Competitive Strategies for Walking in Streets for a Simple Robot Using Local Information

Azadeh Tabatabaei* Mohammad Aletaha† Mohammad Ghodsi‡

Abstract

We consider the problem of walking in an unknown street, for a robot that has a minimal sensing capability. The robot is equipped with an abstract sensor that only detects the discontinuities in depth information (gaps) and can locate the target point as it enters its visibility region. First, we propose an online deterministic search strategy that generates an optimal search path for the simple robot to reach the target t, starting from s. The path created by this strategy is 9-competitive which is proven to be optimal. In contrast with previously known research, the path is designed without memorizing any portion of the scene that has been seen so far. The robot using local information about the location of some gaps achieves the target t starting from s in a street. Then, we present a randomized search strategy, based on the deterministic strategy. Also, a randomized lower bound on the competitive ratio has been proved.

1 Introduction

Path planning is a basic problem to almost all scopes of computer science; such as computational geometry, online algorithms, robotics, and artificial intelligence [3]. Especially, path planning in an unknown environment for which there is no geometric map of the scene is interesting in many real-life cases. Robot sensors are the only tool for gathering information in an unknown street. The amount of information derived from the environment depends on the capability of the robot. Due to the importance of using a simple robot, including low cost, less sensitive to failure, robust against sensing errors and noise, many types of path planning for simple robots have been studied [1, 5, 9].

In this paper, we consider the problem of walking a simple robot in an unknown street. A simple polygon P with two separated vertices s and t is called a street if the left boundary chain L_{chain} and the right boundary chain R_{chain} constructed on the polygon from s to t are mutually weakly visible. In other words, each point on the left chain can see at least one point on the right chain and vice versa [6], see Figure 1(a). A point robot which its sensor has a minimal capability that can only detect discontinuities in depth information (gaps) and the target point t, starts searching the street. The robot can locate the target as soon as it enters its visibility region. Also, the robot cannot measure any angles or distances, or infer its position, see Figure 1. The goal is to reach the target t using the information gathered through its sensor, starting from s such that the traversed path by the robot is as short as possible.

To evaluate the efficiency of a search strategy for the robot, we use the notion of competitive of the competitive analysis. The competitive analysis for a strategy that leads the robot is the ratio of the (expected) distance traversed by the robot over the shortest distance from s to t, in the worst case.

In this paper, first, we present a deterministic strategy using local information about the location of two special gaps which are updated during the walking. The robot achieves the target, without memorizing environment and without using pebbles, in contrast with previously known research [10]. The search path is optimal; the length of the generated path is at most 9 times longer than the shortest path. Then, we present a randomized strategy that generates a search path similar to the deterministic one. We introduced the deterministic strategy and the idea of randomization of that previously in [12].

Related Works: Klein proposed the first competitive algorithm for walking in streets problem for a robot that was equipped with a 360 degrees vision system [6]. Also, Icking, et al. presented an optimal search strategy for the problem with the competitive factor of $\sqrt{2}$ [4]. Many online strategies for patrolling unknown environments such as streets, generalized streets, and star polygons are presented in [3, 7, 13].

The limited sensing model (gap sensor) that our robot is equipped with, in this research, was first introduced by Tovar, et al. [14]. They offered Gap Navigation Tree (GNT) to maintain and update the gaps seen along a navigating path. Some strategies, using GNT for exploring unknown environments, presented in [8, 15].

Tabatabaei, et al. gave a deterministic algorithm for the simple robot to reach the target t in a street and a generalized street, starting from s. The robot us-

*Department of Computer Engineering, University of Science and Culture, Tehran, Iran, a.tabatabaei@usc.ac.ir

†Department of Computer Engineering, Sharif University of Technology, Tehran, Iran, mohammadaletaha@ce.sharif.edu

‡Sharif University of Technology and Institute for Research in Fundamental Sciences (IPM), Tehran, Iran, ghodsi@sharif.edu

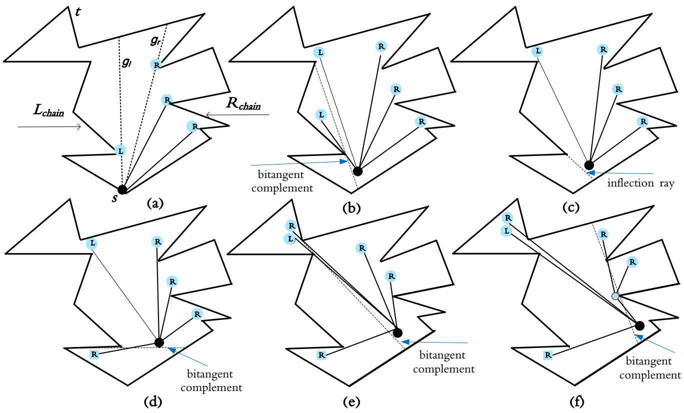

Figure 1: Street polygons and the dynamical changes of the gaps as the robot walks towards a gap in street polygons. The dark circle is the location of the robot, and squares and other circles denote primitive and non-primitive gaps respectively. (a) Existing gaps at the start point. (b) A split event. (c) A disappearance event. (d) An appearance event. (e) Another split event. (f) A merge event.

ing some pebbles and memorizing some portion of the streets has seen so far, explores the street. The target t is achieved such that the traversed path is at most 11 times longer than the shortest path by using one pebble. Also, they showed, allowing the use of many pebbles reduces the factor to 9 [10, 11].

Another minimal sensing model was presented by Suri, et al. [9]. They assumed that the simple robot can only sense the combinatorial (non-metric) properties of the environment. The robot can locate the vertices of the polygon in its visibility region and can report if there is a polygonal edge between them. Despite the minimal ability, they showed that the robot can accomplish many non-trivial tasks. Then, Disser et al. empowered the robot with a compass to solve the mapping problem in polygons with holes [2].

2 Preliminaries

2.1 The Sensing Model and Motion Primitives

The robot has an abstract sensor that reports a cyclically order list of discontinuities in the depth information (gaps) in its visibility region, see Figure 1(a). All the gaps and the target can be located by the robot as they enter in the robots omnidirectional and unbounded field of view. Each gap has a label of L (left) or R (right) which displays the direction of the part of the scene that is hidden behind the gap, see Figure 1.

The robot can orient its heading to each gap and moves towards the gap in an arbitrary number of steps, e.g., two steps towards gap g_x. Each step is a constant distance which is already specified for the robot by its manufacturer, it puts a stepper motor on the robot that specifies its step size, for example 1mm, 2mm,..., also, the robot moves towards the target as it enters its visibility region.

While the robot moves, combinatorial changes occur in the visibility region of the robot called critical events. There are four types of critical events: appearances, disappearances, merges and splits of gaps. Appearance and disappearance events occur when the robot crosses inflection rays. Each gap that appears during the movement, corresponds to a portion of the environment that was already visible, but now is not visible. such gaps are called primitive gaps and all the others are non-primitive gaps. Merge and split events occur when the robot crosses bitangent, as illustrated in Figure 1.

2.2 Known Properties

At each point of the search path, if the target is not visible, the robot reports a set of gaps with the labels of L or R (l-gap and r-gap for abbreviation) cyclically. Let g_l be a non-primitive l-gap that is on the right side of the other left gaps, and g_r be a non-primitive r-gap that is in the left side of the other right gaps, see Figure 1(a). Each of the two gaps is called the most advanced gap. The two gaps have a fundamental role in path planning for the simple robot.

Theorem 1 *[4, 10] While the target is not visible, it is hidden behind one of the two gaps, g_l or g_r.*

From Theorem 1, if there exist only one of the two gaps (g_r and g_l) then the goal is hidden behind the gap. Thus, there is no ambiguity and the robot moves towards the gap, see Figure 2(a). When both of g_r and g_l exist, a funnel case arises, see Figure 2(b). At each funnel case, the robot does not know that the shortest path is along which ofg_r and g_l. So, usually, a detour from the shortest path is unavoidable.

2.3 Essential Information

All we maintain during the search strategy is the location of g_l and g_r. As the robot moves in the street, the critical events that change the structure of the robot's visibility region may dynamically change g_l and g_r. Also, by the robot movement, a funnel case may end or a new funnel may start. We refer to the point, in which a funnel ends a *critical point* of the funnel.

The following events update the location of g_l and g_r as well as a funnel situation when the robot moves towards g_l or g_r.

1. When g_r/g_l splits into g_r/g_l and another r-gap/l-gap, then g_r/g_l will be replaced by the r-gap/l-gap, (point 1 in Figure 2(b)).

2. When g_r/g_l splits into g_r/g_l and another l-gap/r-gap, then l-gap/r-gap will be set as g_l/g_r. This point is a critical point in which a funnel situation ends, (point 2 in Figure 2(b)).

3. When g_l or g_r disappears, the robot may achieve a critical point in which a funnel situation ends, (point 3 in Figure 2(a)).

Note that the split and disappearance events may occur concurrently, (point 3 in Figure 2(b)). Furthermore, by moving towards g_r and g_l, these gaps never merge with other gaps.

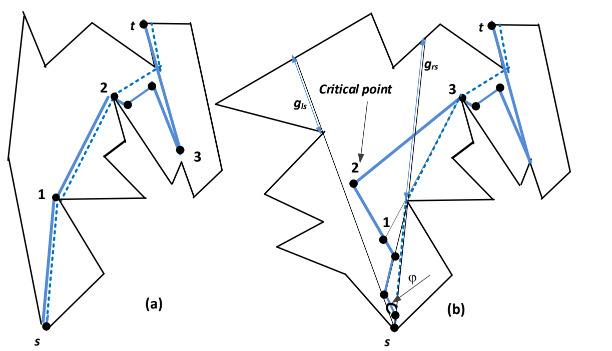

Figure 2: The bold path is the robot search path, the dotted path is shortest path, and v_l and v_r are the corresponding reflex of g_l and g_r respectively. (a) There is only g_r. (b) g_r and g_l are the two most advanced gaps at the start point s, in which a funnel case arises. The angle between the gaps, φ, is the opening angle at the start point.

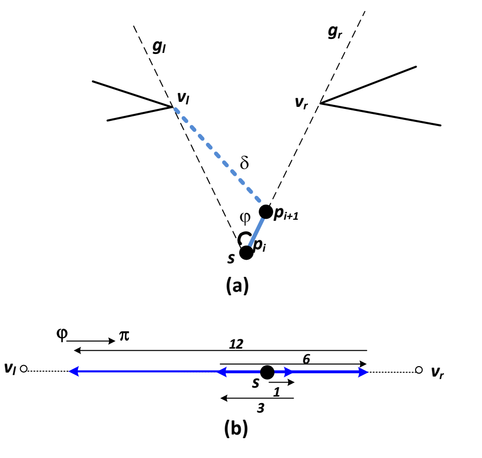

Figure 3: (a) p_ip_{i+1} is a detour from the shortest path. (b) The worst case.

3 Algorithm

Now, we present our strategy for searching the street, from s to t. Since the target is constantly behind one of g_r and g_l, during the search, the location of the two gaps is maintained and dynamically updated as explained in the previous section.

3.1 A Deterministic Strategy

At each point of the search path, especially at the start point s, there are two cases:

- If only one of the two gaps (g_r and g_l) exists, or they are collinear then the goal is hidden behind the gap. The robot moves towards the gap until the target is achieved or a funnel situation arises, see Figure 2(a).
- If there is a funnel case, to bound the detour, the robot moves towards g_r and g_l alternatively, as follows:

  Move towards g_r up to one step;
  $d \leftarrow 3$;
  repeat
      Move towards g_l up to d steps;
      if Critical point not achieved **then**
          $d \leftarrow 2.d$;
          Move towards g_r up to d steps;
      end if
      $d \leftarrow 2.d$;
  until Critical point of the funnel achieved;

  At the critical point, one of g_r or g_l disappears, or g_r and g_l are collinear. So, the robot moves along the existing gap direction until the target is achieved or a new funnel situation arises, as illustrated in Figure 2(b).

3.2 The Randomized Strategy

Now, we present a randomized search strategy based on the above deterministic strategy. The difference between them is using a random variable at the beginning of the above algorithm (in the funnel case). We choose random variable X from $\{0, 1\}$ u.a.r to lead the robot towards g_r or g_l at the first movement while in the deterministic strategy, the robot moves towards g_r.

3.3 Correctness and Analysis

Throughout the search, the robot path coincides with the shortest path unless a funnel case arises. Then, to prove the competitive ratio of our strategy, we compare the length of the path and the shortest path in a funnel case. In the case, the angle between g_r and g_l that is always smaller than π is called the opening angle [4], see Figure 2(b). In lemma 3, we show that our robot detour from the shortest path depends on the size of the angle.

Also, we inspire from the doubling strategy by Baeza-Yates, et al. [1] to compute the competitive ratio of our strategy. In the strategy, a robot moves back and forth on a line such that the distance to the start point doubles at each movement until the target is reached.

Theorem 2 *[1] The doubling strategy for searching a point on a line has a competitive factor of 9, and this is optimal.*

Lemma 3 *By our strategy, the detour from the shortest path for a small opening angle, in the funnel case, is shorter than detour for a large opening angle.*

Theorem 4 *Our deterministic strategy guarantees a path at most 9 times longer than the shortest path. Also, the strategy is optimal.*

The proof of Theorem 4 shows that our deterministic strategy to reach the goal in street is a planar generalization of the doubling strategy for search a point on a line.

Theorem 5 *The randomized strategy generates a search path to achieve target t in the street, starting from s, with an expected competitive ratio of 7.*

3.4 Randomized Lower Bound

To achieve a randomized lower bound of the competitive ratio we consider a special funnel case which it's opening angle is very closed to π. so we can consider it as a problem of searching on the line, see Figure 3(b). Kao, Reif, and Tate [5] proved that the randomized lower bound of the competitive ratio for searching on the line is $1 + (1 + r)/\ln r$ where r is the multiplication factor of the randomized SmartCow algorithm and it is optimal. If we let r=3.59112 we can achieve the expected competitive factor of 4.59112 which is optimal and no other strategy can achieve this bound.

Theorem 6 *There is no on-line randomized strategy for walking in the streets for a simple robot that achieves an expected competitive ratio of less than 4.59112.*

4 Conclusions

In this paper, we have improved the previously known strategy for walking in streets for a simple robot. The point robot can only detect the gaps and the target in the environment. The robot using local information about the location of some gaps, along a 9-competitive optimal path achieves the target t starting from s in a street. Also, based on the improved strategy, a randomized strategy that has better performance is proposed. The expected length of the generated path by the random strategy is 7 times longer than the shortest path. Moreover, a randomized lower bound of 4.59112 is proved. It would be absorbing if there are competitive search strategies for more general classes of polygons.

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Appendix

Proof of lemma 3

In each funnel case, the robot moves some steps towards g_r or g_l, alternatively.

In the alternative movement, one of the directions is correct and the other is a deviation. Assume that at point p_i when a funnel case arises the robot moves toward g_r while the target is behind g_l. The robot achieves point p_{i+1}. In order to achieve the target, it should traverse at least distance $\delta = \sqrt{p_i p_{i+1}^2 + p_i v_l^2 - 2 p_i p_{i+1} p_i v_l \cos\varphi}$, by the law of cosines, see Figure 3(a). It can be verified that δ is strictly increasing as a function of φ by taking the derivative with respect to φ where $0 \leq \varphi < \pi$.

Proof of Theorem 4

In a funnel case, when the opening angle φ is adequately near to π, the simple robot can only move towards left or right. Searching the target in the street in the limited case is similar to searching a line. So walking in street is at least as hard as searching a point on a line. Then, the competitive ratio of 9 is the lower bound for leading the robot in street, see Figure 3(b). From Lemma 3, there is a further deviation from the shortest path for large opening angles. The angle never exceeds π. Then, for computing a competitive factor, we consider it equals π. Starting from s, the robot moves one step towards g_r, then moves $1+2$ steps towards g_l, and again moves forth $2+2^2$ steps towards g_r, moves back 2^2+2^3 steps towards g_l, and so on. In other words, the robot moves back and forth on the line that contains g_l and g_r such that the distance to the start point s doubles until the critical point is reached. By Theorem 2, the competitive factor for the search strategy is 9. Then, the problem of walking in street polygons for a simple robot in the worst-case coincides with the searching a point on a line problem. So, the ratio of 9 is optimal.

Proof of Theorem 5

As shown in Theorem 4, in the worst case, when φ comes close to π, our problem is similar to the problem of searching on the line and our deterministic strategy coincides with the doubling strategy. In the first randomized strategy by choosing the direction of the first movement u.a.r, we have two cases depend on which direction is selected and each of which makes the robot traveling different distances. In the worst case, the critical point is on the $n = 2^k + \delta$ (where k is an integer and δ is a real value satisfying $0 < \delta < 1$) from the origin and the greatest distance for search is taken. Let m be the first stage where robot travels distance at least 2^k on the same path as the critical point exists. The value m satisfies $m \in \{k, k+1\}$. At the beginning of the search, the algorithm chooses a random direction, so $Prob(m = c) = \frac{1}{2}$ for $c = k, k+1$. If D is the random variable denoting the distance traveled by the randomized strategy, then it is easy to see that when $m = c$ we have

$$D = 2\sum_{i=0}^{c} 2^i + n$$

and the expected values calculated as

$$E[D|m = c] = D$$

$$E[D] = \sum_{c=k}^{k+1} Prob(m = c)E[D|m = c]$$

Thus, the resulting expected distance traveled is

$$\begin{aligned}
E[D] &= \frac{1}{2}[2\sum_{i=0}^{k+1} 2^i + n] + \frac{1}{2}[2\sum_{i=0}^{k} 2^i + n] \\
&= \frac{1}{2}[2(2^{k+2} - 1) + 2^k + \delta] + \frac{1}{2}[2(2^{k+1} - 1) + 2^k + \delta] \\
&= \frac{1}{2}[(9)2^k - 2 + \delta] + \frac{1}{2}[(5)2^k - 2 + \delta] \\
&= \frac{1}{2}[(14)2^k + 2\delta - 4] = (7)2^k + \delta - 2 \\
&\leq 7(2^k + \delta) = 7n
\end{aligned}$$