R *et al* 2006 New computational approaches to the analysis of interbeat intervals in human subjects *Comput. Sci. Eng.* **8** 54
- [2] Zhilin Qu *et al* 2014 Nonlinear and stochastic dynamics in the heart *Phys. Rep.* **543** 61
- [3] Valenza G, Citi L and Barbieri R 2014 Estimation of instantaneous complex dynamics through Lyapunov exponents: a study on heartbeat dynamics *PLoS One* **9** 105, 622
- [4] Peng C-K, Mietus J, Hausdorff J M, Havlin S, EStanley H and Goldberger A L 1993 *Phys. Rev. Lett.* **70** 1343
- [5] Bernaola-Galvan P, Ivanov P Ch, Amaral L N and Stanley H E 2001 *Phys. Rev. Lett.* **87** 168105
- [6] Ashkenazy Y, Ivanov P, Havlin S, Peng C-K, Goldberger A L and Stanley H E 2001 *Phys. Rev. Lett.* **86** 1900
- [7] Ivanov P Ch, Bunde A, Amaral L A N, Havlin S, Fritsch-Yelle J, Baevisky R M, Stanley H E and Goldberger A L 1999 *Europhys. Lett.* **48** 594
- [8] Ivanov P Ch, Amaral L A N, Goldberger A L, Havlin S, Rosenblum M G, Struzik Z and Stanley H E 1999 *Nature* **399** 461
- [9] Peng C-K, Havlin S, Stanley H E and Goldberger A L 1995 *Chaos* **5** 82
- [10] Peng C-K, Buldyrev S V, Havlin S, Simons M, Stanley H E and Goldberger A L 1994 *Phys. Rev. E* **49** 1685
- [11] Tabar M R R 2019 *Analysis and Data-Based Reconstruction of Complex Nonlinear Dynamical Systems: Using the Methods of Stochastic Processes* (Cham: Springer)
- [12] Friedrich R, Peinke J, Sahimi M and Rahimi Tabar M R 2011 Approaching complexity by stochastic methods: from biological systems to turbulence *Phys. Rep.* **506** 87
- [13] Peinke J, Rahimi Tabar M R and Wächter M 2019 *Ann. Rev. Condens. Matter Phys.* **10** 110
- [14] Anvari M, Reza Rahimi Tabar M, Peinke J and Lehnertz K 2016 Disentangling the stochastic behavior of complex time series *Sci. Rep.* **6** 35435
- [15] Ivanov P Ch, Amaral L A N, Goldberger A L and Stanley H E 1998 *Europhys. Lett.* **43** 363
- [16] Ivanov P C *et al* 2001 From $1/f$ noise to multifractal cascades in heartbeat dynamics *Chaos* **11** 641
- [17] Bogachev M I *et al* 2009 Statistics of return intervals between long heartbeat intervals and their usability for online prediction of disorders *New J. Phys.* **11** 063036
- [18] Sokolova A, Bogachev M I and Bunde A 2011 *Phys. Rev. E* **83** 021918
- [19] Witt A, Ehlers F and Luther S 2017 Extremes of fractional noises: a model for the timings of arrhythmic heart beats in post-infarction patients *Chaos* **27** 092942

- [20] Risken H 1996 *The Fokker–Planck Equation* (Berlin: Springer)
- [21] Lehnertz K, Zabawa L and Reza Rahimi Tabar M 2018 Characterizing abrupt transitions in stochastic dynamics *New J. Phys.* **20** 113043
- [22] Anvari M *et al* 2016 Short term fluctuations of wind and solar power systems *New J. Phys.* **18** 063027
- [23] Ghasemi F, Sahimi M, Peinke J and Rahimi Tabar M R 2006 Analysis of non-stationary data for heart-rate fluctuations in terms of drift and diffusion coefficients *J. Biol. Phys.* **32** 117
- [24] Renat Y, Hänggi P and Gafarov F 2002 Quantification of heart rate variability by discrete nonstationary non-Markov stochastic processes *Phys. Rev. E* **65** 046107
- [25] Lamouroux D and Lehnertz K 2009 *Phys. Lett. A* **373** 3507
- [26] Beck C and Cohen E G D 2003 Superstatistics *Physica A* **322** 267
- [27] Bogachev M I *et al* 2017 Superstatistical model of bacterial DNA architecture *Sci. Rep.* **7** 43034
- [28] Hennig T *et al* 2006 Exponential distribution of long heart beat intervals during atrial fibrillation and their relevance for white noise behaviour in power spectrum *J. Biol. Phys.* **32** 383
- [29] Manshour P, Anvari M, Reinke N, Sahimi M and Rahimi Tabar M R 2016 *Sci. Rep.* **6** 27452