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Logarithmic conformal field theories with continuous weights

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Abstract

We study the logarithmic conformal field theories in which conformal weights are continuous subset of real numbers. A general relation between the correlators consisting of logarithmic fields and those consisting of ordinary conformal fields is investigated. As an example the correlators of the Coulomb-gas model are explicitly studied. © 1998 Elsevier Science B.V.

1. Introduction

It has been shown by Gurarie [1], that conformal field theories (CFT) whose correlation functions exhibit logarithmic behaviour, can be consistently defined and if in the OPE of two given local fields which has at least two fields with the same conformal dimension, one may find some operators with a special property, known as logarithmic operators. As discussed in [1], these operators with the ordinary operators form the basis of the Jordan cell for the operators L_i.

The logarithmic fields (operators) in CFT were first studied by Gurarie in the $c=-2$ model [1]. After Gurarie, thes logarithms have been found in a multitude of others models such as the WZNW-model on GL(1,1) [2], the gravitationally dressed CFT [3], $c_{p,1}$ and non-minimal $c_{p,q}$ models [2,4–6], critical disorderd models [7,8], and the WZNW-models at level 0 [9,10]. They play a role in the study of critical polymers and percolation [11,12], 2D-MHD turbulence [13–15], 2D-turbulence [16,17] and quantum Hall states [18–20]. They are also important for studying the problem of recoil in the string theory and D-branes [9,21–24], as well as target space symmetries in string theory [9]. The representation theory of the Virasoro algebra for LCFT was developed in [25]. The origin of the LCFT has been discussed in [26–28]. The modular invariant partition functions for $c_{\text{eff}}=1$ and the fusion rules of logarithmic conformal field theories (LCFT) are considered in [4], see also [29] about consequences for Zamolodchikov's C-theorem. Structure of the LCFT in D-dimensions has been discussed in [30].

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The basic properties of logarithmic operators are that, they form a part of the basis of the Jordan cell for L_i's and in the correlator of such fields there is a logarithmic singularity [1]. It has been shown that in rational minimal models such a situation, i.e. two fields with the same dimensions, doesn't occur [14].

In a previous paper [27] assuming conformal invariance we have explicitly calculated two- and three-point functions for the case of more than one logarithmic field in a block, and more than one set of logarithmic fields for the case where conformal weights belong to a discrete set. Regarding logarithmic fields *formally* as derivations of ordinary fields with respect to their conformal dimension, we have calculated n-point functions containing logarithmic fields in terms of those of ordinary fields (see also [31], about the role of such derivative in the OPE coefficients of LCFT).

We have done these when conformal weights belong to a discrete set. In [28], there is an attempt to understand the meaning of derivation CFT with respect to conformal weights. Here, we want to consider logarithmic conformal field theories with continuous weights. The simplest example of such theories is the free field theory. The structure of this article is as follows. In Section 2 we study conformal theories, in which conformal weights belong to a continuous subset of real numbers, and calculate the correlators of these theories. Specifically, we show that one can calculate the two-point functions of logarithmic fields in terms of those of ordinary fields by derivation. This is not possible in the case of discrete weights. In Section 3 we consider the Coulomb-gas model as an example.

2. Correlators of a logarithmic CFT with continuous weights

In [27], it was shown that if there are quasi-primary fields in a conformal field theory, there arises logarithmic terms in the correlators of the theory. By quasi-primary fields, it is meant a family of operators satisfying

$$\begin{aligned}\left[L_n,\Phi^{(j)}(z)\right] &= z^{n+1}\partial_z\Phi^{(j)}(z)+(n+1)z^n\Delta\Phi^{(j)}(z)\\ &\quad +(n+1)z^n\Delta\Phi^{(j-1)}(z),\end{aligned}\tag{1}$$

where Δ is the conformal weight of the family. Among the fields $\Phi^{(j)}$, the field $\Phi^{(0)}$ is primary. It was shown that one can interpret the fields $\Phi^{(j)}$, *formally*, as the j-th derivative of a field with respect to the conformal weight:

$$\Phi^{(j)}(z)=\frac{1}{j!}\frac{\mathrm{d}^j}{\mathrm{d}\Delta^j}\Phi^{(0)}(z),\tag{2}$$

and use this to calculate the correlators containing $\Phi^{(j)}$ in terms of those containing $\Phi^{(0)}$ only. The transformation relation (1), and the symmetry of the theory under the transformations generated by $L_{\pm1}$ and L_0, were also exploited to obtain two-point functions for the case where conformal weights belong to a discrete set. There were two features in two-point functions. First, for two families Φ_1 and Φ_2, consisting of n_1+1 and n_2+1 members, respectively, it was shown that the correlator $\langle\Phi_1^{(i)}\Phi_2^{(j)}\rangle$ is zero unless $i+j\ge\max(n_1,n_2)$. (It is understood that the conformal weights of these two families are equal. Otherwise, the above correlators are zero.) Another point was that one could not use the derivation process with respect to the conformal weights to obtain the two-point functions of these families from $\langle\Phi_1^{(0)}\Phi_2^{(0)}\rangle$, since the correlators contain a multiplicative term $\delta_{\Delta_1,\Delta_2}$, which can not be differentiated with respect to the conformal weight.

Now, suppose that the set of conformal weights of the theory is a continuous subset of the real numbers. First, reconsider the arguments resulted to the fact that $\langle\Phi_1^{(i)}\Phi_2^{(j)}\rangle$ is equal to zero for $i+j\ge\max(n_1,n_2)$. These came from the symmetry of the theory under the action of $L_{\pm1}$ and L_0. Symmetry under the action of L_{-1} results in

$$\begin{aligned}\langle\Phi_1^{(i)}(z)\Phi_2^{(j)}(w)\rangle &= \langle\Phi_1^{(i)}(z-w)\Phi_2^{(j)}(0)\rangle\\ &=: A^{ij}(z-w).\end{aligned}\tag{3}$$

We also have

$$\begin{aligned}&\langle\left[L_0,\Phi_1^{(i)}(z)\Phi_2^{(j)}(0)\right]\rangle\\ &\quad=(z\partial+\Delta_1+\Delta_2)A^{ij}(z)+A^{i-1,j}(z)\\ &\qquad+A^{i,j-1}(z)=0,\end{aligned}\tag{4}$$

and

$$\begin{aligned}&\langle\left[L_1,\Phi_1^{(i)}(z)\Phi_2^{(j)}(0)\right]\rangle\\ &\quad=\left(z^2\partial+2z\Delta_1\right)A^{ij}(z)+2zA^{i-1,j}(z)=0.\end{aligned}\tag{5}$$

These show that

$$(\Delta_1-\Delta_2)\,A^{ij}(z)+A^{i-1,j}(z)-A^{i,j-1}(z)=0. \tag{6}$$

If $\Delta_1\neq\Delta_2$, it is easily seen, through a recursive calculation, that A^{ij}'s are all equal to zero. This shows that the support of these correlators, as distribution of Δ_1 and Δ_2, is $\Delta_1-\Delta_2=0$. So, one can use the ansatz

$$A^{ij}(z)=\sum_{k\geq 0}A^{ij}_k(z)\,\delta^{(k)}(\Delta_1-\Delta_2). \tag{7}$$

Inserting this in (6), and using $x\delta^{(k+1)}(x)=-(k+1)\delta^{(k)}(x)$, it is seen that

$$\sum_{k\geq 0}\left[-(k+1)\,A^{ij}_{k+1}(z)+A^{i-1,j}_k(z)-A^{i,j-1}_k(z)\right]\delta^{(k)}(\Delta_1-\Delta_2)=0, \tag{8}$$

or

$$(k+1)\,A^{ij}_{k+1}(z)=A^{i-1,j}_k(z)-A^{i,j-1}_k(z),\quad k\geq 0. \tag{9}$$

This equation is readily solved:

$$A^{ij}_k(z)=\frac{1}{k!}\sum_{l=0}^{k}\binom{k}{l}A_0^{i-k+l,j-l}(z), \tag{10}$$

where A^{ij}_0's remain arbitrary. Also note that A^{ij}_k's with a negative index are zero. We now put (7) in (4). This gives

$$(z\partial+\Delta_1+\Delta_2)\,A^{ij}_k(z)+A^{i-1,j}_k(z)+A^{i,j-1}_k(z)=0, \tag{11}$$

Using (10), it is readily seen that it is sufficient to write (11) only for $k=0$. This gives

$$(z\partial+\Delta_1+\Delta_2)\,A^{ij}_0(z)+A^{i-1,j}_0(z)+A^{i,j-1}_0(z)=0. \tag{12}$$

Putting the ansatz

$$A^{ij}_0(z)=z^{-(\Delta_1+\Delta_2)}\sum_{m=0}^{i+j}\alpha^{ij}_m(\ln z)^m \tag{13}$$

in (12), one arrives at

$$(m+1)\,\alpha^{ij}_{m+1}+\alpha^{i-1,j}_m+\alpha^{i,j-1}_m=0, \tag{14}$$

the solution to which is

$$\alpha^{ij}_m=\frac{(-1)^m}{m!}\sum_{s=0}^{m}\binom{m}{s}\alpha_0^{i-m+s,j-s}. \tag{15}$$

From this

$$A^{ij}_0(z)=z^{-(\Delta_1+\Delta_2)}\sum_{m=0}^{i+j}(\ln z)^m\frac{(-1)^m}{m!}\times\sum_{s=0}^{m}\binom{m}{s}\alpha_0^{i-m+s,j-s}, \tag{16}$$

and

$$A^{ij}_k(z)=\left[\frac{1}{k!}\sum_{l=0}^{k}(-1)^l\binom{k}{l}\sum_{m=0}^{i+j-k}(\ln z)^m\frac{(-1)^m}{m!}\times\sum_{s=0}^{m}\binom{m}{s}\alpha_0^{i-k-m+l+s,j-l-s}\right]z^{-(\Delta_1+\Delta_2)}. \tag{17}$$

So we have

$$A^{ij}(z)=z^{-(\Delta_1+\Delta_2)}\sum_{k\geq 0}\delta^{(k)}(\Delta_1-\Delta_2)\times\left[\frac{1}{k!}\sum_{l=0}^{k}(-1)^l\binom{k}{l}\sum_{m=0}^{i+j-k}(\ln z)^m\frac{(-1)^m}{m!}\times\sum_{s=0}^{m}\binom{m}{s}\alpha_0^{i-k-m+l+s,j-l-s}\right], \tag{18}$$

or

$$A^{ij}(z)=z^{-(\Delta_1+\Delta_2)}\sum_{p,q,r,s\geq 0}\frac{(-1)^{q+r+s}}{p!q!r!s!}\times\alpha^{i-p-r,j-q-s}(\ln z)^{r+s}\delta^{(p+q)}(\Delta_1-\Delta_2), \tag{19}$$

where

$$\alpha^{ij}:=\alpha^{ij}_0. \tag{20}$$

These constants, defined for nonnegative values of i and j, are arbitrary and not determined from the conformal invariance only.

Now differentiate (19) formally with respect to

Δ_1. In this process, α^{ij}'s are also assumed to be functions of Δ_1 and Δ_2. This leads to

$$\frac{\partial A^{ij}(z)}{\partial \Delta_1} = z^{-(\Delta_1+\Delta_2)} \sum_{p,q,r,s} \frac{(-1)^{q+r+s}}{p!q!r!s!} \times \left\{ \frac{\partial \alpha^{i-p-r,j-q-s}}{\partial \Delta_1} (\ln z)^{r+s} \delta^{(p+q)}(\Delta_1-\Delta_2) + \alpha^{i-p-r,j-q-s}\left[(\ln z)^{r+s}\delta^{(p+q+1)}(\Delta_1-\Delta_2) - (\ln z)^{r+s+1}\delta^{(p+q)}(\Delta_1-\Delta_2)\right]\right\}, \qquad (21)$$

or

$$\frac{\partial A^{ij}(z)}{\partial \Delta_1} = z^{-(\Delta_1+\Delta_2)} \sum_{p,q,r,s} \frac{(-1)^{q+r+s}}{p!q!r!s!} \times (\ln z)^{r+s}\delta^{(p+q)}(\Delta_1-\Delta_2) \times \left[(p+r)\alpha^{i-p-r,j-q-s} + \frac{\partial \alpha^{i-p-r,j-q-s}}{\partial \Delta_1}\right]. \qquad (22)$$

Comparing this with $A^{i+1,j}$, it is easily seen that

$$A^{i+1,j} = \frac{1}{i+1}\frac{\partial A^{ij}}{\partial \Delta_1}, \qquad (23)$$

provided

$$\frac{\partial \alpha^{i-p-r,j-q-s}}{\partial \Delta_1} = (i+1-p-r)\alpha^{i+1-p-r,j-q-s}. \qquad (24)$$

Note, however, that the left hand side of (24) is just a *formal differentiation*. That is, the functional dependence of α^{ij}'s on Δ_1 and Δ_2 is not known, and their derivative is just another constant. Repeating this procedure for Δ_2, we finally arrive at

$$\alpha^{ij} = \frac{1}{i!j!}\frac{\partial^i}{\partial \Delta_1^i}\frac{\partial^j}{\partial \Delta_2^j}\alpha^{00}, \qquad (25)$$

and

$$A^{ij} = \frac{1}{i!j!}\frac{\partial^i}{\partial \Delta_1^i}\frac{\partial^j}{\partial \Delta_2^j}A^{00}. \qquad (26)$$

These relations mean that one can start from A^{00}, which is simply

$$A^{00}(z) = z^{-(\Delta_1+\Delta_2)}\delta(\Delta_1-\Delta_2)\alpha^{00}, \qquad (27)$$

and differentiate it with respect to Δ_1 and Δ_2, to obtain A^{ij}. In each differentiation, some new constants appear, which are denoted by α^{ij}'s but with higher indices. Note also that the definition is self-consistent. So that this formal differentiation process is well-defined.

One can use this two-point functions to calculate the one-point functions of the theory. We simply put $\Phi_2^{(0)} = 1$. So, $\Delta_2 = 0$,

$$\langle \Phi^{(0)}(z)\rangle = \beta^0 \delta(\Delta), \qquad (28)$$

and

$$\langle \Phi^{(i)}(z)\rangle = \sum_{k=0}^{i} \frac{\beta^{n-k}}{k!}\delta^k(\Delta), \qquad (29)$$

where

$$\beta^i := \frac{1}{i!}\frac{\mathrm{d}^i \beta^0}{\mathrm{d}\Delta^i}. \qquad (30)$$

The more than two-point function are calculated exactly the same as in [27].

3. The Coulomb-gas model as an example of LCFT

As an explicit example of the general formulation of the previous section, consider the Coulomb-gas model characterized by the action [26]

$$S = \frac{1}{4\pi}\int \mathrm{d}^2 x \sqrt{g}\left[-g^{\mu\nu}(\partial_\mu \Phi)(\partial_\nu \Phi) + iQR\Phi\right], \qquad (31)$$

where Φ is a real scalar field, Q is the charge of the theory, R is the scalar curvature of the surface and the surface itself is of a spherical topology, and is everywhere flat except at a single point.

Defining the stress tensor as

$$T^{\mu\nu} := -\frac{4\pi}{\sqrt{g}}\frac{\delta S}{\delta g_{\mu\nu}}, \tag{32}$$

it is readily seen that

$$T^{\mu\nu} = -(\partial^{\mu}\Phi)(\partial^{\nu}\Phi) + \tfrac{1}{2}g^{\mu\nu}g^{\alpha\beta}(\partial_{\alpha}\Phi)(\partial_{\beta}\Phi) - iQ\left[\phi^{;\mu\nu} - g^{\mu\nu}\nabla^{2}\Phi\right], \tag{33}$$

and

$$T(z) := T_{zz}(z) = -(\partial\phi)^{2} - iQ\partial^{2}\phi, \tag{34}$$

where in the last relation the equation of motion has been used to write

$$\Phi(z,\bar{z}) = \phi(z) + \bar{\phi}(\bar{z}). \tag{35}$$

It is well known that this theory is conformal, with the central charge

$$c = 1 - 6Q^{2}. \tag{36}$$

There are, however, some features which need more care in our later calculations. First, this theory can not be normalized so that the expectation value of the unit operator become unity. In fact, using e^{S} as the integration measure, it is seen that

$$\langle 1\rangle \propto \delta(Q) \tag{37}$$

one can, at most, normalize this so that

$$\langle 1\rangle = \delta(Q). \tag{38}$$

Second, ϕ has a z-independent part, which we denote it by ϕ_0. The expectation value of ϕ_0 is not zero. In fact, from the action (31),

$$\langle\phi\rangle = \langle\phi_0\rangle = \frac{1}{N(Q)}\int \mathrm{d}\phi_0\,\phi_0\exp(2iQ\phi_0), \tag{39}$$

where N is determined from (38) and

$$\langle 1\rangle = \frac{1}{N}\int \mathrm{d}\phi_0\exp(2iQ\phi_0). \tag{40}$$

This shows that $N(0) = \pi$, and

$$\langle\phi_0\rangle = \frac{1}{2i}\left[\delta'(Q) + \frac{N'(0)}{N(0)}\delta(Q)\right]. \tag{41}$$

More generally

$$\langle f(\phi_0)\rangle = \frac{1}{N}f\left(\frac{1}{2i}\frac{\mathrm{d}}{\mathrm{d}Q}\right)(N\langle 1\rangle) = \frac{1}{N}f\left(\frac{1}{2i}\frac{\mathrm{d}}{\mathrm{d}Q}\right)[N\delta(Q)]. \tag{42}$$

Third, the normal ordering procedure is defined as following. One can write

$$\phi(z) = \phi_0 + \phi_{+}(z) + \phi_{-}(z), \tag{43}$$

where $\langle 0|\phi_{-}(z) = 0$, $\phi_{+}(z)|0\rangle = 0$, and

$$[\phi_0, \phi_{\pm}] = 0. \tag{44}$$

The normal ordering is so that one puts all '-' parts at the left of all '+' parts. It is then seen that

$$\langle :f[\phi]:\rangle = \langle f(\phi_0)\rangle. \tag{45}$$

Here, the dependence of f on ϕ in the left hand side may be quite complicated; even f can depend on the values of ϕ at different points. In the right hand side, however, one simply changes $\phi(z) \to \phi_0$.

Now consider the two-point function. From the equation of motion, we have

$$\langle\phi(z)\phi(w)\rangle = -\tfrac{1}{2}\ln(z-w)\langle 1\rangle + b; \tag{46}$$

we also have

$$\langle :\phi(z)\phi(w):\rangle = \langle\phi_0^{2}\rangle = -\frac{1}{4N}\frac{\mathrm{d}^{2}}{\mathrm{d}Q^{2}}[N\delta(Q)]. \tag{47}$$

Note that there is an arbitrary term in (46), due to the ultraviolet divergence of the theory. One can use this arbitrariness, combined with the arbitrariness in $N(Q)$, to redefine the theory as

$$\phi(z)\phi(w) = :-\tfrac{1}{2}\ln(z-w) + :\phi(z)\phi(w):, \tag{48}$$

and

$$\langle f(\phi_0)\rangle := f\left(\frac{1}{2i}\frac{\mathrm{d}}{\mathrm{d}Q}\right)\delta(Q); \tag{49}$$

these relations, combined with (45) are sufficient to obtain all of the correlators. One can, in addition, use (34) (in normal ordered form) to arrive at

$$T(z)\phi(w) = \frac{\partial_w \phi}{z-w} - \frac{iQ/2}{(z-w)^2} + \text{r.t.}, \tag{50}$$

and

$$T(z)T(w) = \frac{\partial_w T}{z-w} - \frac{2T(w)}{(z-w)^2} + \frac{(1-6Q^2)/2}{(z-w)^4}. \tag{51}$$

Eq. (50) can be written in the form

$$[L_n, \phi(z)] = z^{n+1}\partial\phi - \frac{iQ}{2}(n+1)z^n. \tag{52}$$

This shows that the operators ϕ and 1 are a pair of logarithmic operators with $\Delta = 0$ (in the sense of (1)). One can easily show that

$$T(z):\mathrm{e}^{i\alpha\phi(w)}: = \frac{\partial_w :\mathrm{e}^{i\alpha\phi(w)}:}{z-w} - \frac{\alpha(\alpha+2Q)/4}{(z-w)^2}:\mathrm{e}^{i\alpha\phi(w)}: + \text{r.t.}, \tag{53}$$

which shows that $:\mathrm{e}^{i\alpha\phi}:$ is a primary field with

$$\Delta_\alpha = \frac{\alpha(\alpha+2Q)}{4}. \tag{54}$$

To this field, however, there corresponds a quasi conformal family (pre-logarithmic operators [26]), whose members are obtained by explicit differentiation with respect to α (α is not the conformal weight but is a function of it):

$$W_\alpha^{(n)} := \phi^n \mathrm{e}^{i\alpha\phi} := (-i)^n \frac{\mathrm{d}}{\mathrm{d}\alpha^n} :\mathrm{e}^{i\alpha\phi}:. \tag{55}$$

To calculate the correlators of W's, it is sufficient to calculate $\langle W_{\alpha_1}^{(0)} \cdots W_{\alpha_k}^{(0)} \rangle$.

One has, using Wick's theorem and (48),

$$\Pi_{j=1}^k :\mathrm{e}^{i\alpha_j\phi(z_j)}: = \mathrm{e}^{1/2 \sum_{1\le i<j\le k} \alpha_i\alpha_j \ln(z_i-z_j)} :\mathrm{e}^{i\sum_{j=1}^k \alpha_j\phi(z_j)}:. \tag{56}$$

From this using (45) and (48), we have

$$\begin{aligned} &\langle \Pi_{j=1}^k W_{\alpha_j}^{(0)}(z_j) \rangle \\ &= \left[\Pi_{1\le i<j\le k}(z_i-z_j)^{\frac{\alpha_i\alpha_j}{2}} \right] \mathrm{e}^{1/2 \sum_{j=1}^k \alpha_j \frac{\mathrm{d}}{\mathrm{d}Q}} \delta(Q) \\ &= \left[\Pi_{1\le i<j\le k}(z_i-z_j)^{\frac{\alpha_i\alpha_j}{2}} \right] \delta\left(Q + \tfrac{1}{2}\sum_{j=1}^k \alpha_j \right). \end{aligned} \tag{57}$$

Obviously, differentiating with respect to any α_i, leads to logarithmic terms for the correlators consisting of logarithmic fields $W_\alpha^{(n)}$. The power of logarithmic terms is equal to the sum of superscripts of the fields $W_\alpha^{(n)}$.

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