A Competitive Strategy for Walking in Generalized Streets for a Simple Robot

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Abstract

In this study, we consider the problem of walking in an unknown generalized street or G-street, for a simple robot. The basic robot is equipped with a sensor that only detect the discontinuities in depth information (gaps). In the former recent researches some competitive strategies for walking the robot in street polygons have been presented. In this research we have empowered the robot by adding a compass to patrol a more general class of polygons. We present an online strategy that generates a search path for the empowered robot in G-streets; a more general class of polygons that contains all streets properly. The empowered robot, using the local information gathered through its sensors and using some pebbles as marker, locates target t, starting from a vertex s in a G-street. Length of the traveled path by the robot is at most 9.06 times longer than the shortest path. The competitive ratio is optimal.

1 Introduction

Exploring an unknown environment is a fundamental problem characterized by researcher in robotics, computational geometry, game theory and online algorithm [9, 12]. An autonomous mobile robot without access to the geometry of the scene depending the information collected through its sensor moves to reach a goal. Variants of robot models, and problems have been studied in this context [1, 4, 7]. We are interested in using a minimalist robot model system for walking in unknown scene.

Our basic robot is a simple point robot with the sensing model of gap sensor. At each point the robot locates the depth discontinuities (gaps) of its visibility region in a circularly ordered, (Figure 1). All times the robot can track the gaps and detects each topological changes of the gaps. These changes are the appearance, disappearance, merging, or splitting of gaps which are called *critical events*. While the robot traverses an environment, it can rotate as often as each of the critical events arises, or a target point enters in its visibility region.

In order to measure the performance of an online search strategy, the notation of competitive analysis is used. The competitive ratio is the worst case ratio of the path traveled by the robot in the unknown environment to shortest path. Tabatabaei and Ghodsi designed an online strategy for the simple robot to walk in streets. By the strategy the robot explores a street from a vertex s to a vertex t such that the traveled distances by the robot is at most 9 times longer than the shortest path [15]. A street polygon is characterized by the feature that the two boundary chains from s to t are mutually weakly visible.

In this study, our goal is equipping the simple robot with a smallest set of additional capabilities to empower the robot for searching more general classes of polygons by a competitive search strategy. So, we consider the following extension of the simple robot. The robot carries a compass that denotes to it the north, west, south and east directions. It can moves toward the directions, in additional to the gap tracking. Also, it can put a pebble for marking anywhere. We present an online search strategy for exploring a generalized street environment, from a vertex s to a vertex t, for the empowered robot with the competitive ratio of 9.06. A generalized street is a polygon for which every point on its boundary is visible from a point on a horizontal line segment that connects the two boundary chains from s to t, see Figure 1.

Our robot sensing model is strongly weaker than model of the robot in the previous research. Datta and Icking presented an optimal online strategy with a competitive ratio of 9.06 for searching a generalized street [2]. Datta and Icking robot equipped with a 360 degrees vision also, it memorized the map of the scene has seen so far while our robot using the local information gathered through its sensor walks in the street. The ratio of 9.06 is optimal, Lopez-Ortiz and Schuierer have shown the lower bound of 9.06 in [9].

Related Works: Klein presented the first competitive strategy for walking in streets problem for a robot that was equipped with a 360 degrees vision system [8]. Many online algorithms for patrolling unknown environments such as street, generalized street, and star polygons are proposed in [6, 10].

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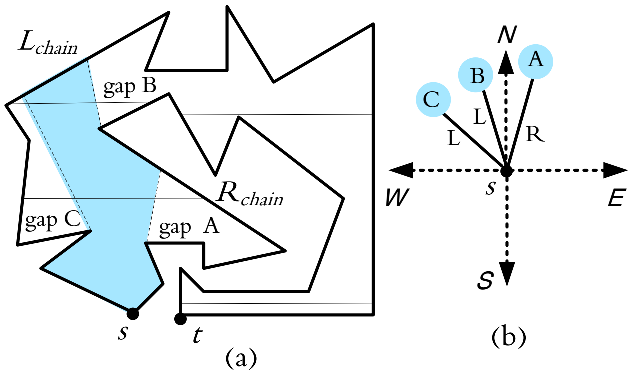

Figure 1: (a) A G-street polygon. The colored region is the visibility polygon of the point robot at the start point s. (b) The position of discontinuities in the depth information (gaps) reported by the sensor and directions of the compass.

The basic sensing model (gap sensor) that our robot is equipped with, in this study, was first introduced by Tovar, Murrieta-Cid, and LaValle [17]. They offered Gap Navigation Tree (GNT) as a means to record and update the gaps seen along an exploring path. Other researcher presented some strategies, using GNT, for searching unknown environments [5, 11, 13]. An optimal search strategy with minimum number of turns, for the basic simple robot equipped with the gap sensor, presented in [16].

Another minimal sensing model offered by Suri, Vicari, and Widmayer [14]. They assumed that the simple robot can only sense the combinatorial (non-metric) properties of the environment. The robot can locate the vertices of the polygon in its visibility region, and can report if there is a polygonal edge between them. Despite of the minimal capability, they showed that the robot performs many non-trivial tasks. Then, Disser, Ghosh, Mihalak, and Widmayer empowered the robot with a compass to solve the mapping problem in polygons with holes [3].

2 preliminaries

2.1 Workspace

Generalized street polygons are considered as the robot workspace. So, we briefly repeat its definitions, and some of its properties. In a simple polygon P with two vertices s and t the counter-clockwise polygonal chain from s to t is called the *right chain* or R_{chain}, and the clockwise one from s to t is called the left chain or L_{chain}.

Defnition 1 *[8] A polygon is a street polygon if each point on the left chain is visible from at least one point on the right chain and vice versa.*

Defnition 2 *[2] A chord is a horizontal line segment inside a polygon P such that its both end points are on the boundary of P. The chord is called an LR-chord when it touches both the L_{chain} and R_{chain}.*

Defnition 3 *[2] A simple polygon in the plane is called a generalized street or G-street if for every boundary point $p \in L \cup R$, there exists an LR-chord c such that p is visible from a point on c.*

Figure 1 displays an example of a G-street, and Some of its LR-chords, the horizontal lines. Class of the generalized street polygons is strictly larger than class of the street polygons that our empowered robot explores it.

2.2 Sensing Model and Motion

From the vertex s in an unknown G-street, a simple robot starts walking to achieve the target t. The robot based on the local information collected by its sensor explores the scene. The basic robot is a point robot equipped with a sensor that detects each discontinuity in depth information that referred as gaps. The sensor reports a cyclically ordered location of the gaps in its visibility region. Also, the robot assigns a label of L or R (left or right) to each gap based on the direction of the hidden region that is behind the gap [17], (Figure 1). The robot can only track the gaps and detects their topological changes. These changes are: appearance, disappearance, merging, and splitting of gaps. The appearance and disappearance events arise when the robot crosses the inflection rays, (at point 3 in Figure 2 gap $R - EG_1$ disappears). The merge and split events occur when the robot crosses a bitangent complement, (at point 2 in Figure 2 gap $L - EG_1$ splits). The robot can move along a straight line towards gaps to cover the region hidden behind it. The robot may rotate as a critical event occurs, or as soon as the target enters in the robot's visibility region. By equipping the robot with a compass, the robot is empowered; such that it can detects and tracks the north, west, south and east directions, see Figure 1. Also, some pebbles are available to the robot as marker.

2.3 Main Strategy for Walking in rectilinear G-streets

Now, we present an online search strategy for walking in a rectilinear G-street in which all of the edges are either horizontal or vertical, (Figure 2). From the start vertex s, the robot starts walking in the G-street to reach the target t, using the information collected by the gap sensor and the compass. The goal is to minimize length of the search path.

In contrast with the previous research, our robot has no access to the map of its visibility region has seen so far, and especially angles of the region boundary. Only

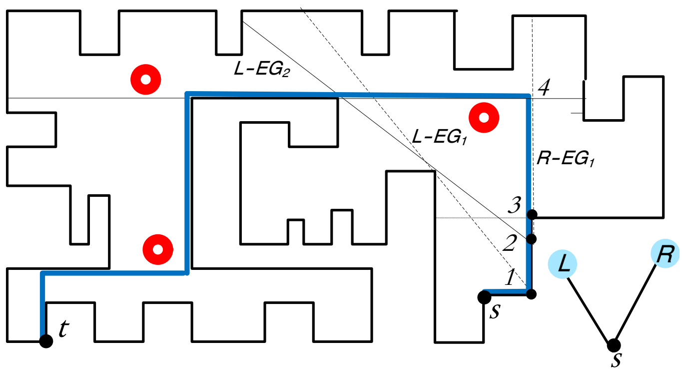

Figure 2: A rectilinear G-street, and located gaps (in unexplored region) at points s and p. $R-EG_i$ and $L-EG_i$ are essential gaps at point i. Colored bold path is the robot search path (traversed path for performing the doubling search strategy to reach the turn points is omitted). Colored discs are the pebbles for marking the regions that the robot comes from.

local information about the gaps location and the compass directions, north, south, east and west, are available to the simple robot.

There is a reflex vertex correspond to each gap. We refer to the horizontal chord that crosses the vertex as a *gap chord*. Each gap chord divides the polygon into three parts, or two parts as shown in figure 3. Also, the edge of the polygon that is collinear with the gap chord is called as *gap edge*.

Defnition 4 *A gap is an essential gap when the target is hidden beyond its gap chord. If the gap is a right/left gap it is denoted by $R-EG/L-EG$.*

From the definition of G-street following Lemma is straightly obtained.

Lemma 1 *The target vertex t or an essential gap is visible from some point on the horizontal line containing the start point.*

Examples of gap chord, gap edge, and essential gap are shown in Figure 3. The important key is that the simple robot how can distinguish if a gap is essential.

Theorem 2 *While the robot searching on the horizontal line, it reports an essential gap as soon as it locates a vertical gap that is collinear with north or south directions of the compass.*

Proof. Assume, d is the gap chord of the detected vertical gap. It divides the polygon into three parts, or two parts. s and t must be in different parts. If they are in the same part which contains the horizontal line, then there is at least one point on edge of gap chord which is not visible from any LR-chord and the polygon is not a G-street, a contradiction. So, d is an LR-chord that each path from s to t intersects it. Then, the vertical gap is an essential gap. See the vertical gap detected at point 1 in Figure 2. □

We refer to the point in theorem 2 as a *turn point*. Note that at the turn point the robot detects a vertical essential gap.

Theorem 3 *If the detected gap, at the turn point is $L-EG$, the first right gap which lies clockwise after the gap is $R-EG$. Analogously, if the gap is $R-EG$, the first left gap which lies counterclockwise after the gap is $L-EG$.*

Proof. Assume that at the turn point, $L-EG$ is detected, also there is a right gap clockwise after the gap essential gap. If the robot continues moving towards right direction, the right gap will coincide with the vertical direction. So, by definition of the essential gap, the right gap is $R-EG$, see the turn point 3 in figure 3. The other case is similar. □

Now, by the above discussion, we can describe our algorithm. At the start point the simple robot searches on the horizontal line containing the start point. Three cases may arise.

- If all of the detected gaps are in the left side of the vertical line containing s, the robot moves toward left to reach the turn point, using the compass.
- If all of the detected gaps are in the right side of vertical line containing the start point, the robot moves toward right to reach the turn point. For example consider point p as start point in figure 3.
- If there are some gaps in both sides of the vertical line, we performs the doubling strategy; the robot walks back and forth on the horizontal line, at each step doubling the distance to the start point, until the turn point is reached (start point in figure 2).

At the turn point, from Theorem 3, the robot can locate the other existing essential gap. Assume, the essential vertical gap is $R-EG$; the other case is similar. The robot moves towards the gap along the vertical direction while maintaining location of $L-EG$. During the walking, $L-EG$ may disappears, also it updates as the robot crosses over bitangent compliment of $L-EG$ and another left gap, see point 2 in Figure 2. The robot continues walking along the vertical direction until the vertical $R-EG$ disappears.

At the event point, the first right gap (if exits) which lies clockwise after the movement direction is current $R-EG$; by a similar argument to the proof of Theorem 3. Now, at the event point, there are three cases:

1. $L - EG$ and $R - EG$ exist. The robot continues walking along the current vertical direction while maintaining and updating the essential gaps until the two gaps disappear (case 3 arises).

2. One of the essential gaps exists (point 2 in Figure 3). The robot continues walking along the current vertical direction until the gap merges with another gap, or the gap disappears. In the former case, the robot turns, and move along the horizontal direction containing the gap until achieves a turn point in which a vertical essential gap is detected, (point 3 in Figure 3). In the later case when the essential gap disappears, case 3 arises.

3. No essential gap exists. The robot puts a pebble in the region where it comes from, (point 4 in Figure 2). Then, same as the start point, it performs the doubling search strategy on the horizontal direction in region containing its current location for achieving a turn point. The detected essential gap must be in a region other than the marked region by the pebble.

The robot repeats the process until the target t is visible. At the point, the robot can, using the compass direction, move a long a rectilinear path to achieve the target t.

2.4 Analysis of the Algorithm

We show that our simple robot, using the local information about location of gaps and the compass achieves the target t starting from s in the G-street. Although our robot is strongly weaker than Datta and Icking robot [2], and its search path differs from the robot path, we demonstrate the competitive ratio of our strategy is 9.06 like their strategy.

Theorem 4 *Our search strategy terminates while a search path to t is generated, starting from s in the G-street.*

Proof. At the start point, if the target is not visible to the robot, it is hidden behind the existing gaps. The robot searches on the horizontal line to reach an essential gap, then tracks the gap which is a correct vertical direction. The robot continues moving a long a direction unless both of $R - EG$ and $L - EG$ disappear. Then, the robot on the horizontal line searches for another essential gaps, and selects again a correct vertical direction which results in being one step closer to the target. So, the strategy terminates. □

Lemma 5 *[1] The doubling strategy for searching a point on a line has a competitive factor of 9 which is optimal.*

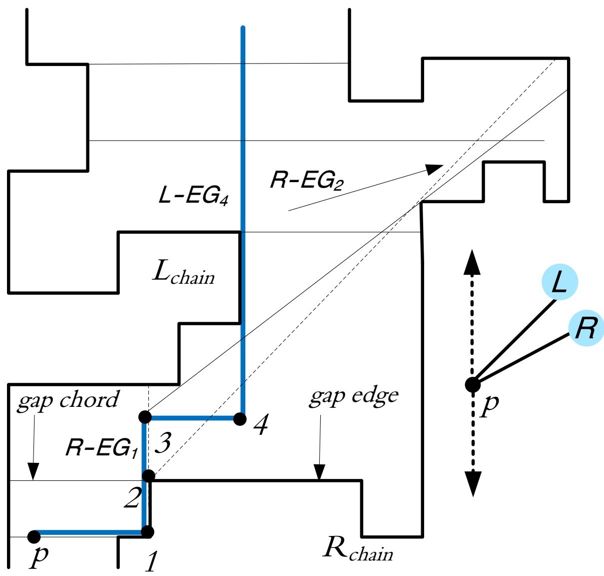

Figure 3: Traversed path between consecutive gap chords, the L_1-shortest path.

Lemma 6 *[9] lower bound of the competitive factor for each online search strategy in G-street is 9.06.*

Theorem 7 *Competitive ratio of our strategy is 9.06, and this is optimal.*

Proof. In order to compute the ratio, we compare length of the generated path with L_1-shortest path. As explained in the strategy our robot always chooses a correct vertical direction (as shown in Figure 3). For detecting an essential gap, the robot either moves along a correct direction or performs the doubling strategy. So, the horizontal traversed path is most 9 times longer than the shortest path. It means that looking for a target in a G-street is at least as hard as searching a point on a line. So, from Lemma 5, the competitive factor of our strategy is 9 in L_1- metric. Now by an argument, similar to the proof of the competitive factor of Datta and Icking strategy [2], we show the competitive ratio of our strategy is 9.06 in L_2- metric. Note that the two paths are different. The length of the L_2-shortest path between two consecutive turn points in which the essential gaps are reported is $\sqrt{{x_1}^2 + {y_1}^2}$). By our strategy, length of the simple robot's path is at most $9x_1 + y_1$. The maximum value of $\frac{9x_1+y_1}{{x_1}^2+{y_1}^2}$ is 9.06. Then, from Lemma 6 our strategy is optimal. □

3 Conclusions

In this paper, we studied the problem of walking in generalized streets for a simple point robot. The basic robot has a minimal sensing model that can only detect the gaps and the target in the street. We have empowered the robot by adding a compass. The robot using local information reported by its sensor explores the scene while in the former research a robot that memorizes the region has seen so far, by its complete vision system, searches the scene. We demonstrated that, despite the weakness in our robot system model, performance of our strategy equals with the optimal strategy for the stronger robot, the competitive ratio of 9.06. Proposing a competitive search strategy for more general classes of polygons and offering other minimal sensing model are attractive problems for future research.

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