# Competitive Strategy for Walking in Streets for an Empowered Simple Robot

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## Abstract

We consider the problem of walking in an unknown street, for a robot that has a minimal sensing capability. The basic robot is equipped with a sensor that only detects the discontinuities in depth information (gaps). In the former recent researches some competitive strategies for walking the robot in street polygons have been presented. In this research we have empowered the robot by adding a compass to reach the target $t$ along a shorter route starting from$s$, in street polygons. We present an online strategy that generates a search path for the empowered robot in streets such that the competitive ratio of our strategy is $3\sqrt{2}$.


## 1 Introduction

Exploring an unknown environment is a fundamental problem characterized by researcher in robotics, computational geometry, game theory and online algorithm [3]. An autonomous mobile robot without access to the geometry of the scene depending the information collected through its sensor moves to reach a goal. Variants of robot models, and problems have been studied in this context [1, 5, 9]. We are interested in using a minimalist robot model system for walking in unknown scene.

Our basic robot is a simple point robot with the sensing model of gap sensor. At each point the robot locates the depth discontinuities (gaps) of its visibility region in a circularly ordered, (Figure 1). All times the robot can track the gaps and detects each topological changes of the gaps. These changes are the appearance, disappearance, merging, or splitting of gaps which are called *critical events*. While the robot traverses an environment, it can rotate as often as each of the critical events arises, or a target point enters in its visibility region.

In order to measure the performance of an online search strategy, the notation of competitive analysis is used. The competitive ratio is the worst case ratio of the path travelled by the robot in the unknown environment to shortest path. Tabatabaei and Ghodsi designed an online strategy for the simple robot to walk in streets. By the strategy the robot explores a street from a vertex $s$ to a vertex $t$ such that the travelled distance by the robot is at most 9 times longer than the shortest path [10]. A street polygon is characterized by the feature that the two boundary chains from $s$ to $t$ ($L_{chain}$ and $R_{chain}$ ) are mutually weakly visible, see Figure 1(a).

In this study, our goal is equipping the simple robot, with a smallest set of additional capabilities, in order to reach the goal along a shorter route. So, we consider the following extension of the simple robot. The robot carries a compass that denotes to it the north, west, south and east directions. It can moves toward the directions, in additional to the gap tracking (Figure 1). We present an online search strategy for exploring the street environment, from a vertex $s$ to a vertex $t$, for the empowered robot with the competitive ratio of $3\sqrt{2}$. The ratio is almost half of the competitive ratio of 9, presented in the previous research for the simple robot, not equipped with a compass [10].

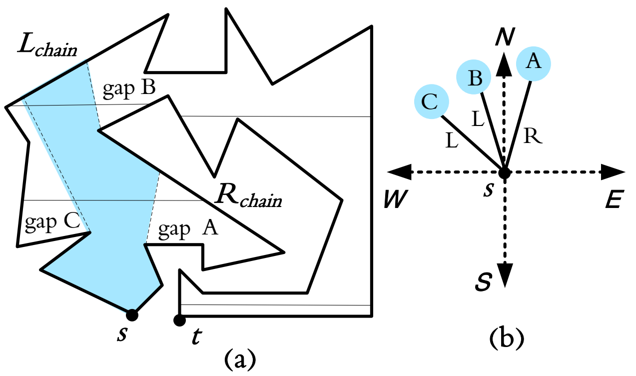


Figure 1: (a) A street polygon. The colored region is the visibility polygon of the point robot at the start point $s$. (b) The position of discontinuities in the depth information (gaps) reported by the sensor and directions of the compass at the start point $s$.

**Related Works:** Klein proposed the first competitive algorithm for walking in streets problem for a robot that was equipped with a 360 degrees vision system [6]. Also, Icking, et al. presented an optimal search strategy for the problem with the competitive factor of $\sqrt{2}$ [4]. Many online strategies for patrolling unknown environment such as street, generalized street, and star polygon are presented in [3, 7].


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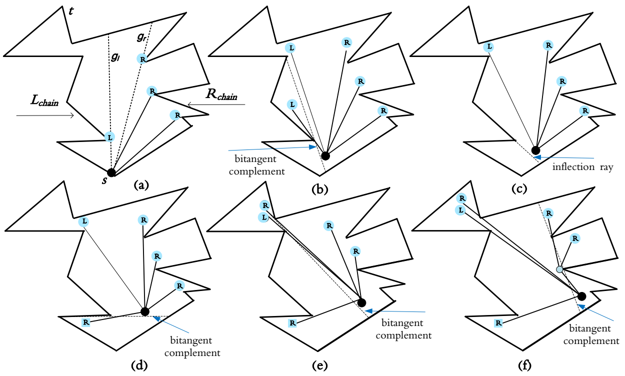


Figure 2: Street polygons, and the dynamically changes of the gaps as the robot walks towards a gap in street polygon. The dark circle is the location of the robot, and squares and other circles denote primitive and non-primitive gaps respectively. (a) Existing gaps at the start point. (b) A split event. (c) A disappearance event. (d) An appearance event. (e) Another split event. (f) A merge event.

The limited sensing model (gap sensor) that our robot is equipped with, in this research, was first introduced by Tovar, et al. [13]. They offered Gap Navigation Tree (GNT) to maintain and update the gaps seen along a navigating path. Some strategies, using GNT for exploring unknown environments, presented in [8, 14].

Tabatabaei, et al. gave a deterministic algorithm for the simple robot to reach the target $t$ in a street and a generalized street, starting from $s$. The robot using some pebbles and memorizing some portion of the streets has seen so far, explores the street. The target $t$ is achieved such that the traversed path is at most 11 times longer than the shortest path by using one pebble. Also they showed, allowing use of many pebbles reduces the factor to 9 [10, 11].

Another minimal sensing model was presented by Suri, et al. [9]. They assumed that the simple robot can only sense the combinatorial (non-metric) properties of the environment. The robot can locate the vertices of the polygon in its visibility region, and can report if there is a polygonal edge between them. Despite of the minimal ability, they showed that the robot can accomplish many non-trivial tasks. Then, Disser et al. empowered the robot with a compass to solve the mapping problem in polygons with holes [2].

## 2 Preliminaries

### 2.1 The Sensing Model and Motion Primitives

The basic robot has an abstract sensor that reports a cyclically ordered of discontinuities in the depth information (gaps) in its visibility region. All the gaps and the target can be located by the robot as they enter in the robots omnidirectional and unbounded field of view. Each gap has a label of $L$ (left) or $R$ (right) which displays the direction of the part of the scene that is hidden behind the gap, see Figure 2.

The robot can orient its heading to each gap and moves towards the gap in an arbitrary number of steps, e.g., two steps towards gap $g_x$. Each step is a constant distance which is already specified for the robot by its manufacturer. By equipping the robot with a compass, the robot is empowered; such that it can detects and tracks the north, west, south and east directions to desired number of steps, see Figure 1. Also the robot moves towards the target as it enters in its visibility region.

While the robot moves, combinatorial changes occur in the visibility region of the robot called critical events. There are four types of critical events: appearances, disappearances, merges, and splits of gaps. Appearance and disappearance events occur when the robot crosses inflection rays. Each gap that appears during the movement, corresponds to a portion of the environment that was already visible, but now is not visible. such gaps are called primitive gaps and all the others are non-primitive gaps. Merge and split events occur when the robot crosses bitangent, as illustrated in Figure 2.

### 2.2 Known Properties

At each point of the search path, if the target is not visible, the robot reports a set of gaps with the labels of L or R ($l$-gap and $r$-gap for abbreviation) cyclically. Let $g_l$ be a non-primitive $l$-gap that is in the right side of the other left gaps, and $g_r$ be a non-primitive $r$-gap that is in the left side of the other right gaps, see Figure 2(a). Each of the two gaps is called the most advanced gap. The two gaps have a fundamental role in path planning for the simple robot.

**Theorem 1** *[4, 10] While the target is not visible, it is hidden behind one of the two gaps, $g_l$ or $g_r$.*

From Theorem 1, if there exist only one of the two gaps ($g_r$ and $g_l$) then the goal is hidden behind of the gap. Thus, there is no ambiguity and the robot moves towards the gap, see Figure 3(a). When both of $g_r$ and $g_l$ exist, a funnel case arises, the angle between $g_r$ and $g_l$ that is always smaller than $\pi$ is called the opening angle [4], see Figure 3(b). At each funnel case, usually, a detour from the shortest path is unavoidable.

### 2.3 Essential Information

All we maintain during the search strategy is location of $g_l$ and $g_r$. As the robot moves in the street, the critical events that change the structure of the robot's visibility region may dynamically change $g_l$ and $g_r$. Also, by the robot movement, a funnel case may end or a new funnel may start. We refer to the point, in which a funnel ends a *critical point* of the funnel.

The following events update the location of $g_l$ and $g_r$ as well as a funnel situation when the robot moves towards $g_l$ or $g_r$.

1. When $g_r/g_l$ splits into $g_r/g_l$ and another $r$-gap/$l$-gap, then $g_r/g_l$ will be replaced by the $r$-gap/$l$-gap, ( Figure 2(b)).
2. When $g_r/g_l$ splits into $g_r/g_l$ and another $l$-gap/$r$-gap, then $l$-gap/$r$-gap will be set as $g_l/g_r$. This point is a critical point in which a funnel situation ends, (Figure 2(e)).
3. When $g_l$ or $g_r$ disappears, the robot may achieve a critical point in which a funnel situation ends, (point 2 in Figure 3(b)).

Note that the split and disappearance events may occur concurrently, (point 3 in Figure 3(b)). Furthermore, by moving towards $g_r$ and $g_l$, these gaps never merge with other gaps.

## 3 Algorithm

Now, we present our strategy for searching the street, from $s$ to $t$. Since the target is constantly behind one of $g_r$ and $g_l$, during the search, the location of the two gaps are maintained and dynamically updated as explained in the previous section.

### 3.1 Main Strategy

At each point of the search path, especially at the start point $s$, there are two cases:

- If only one of the two gaps ($g_r$ and $g_l$) exists, or they are collinear then the goal is hidden behind the gap, see Figure 3(a). The robot moves towards the gap until the target is achieved or a funnel situation arises, (point 2 in Figure 3(a)).
- If both of $g_r$ and $g_l$ exist, a funnel case arises. The possible locations of the compass directions are as follows:
  1. One of the directions of the compass lies in the opening angle (start point s in Figure 3(b)). By our strategy the robot moves along the direction.
  2. Two directions of the compass lie in the opening angle (point 2 in Figure 3(b)). In order to bound the detour, the robot moves one step towards one of the compass direction, and moves one step towards the other, alternatively.
  3. None of the compass directions lies in the opening angle (point 3 in Figure 3(b)). The robot moves towards $g_r$ or $g_l$.

The robot continues to move along the selected path until the position of the gaps changes (point 1 in Figure 3(b)), or the critical point of the funnel achieved (point 2 in Figure 3(b)).

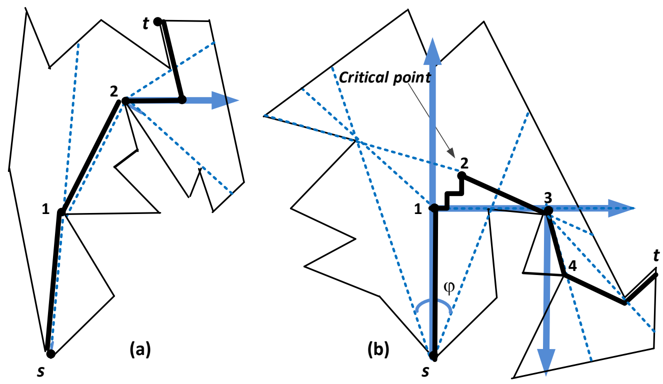


Figure 3: Bold path is the robot search path. (a) There is only $g_r$. (b) $g_r$ and $g_l$ are the two most advanced gaps at the start point $s$. The angle between the gaps, $\varphi$, is the opening angle at the start point.

### 3.2 Correctness and Analysis

Throughout the search, the robot path coincides with the shortest path unless a funnel case arises. Then, in order to prove the competitive ratio of our strategy, we compare length of the path and the shortest path in a funnel case. We use the concept of opening angle to calculate the competitive ratio.

**Lemma 2** *[12] By our strategy, the detour from shortest path for a small opening angle, in the funnel case, is shorter than detour for a large opening angle.*

**Theorem 3** *Our deterministic strategy guarantees a path at most $3\sqrt{2}$ times longer than the shortest path, in the street from $s$ to $t$.*

**Proof.** According to the lemma 2, when there are two directions of the compass in the opening angle, the greatest deviation from the shortest arises. We show the directions with X and Y. Without loss of generality, assume that moving towards $g_r$ coincides with the shortest path. The robot alternately moves one step towards X and moves one step towards Y until it reaches critical point (point $p(x', y')$ in Figure 4). At the critical point, the robot moves towards the only existing advanced gap (point $q(x, y)$). If we compare length of the robot search path with $L_1$-shortest path, moving along one of the directions (for example X) is correct and moving along the other is deviation from the $L_1$-shortest. The robot traverses a maximum length of $|x'|+|y'|+|y'|+|x|+|y|$ to reach point $q$, where $|x'| = |y'| \leq |x|$. So the robot's path length is less than $3(|x|+|y|)$, 3 times longer than the $L_1$-shortest path. Then, the strategy achieves a competitive ratio of $3\sqrt{2}$ in the $L_2$-metric. $\square$

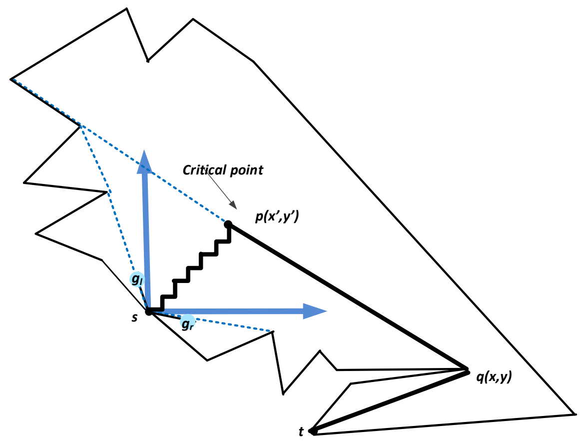


Figure 4: Bold path is the robot search path.

## 4 Conclusions

In this paper, we studied the problem of walking in street environment for a simple point robot. The basic robot has a minimal sensing model that can only detect the gaps and the target in the street. We have empowered the robot by adding a compass. We presented an online competitive strategy that generates a search path for the empowered robot to reach the target. Length of the path is at most 3 times longer than length of the shortest path. The length of this path is almost half the presented path in the previous research.

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