General Physics I

chapter 10

Sharif University of Technology
Mehr 1401 (2022-2023)

M. Reza Rahimi Tabar

Chapter 10 Rotation

- What Is Physics?
- The Rotational Variables
- Are Angular Quantities Vectors?
- Rotation with Constant Angular Acceleration
- Relating the Linear and Angular Variables
- **Kinetic Energy of Rotation**
- **Calculating the Rotational Inertia**
- Torque
- **Newton's Second Law for Rotation**
- Work and Rotational Kinetic Energy

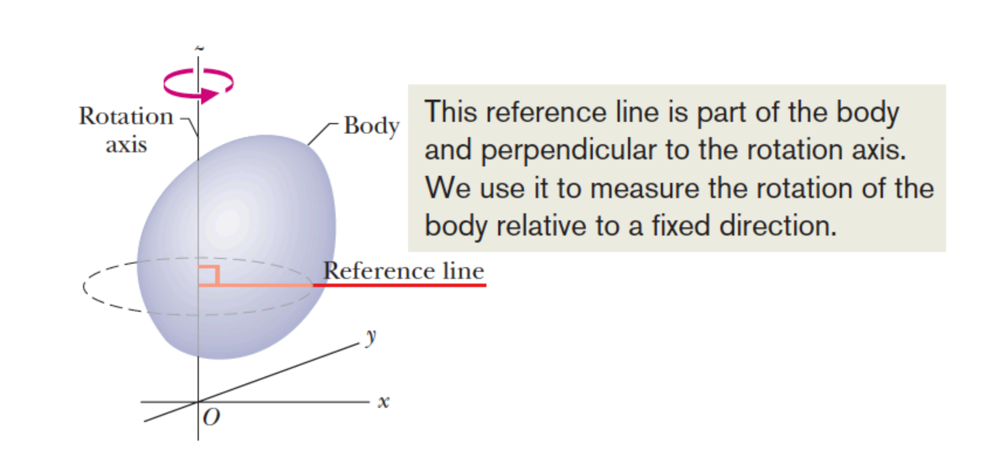
What is Physics?

- Translation, in which an object moves along a straight or curved line.
- We now turn to the motion of rotation, in which an object turns about an axis.
- Rotation is the key to many fun activities,
- Metal failure in aging airplanes.
- Introducing new quantity called **rotational inertia** instead of just **mass**.

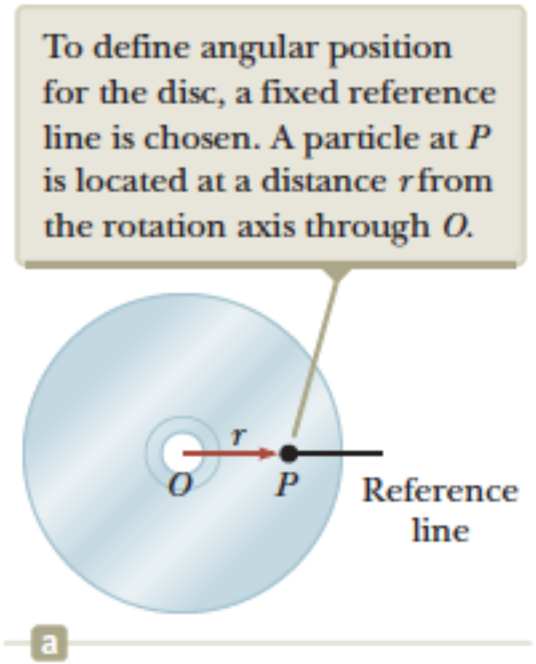
The Rotational Variables

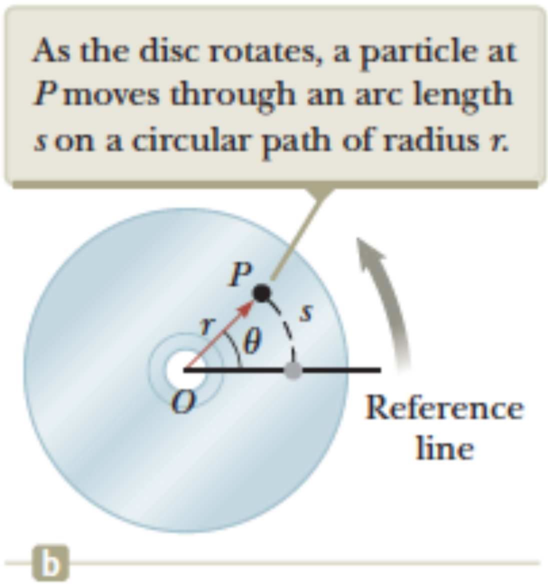
- A **rigid body** is a body that can rotate with all its parts locked together and without any change in its shape.

- A fixed axis means that the rotation occurs about an axis that does not move.



Angular Position



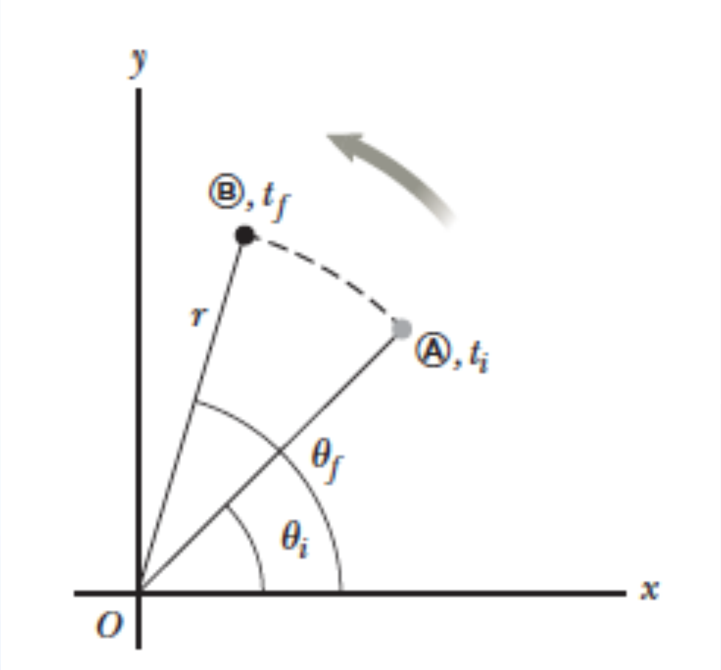


$$s = r\theta$$

$$\theta = \frac{s}{r}$$

$$\theta(\text{rad}) = \frac{\pi}{180°}\theta(\text{deg})$$

Angular Displacement & Angular Velocity



$$\Delta\theta \equiv \theta_f - \theta_i$$

Average angular speed ▶

$$\omega_{avg} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Angular velocity & Angular accelration

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

◀ **Instantaneous angular speed**

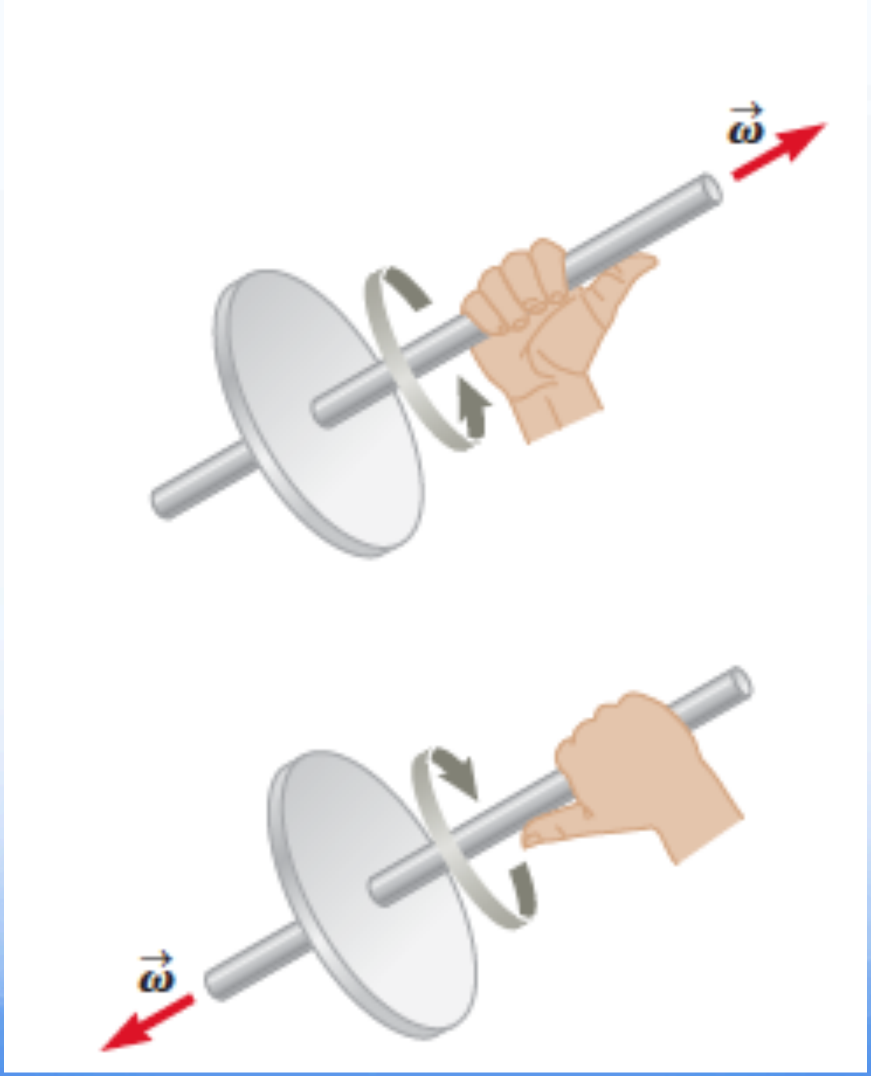
$$\alpha_{\text{avg}} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta \omega}{\Delta t}$$

◀ **Average angular acceleration**

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

◀ **Instantaneous angular acceleration**

The right-hand rule establishes the direction of the angular velocity vector:



Rotation with Constant Angular Acceleration

$$d\omega = \alpha \, dt$$

$t_i = 0$ to $t_f = t$

$$\omega_f = \omega_i + \alpha t \quad \text{(for constant } \alpha\text{)}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2 \quad \text{(for constant } \alpha\text{)}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \quad \text{(for constant } \alpha\text{)}$$

$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)t \quad \text{(for constant } \alpha\text{)}$$

Equivalence:

$$\Delta\theta \Leftrightarrow \Delta x$$

$$\omega_0 \Leftrightarrow v_0$$

$$\omega_f \Leftrightarrow v_f$$

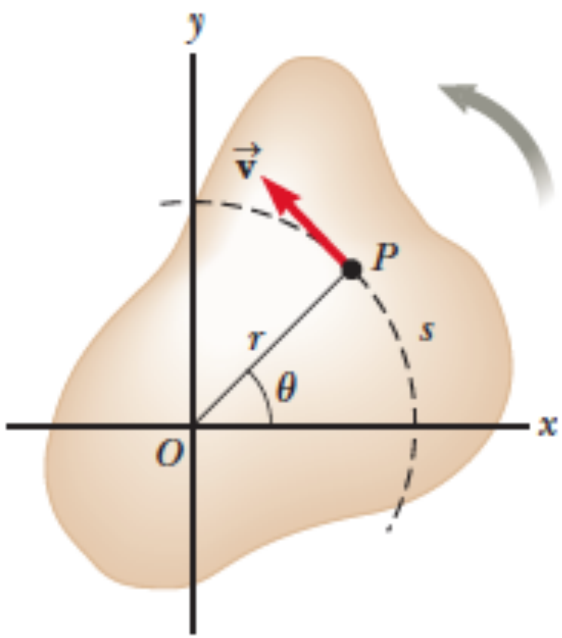
$$\alpha \Leftrightarrow a$$

$$t \Leftrightarrow t$$

Linear and Rotational Motion Analogies

Rotational Motion	Linear Motion
$\Delta\theta = \frac{(\omega_0 + \omega_f)}{2} t$	$\Delta x = \frac{(v_0 + v_f)}{2} t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\frac{\omega_f^2}{2} = \frac{\omega_0^2}{2} + \alpha\Delta\theta$	$\frac{v_f^2}{2} = \frac{v_0^2}{2} + a\Delta x$
$\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$	$\Delta x = v_f t - \frac{1}{2}at^2$

Relating the linear and Angular Variables



$$v = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

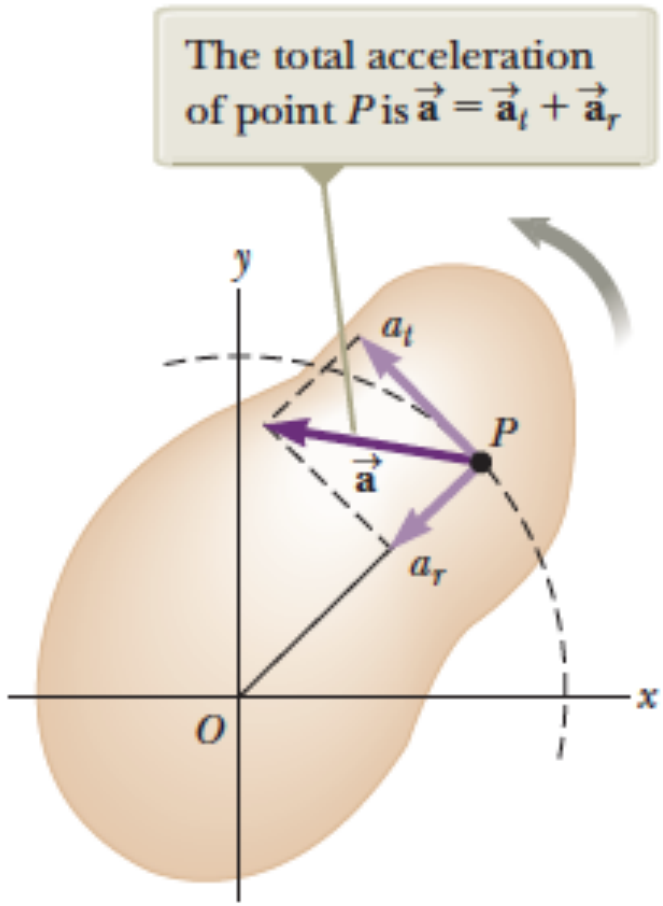
$$d\theta/dt = \omega$$

$$v = r\omega$$ ✔

Relation between tangential acceleration and angular acceleration ▶

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$a_t = r\alpha$$ ✔



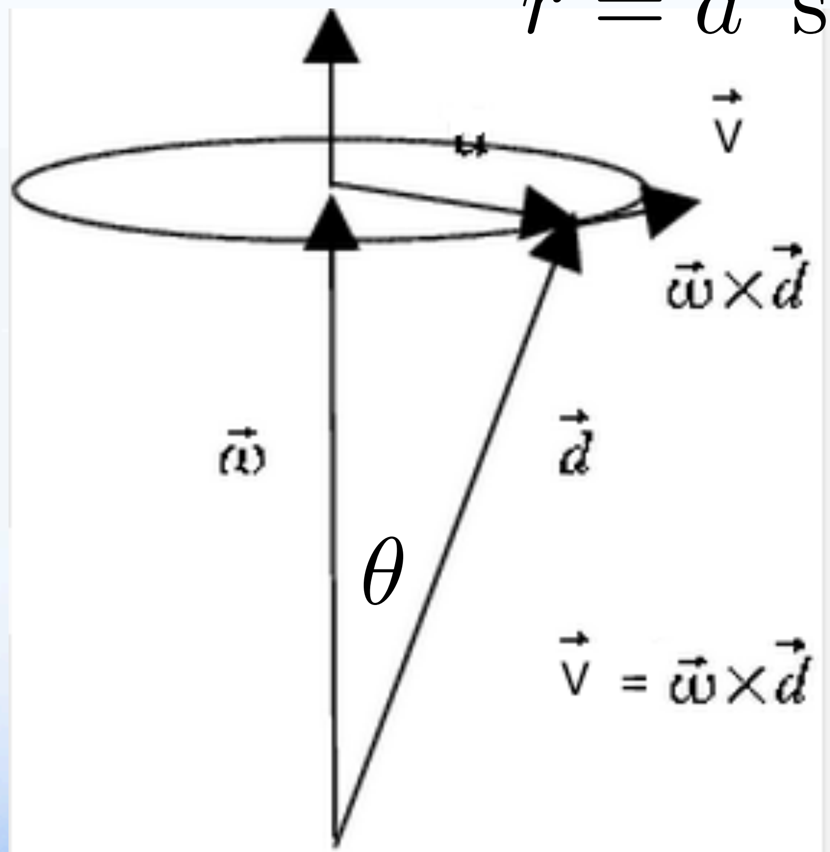
$$a_t = \frac{v^2}{r} = r\omega^2$$

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_r$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

Angular and linear velocities relationship

$$r = d \ \sin(\theta)$$



Zusammenfassung

Summary

$$\omega = 2\pi / T \qquad \omega = \dot{\varphi}$$

$$v = \omega \cdot r \qquad \vec{v} = \vec{\omega} \times \vec{r}$$

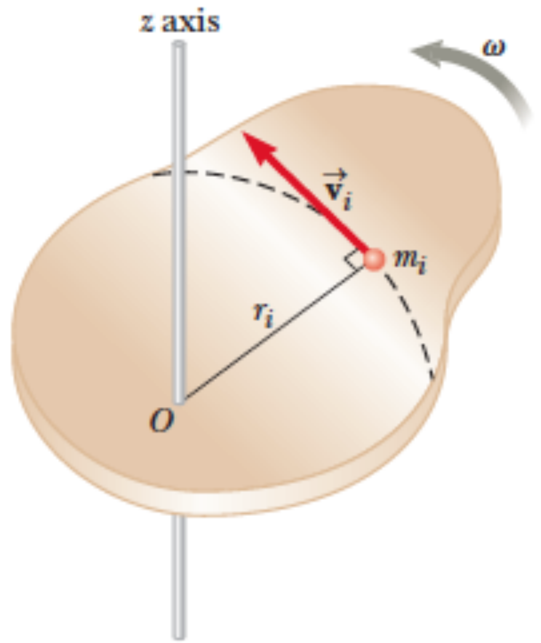
$$a_r = \omega^2 r = v^2 / r$$

$$\vec{a}_r = \vec{\omega} \times \vec{v}$$

$$\alpha = \dot{\omega} \qquad \alpha = \ddot{\varphi}$$

$$a_t = \alpha \cdot r \qquad \vec{a}_t = \vec{\alpha} \times \vec{r}$$

Kinetic Energy of Rotation



$$K_i = \tfrac{1}{2} m_i v_i^2$$

$$K_R = \sum_i K_i = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} \sum_i m_i r_i^2 \omega^2$$

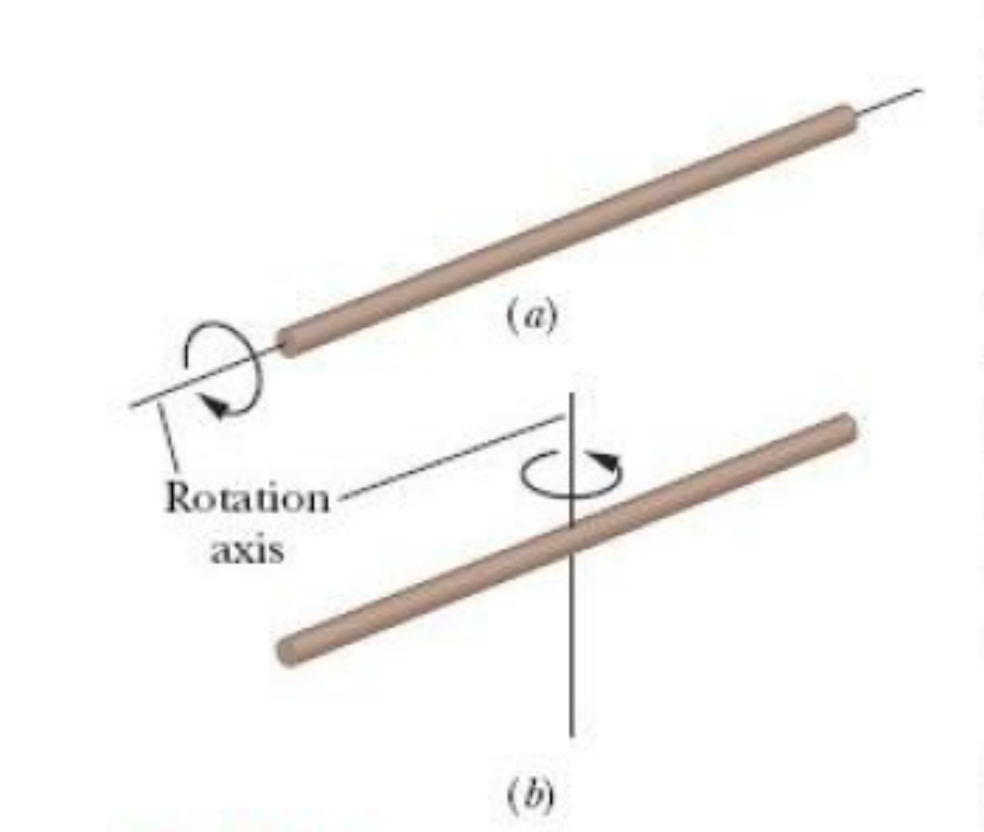
$$K_R = \tfrac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

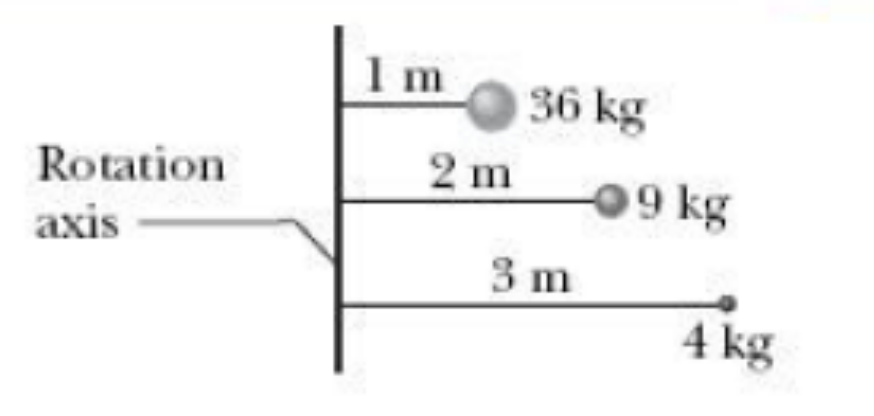
$$I \equiv \sum_i m_i r_i^2$$ ✔

$$K_R = \tfrac{1}{2} I \omega^2$$ ✔

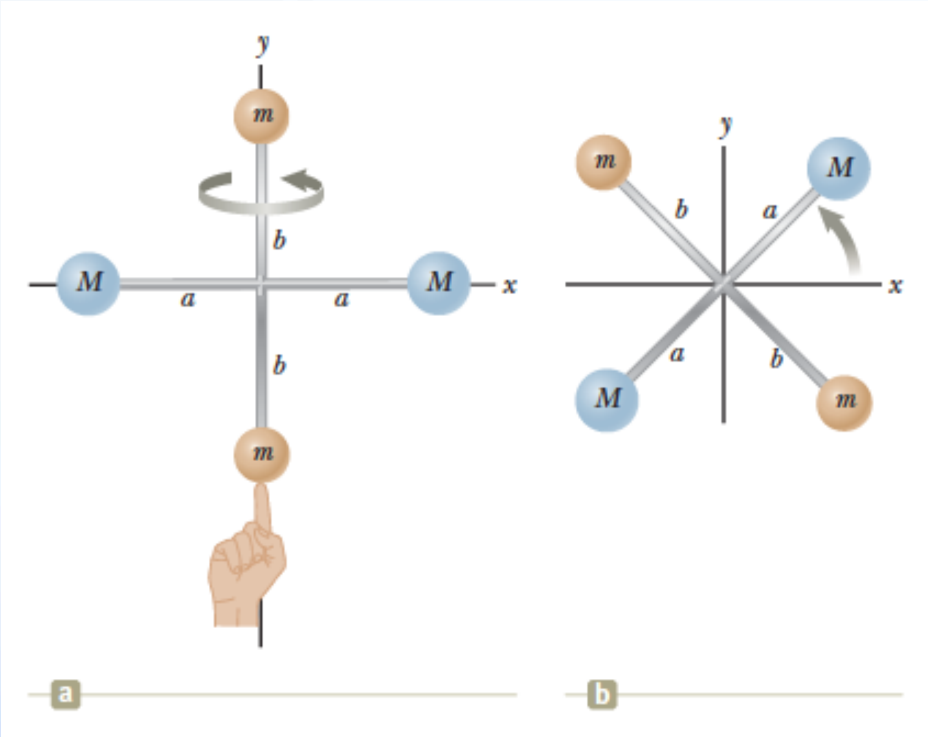
Calculating the Rotational Inertia

- Example:

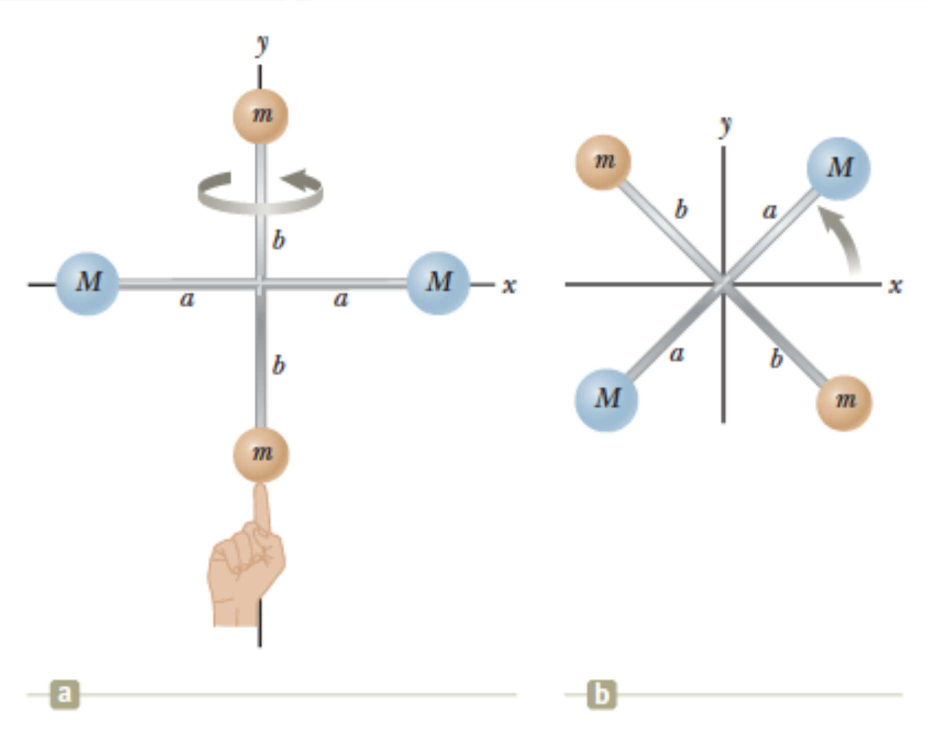




Example:



Example:



$$I_y = \sum_i m_i r_i^2 = Ma^2 + Ma^2 = \boxed{2Ma^2}$$

$$K_R = \frac{1}{2} I_y \omega^2 = \frac{1}{2}(2Ma^2)\omega^2 = \boxed{Ma^2\omega^2}$$

$$I_z = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2 = \boxed{2Ma^2 + 2mb^2}$$

$$K_R = \frac{1}{2} I_z \omega^2 = \frac{1}{2}(2Ma^2 + 2mb^2)\omega^2 = \boxed{(Ma^2 + mb^2)\omega^2}$$

Moment of Inertia

$$I \equiv \sum_i m_i r_i^2$$

Moment of inertia of a rigid object ▶

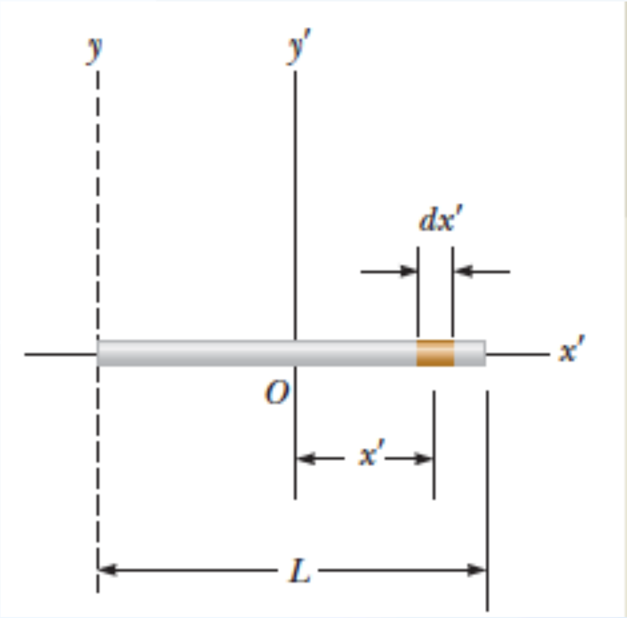
$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \, \Delta m_i = \int r^2 \, dm$$

✔

$$I = \int \rho r^2 \, dV$$

Example:

Calculate the moment of inertia of a uniform rigid rod of length L and mass M (Fig. 10.9) about an axis perpendicular to the rod (the y' axis) and passing through its center of mass.



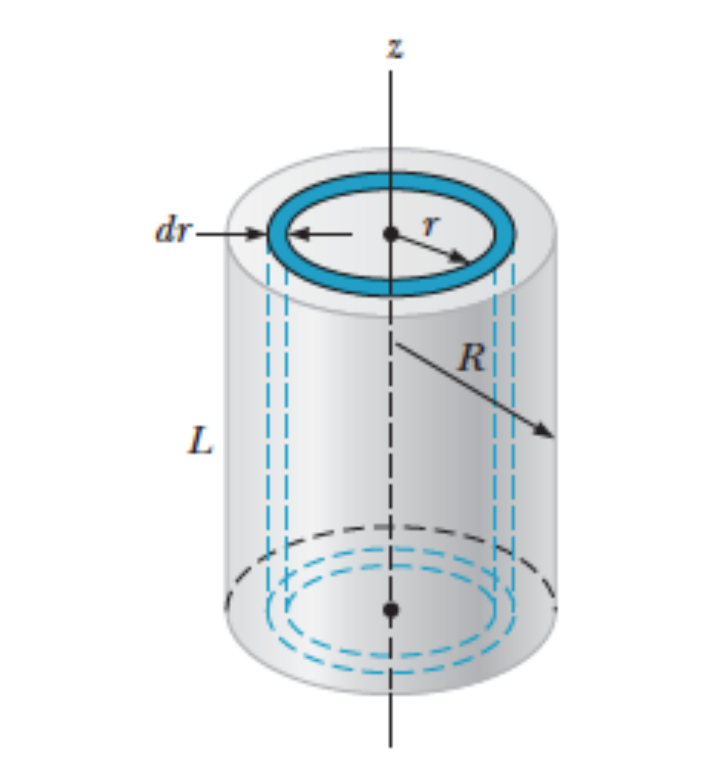
$$dm = \lambda\, dx' = \frac{M}{L}\, dx'$$

$$I_y = \int r^2\, dm = \int_{-L/2}^{L/2} (x')^2 \frac{M}{L}\, dx' = \frac{M}{L} \int_{-L/2}^{L/2} (x')^2\, dx'$$

$$= \frac{M}{L}\left[\frac{(x')^3}{3}\right]_{-L/2}^{L/2} = \tfrac{1}{12} ML^2$$

Example:

A uniform solid cylinder has a radius R, mass M, and length L. Calculate its moment of inertia about its central axis (the z axis in Fig. 10.10).



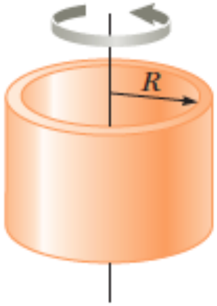
$$dm = \rho\, dV = \rho L(2\pi r)\, dr$$

$$I_z = \int r^2\, dm = \int r^2[\rho L(2\pi r)\, dr] = 2\pi\rho L \int_0^R r^3\, dr = \tfrac{1}{2}\pi\rho L R^4$$

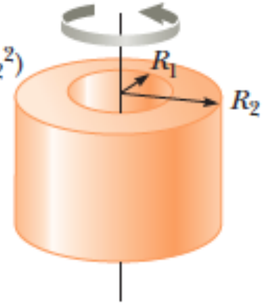
$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$$

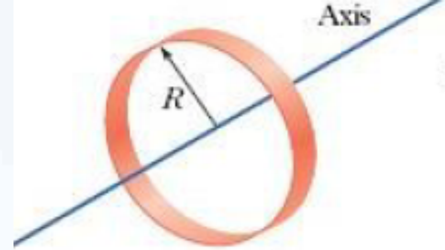
$$I_z = \tfrac{1}{2}\pi\left(\frac{M}{\pi R^2 L}\right) L R^4 = \boxed{\tfrac{1}{2}MR^2}$$

Hoop or thin cylindrical shell
$I_{CM} = MR^2$



Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$

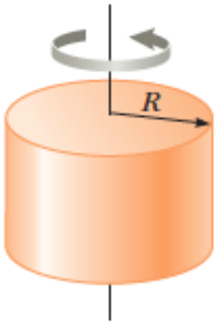




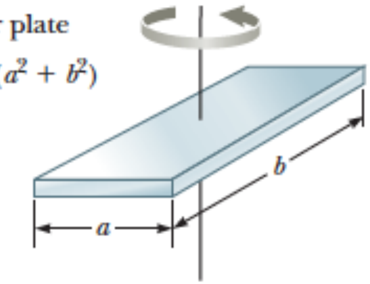
Hoop about central axis

$I = MR^2$

Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



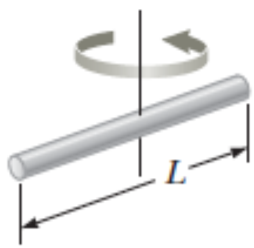
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



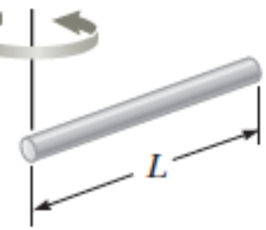
Hoop about any diameter

$I = \frac{1}{2}MR^2$

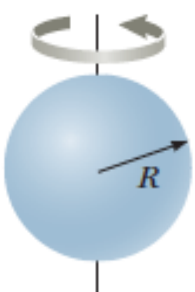
Long, thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$



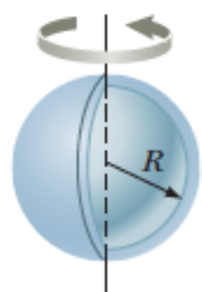
Long, thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$



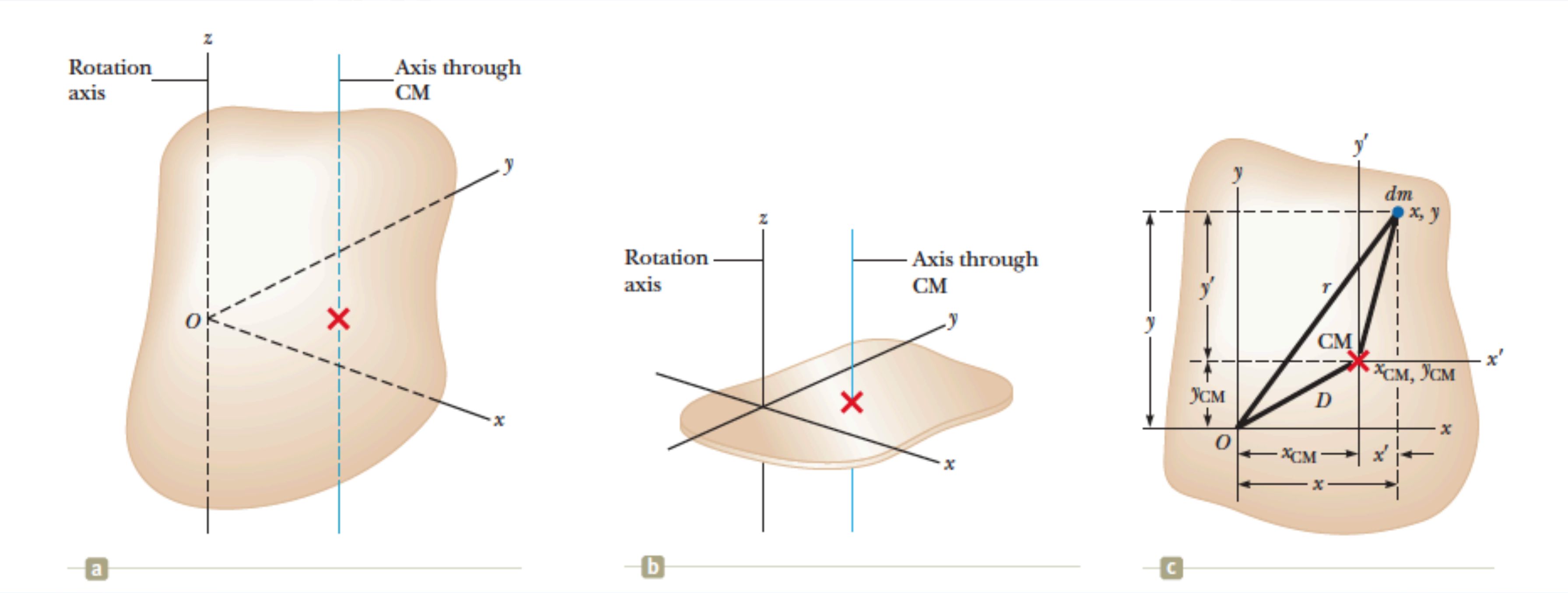
Solid sphere
$I_{CM} = \frac{2}{5}MR^2$



Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$



The parallel-axis theorem:



$$r = \sqrt{x^2 + y^2}$$

$$I = \int r^2\, dm = \int (x^2 + y^2)\, dm$$

$$I = \int [(x' + x_{CM})^2 + (y' + y_{CM})^2]\, dm$$

$$= \int [(x')^2 + (y')^2]\, dm + 2x_{CM} \int x'\, dm + 2y_{CM} \int y'\, dm + (x_{CM}{}^2 + y_{CM}{}^2) \int dm$$

$$\int x'\, dm = \int y'\, dm = 0. \qquad \int dm = M$$

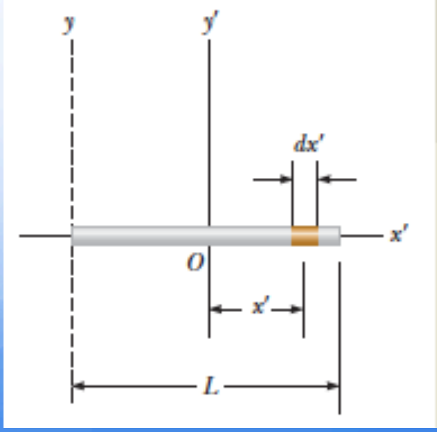
$$D^2 = x_{CM}^2 + y_{CM}^2.$$

$$I = I_{CM} + MD^2$$

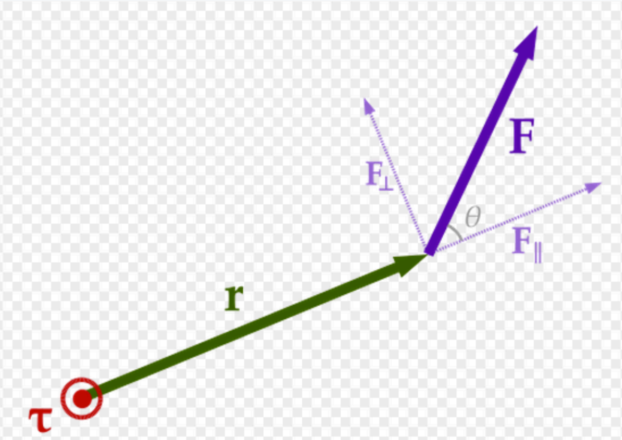
◀ **Parallel-axis theorem** ✔

Consider once again the uniform rigid rod of mass M and length L shown in Figure 10.9. Find the moment of inertia of the rod about an axis perpendicular to the rod through one end (the y axis in Fig. 10.9).

$$I = I_{CM} + MD^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$



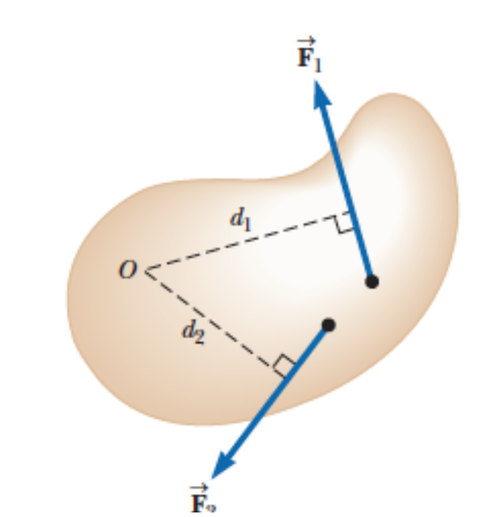
Torque



$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F},$$

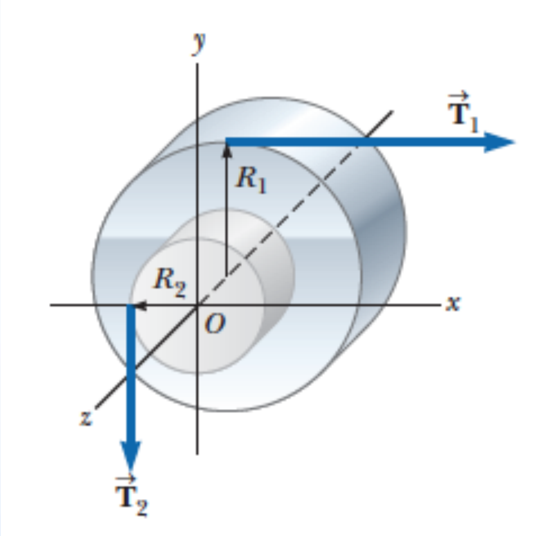
$$\tau = rF\sin\theta,$$

$$\tau = rF_{\perp}$$

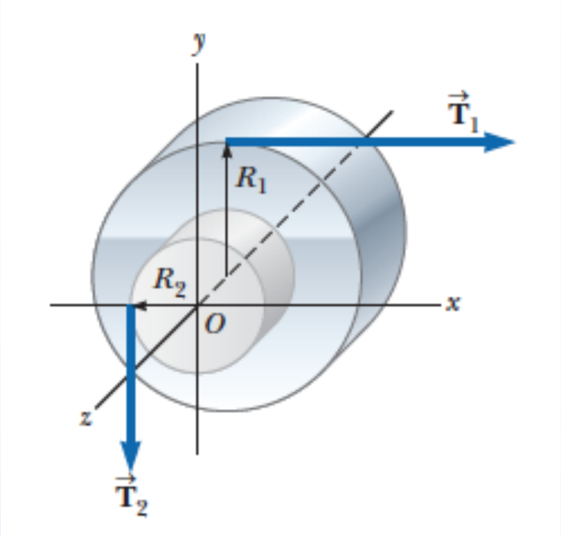


$$\sum \tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$$

Example:

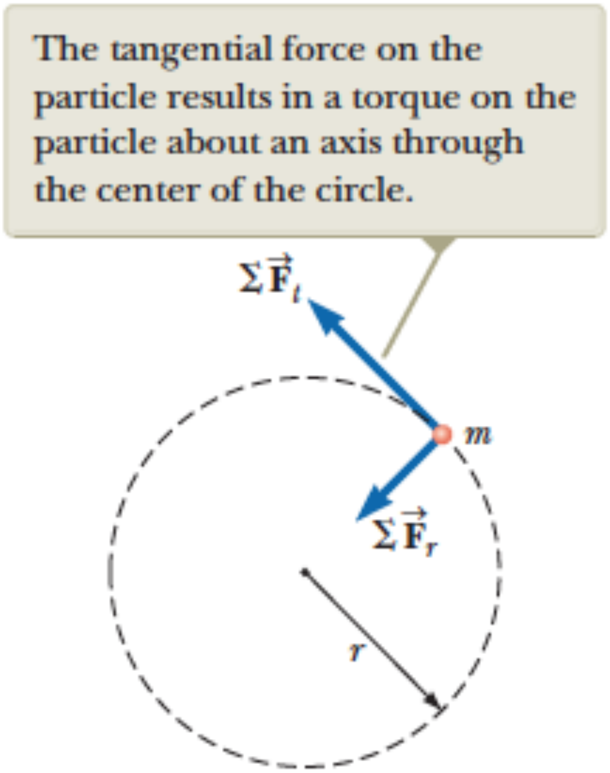


Example:



$$\sum \tau = \tau_1 + \tau_2 = R_2T_2 - R_1T_1$$

Newton's Second Law for Rotation



$$\sum F_t = ma_t$$

$$\sum \tau = \sum F_t r = (ma_t)r$$

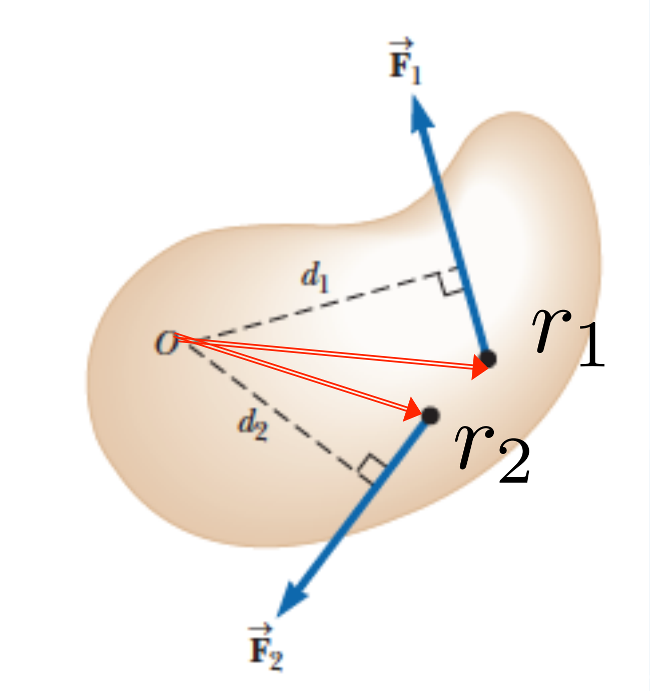
$$a_t = r\alpha$$

$$\sum \tau = (mr\alpha)r = (mr^2)\alpha$$

Torque on a rigid object is proportional to angular acceleration ▶

$$\sum \tau = I\alpha$$

✔



$$\tau_i = F_{t_i} r_i = m_i a_{t_i} t_i = m_i (r_i \alpha) r_i = m_i r_i^2 \alpha$$

$$\sum_i \tau_i = \sum_i m_i r_i^2 \; \alpha = (\sum_i m_i r_i^2) \; \alpha = I \; \alpha$$

Newton's Second Law for Rotation

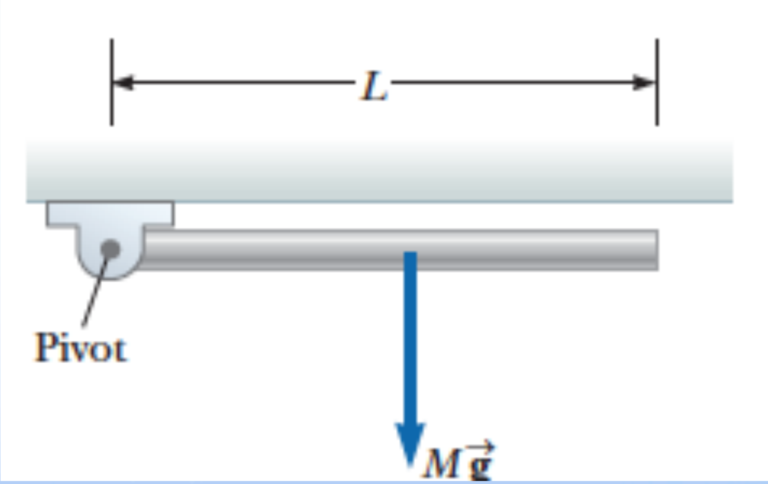
Torque on a rigid object is proportional to angular acceleration ▶

$$\sum \tau_{ext} = I\alpha$$

✔

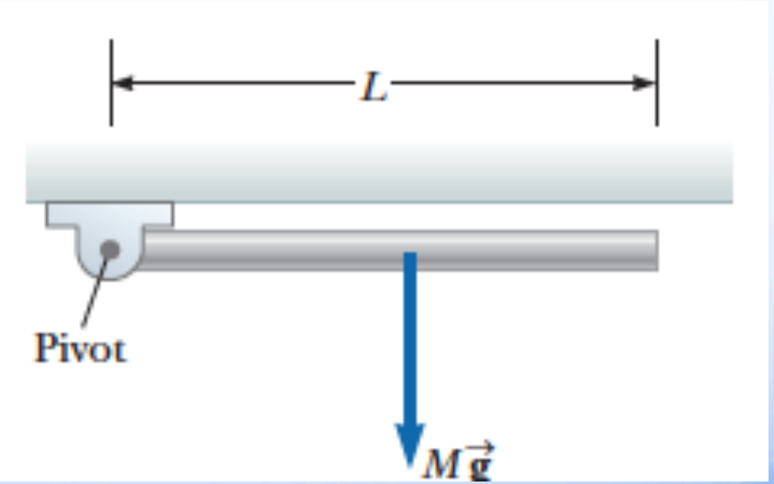
Example:

A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane as in Figure 10.17. The rod is released from rest in the horizontal position. What are the initial angular acceleration of the rod and the initial translational acceleration of its right end?



Example:

A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane as in Figure 10.17. The rod is released from rest in the horizontal position. What are the initial angular acceleration of the rod and the initial translational acceleration of its right end?



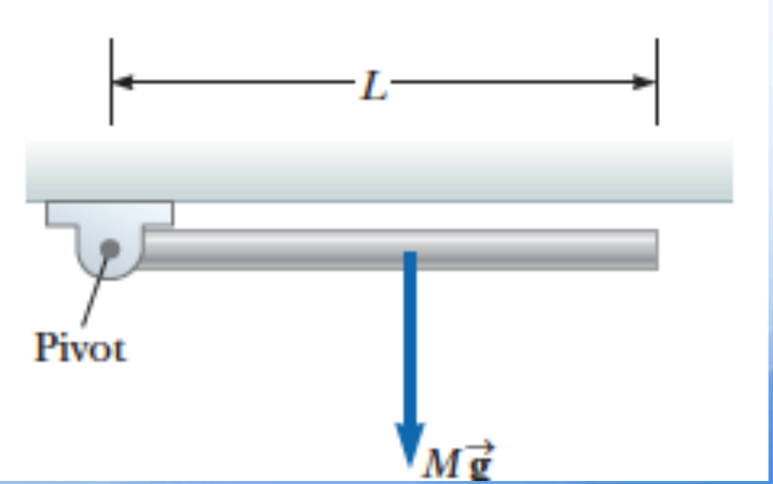
– Determine "rotation axis"
– Write down the torque of external forces
– Calculate I about the rotation axis
– Use $\sum \tau_{ext} = I\alpha$

$$\sum \tau_{ext} = Mg\left(\frac{L}{2}\right)$$

$$(1)\ \alpha = \frac{\sum \tau_{ext}}{I} = \frac{Mg(L/2)}{\frac{1}{3}ML^2} = \boxed{\frac{3g}{2L}}$$

$$a_t = L\alpha = \boxed{\tfrac{3}{2}g}$$

$$a_t = r\alpha = \frac{3g}{2L}r$$

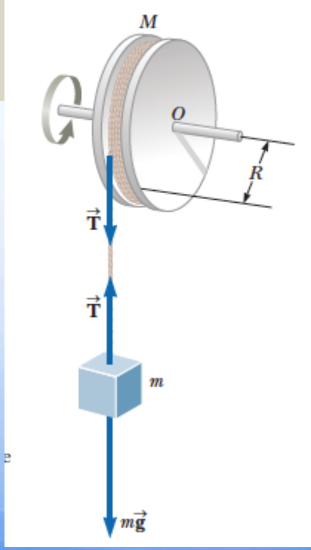


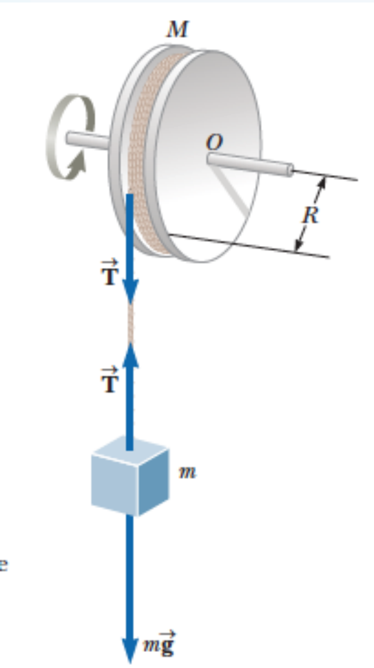
$$a_t = g = \frac{3g}{2L}r$$

$$r = \tfrac{2}{3}L$$

Example:

A wheel of radius R, mass M, and moment of inertia I is mounted on a frictionless, horizontal axle as in Figure 10.19. A light cord wrapped around the wheel supports an object of mass m. When the wheel is released, the object accelerates downward, the cord unwraps off the wheel, and the wheel rotates with an angular acceleration. Calculate the angular acceleration of the wheel, the translational acceleration of the object, and the tension in the cord.





$$\sum \tau_{\text{ext}} = I\alpha$$

$$(1)\ \alpha = \frac{\sum \tau_{\text{ext}}}{I} = \frac{TR}{I}$$

$$\sum F_y = mg - T = ma$$

$$(2)\ a = \frac{mg - T}{m}$$

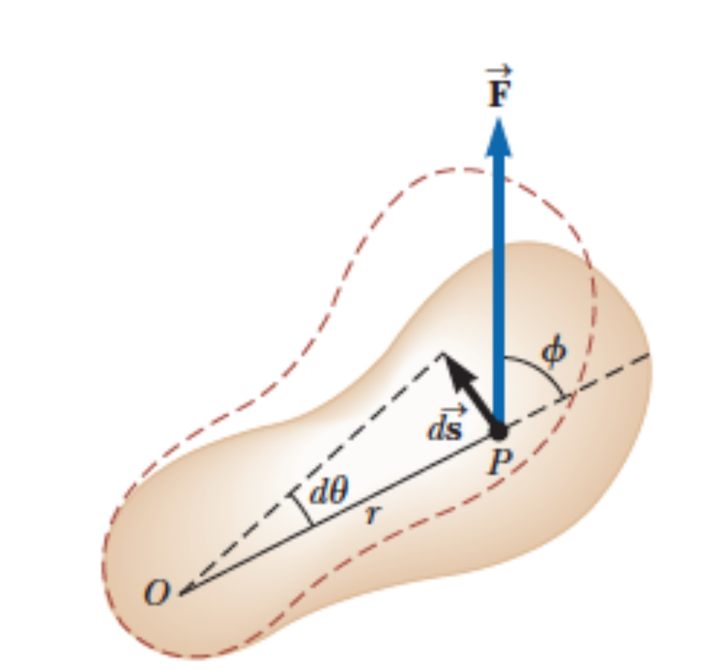
$$(3)\ a = R\alpha = \frac{TR^2}{I} = \frac{mg - T}{m}$$

$$(4)\ T = \frac{mg}{1 + (mR^2/I)}$$

$$(5)\ a = \frac{g}{1 + (I/mR^2)}$$

$$\alpha = \frac{a}{R} = \frac{g}{R + (I/mR)}$$

Work and Rotational Kinetic Energy



$$dW = \vec{F} \cdot d\vec{s} = (F \sin \phi) r \, d\theta$$

$$dW = \tau \, d\theta$$

✔

$$\frac{dW}{dt} = \tau \frac{d\theta}{dt}$$

$$P = \frac{dW}{dt} = \tau\omega$$

$$P = Fv$$

The work-kinetic energy theorem

$$\sum \tau_{ext} = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt} = I\frac{d\omega}{d\theta}\omega$$

$$\Sigma \tau_{ext}\, d\theta = dW$$

$$dW = \tau\, d\theta$$

$$\sum \tau_{ext}\, d\theta = dW = I\omega\, d\omega$$

Work–kinetic energy theorem for rotational motion ▶

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2$$

✔

Some Corresponding Relations for Translational and Rotational Motion

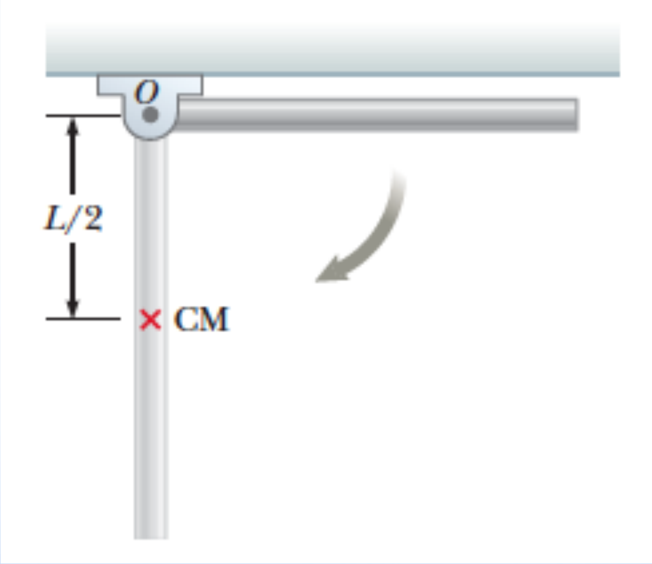
Rotational Motion About a Fixed Axis	Translational Motion
Angular speed $\omega = d\theta/dt$	Translational speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Translational acceleration $a = dv/dt$
Net torque $\Sigma\tau_{\text{ext}} = I\alpha$	Net force $\Sigma F = ma$
If α = constant $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If a = constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $P = \tau\omega$	Power $P = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

✔

Example:

A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end (Fig 10.21). The rod is released from rest in the horizontal position.

(A) What is its angular speed when the rod reaches its lowest position?

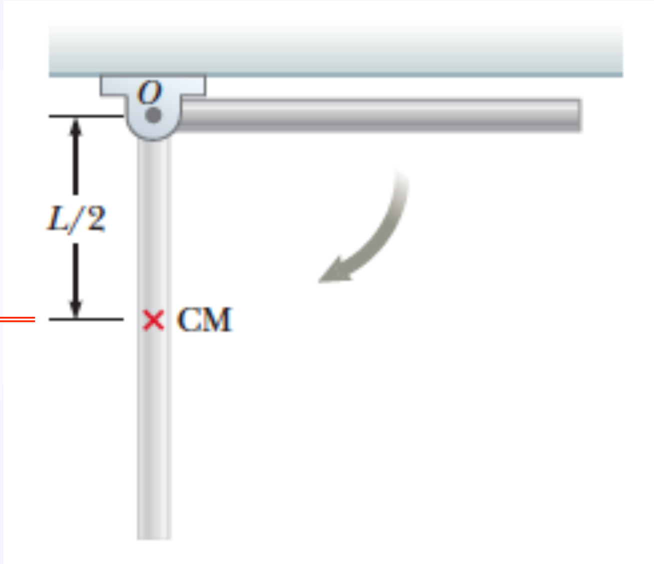


(B) Determine the tangential speed of the center of mass and the tangential speed of the lowest point on the rod when it is in the vertical position.

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}I\omega^2 + 0 = 0 + \tfrac{1}{2}MgL$$

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \sqrt{\frac{3g}{L}}$$

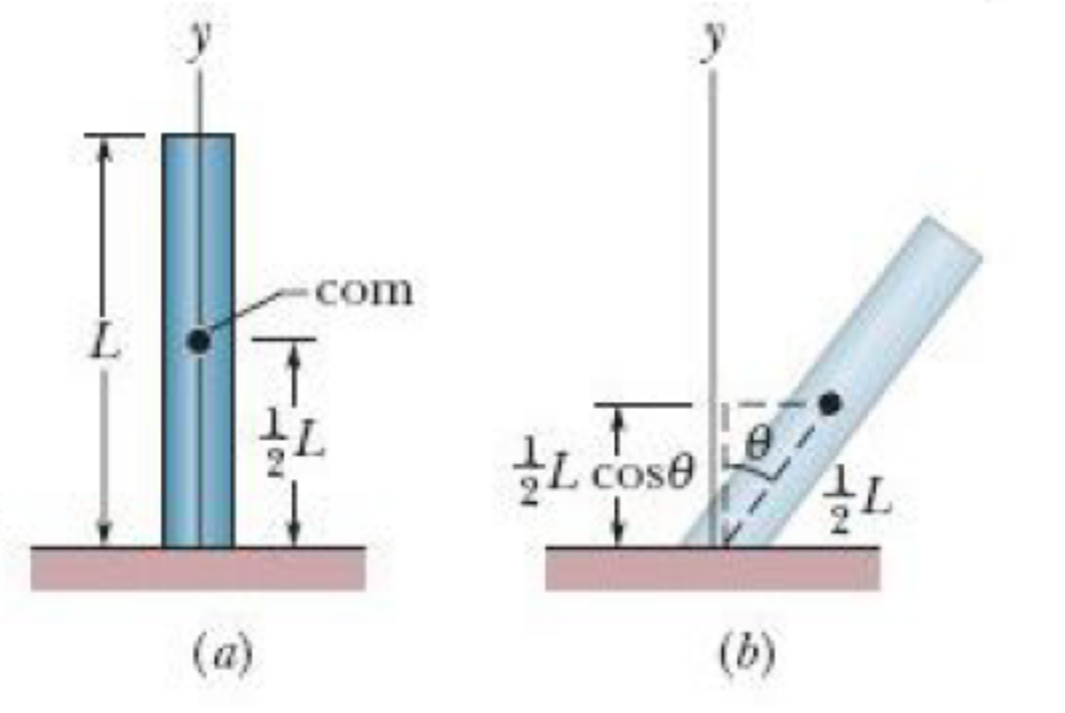


$$v_{CM} = r\omega = \frac{L}{2}\omega = \tfrac{1}{2}\sqrt{3gL}$$

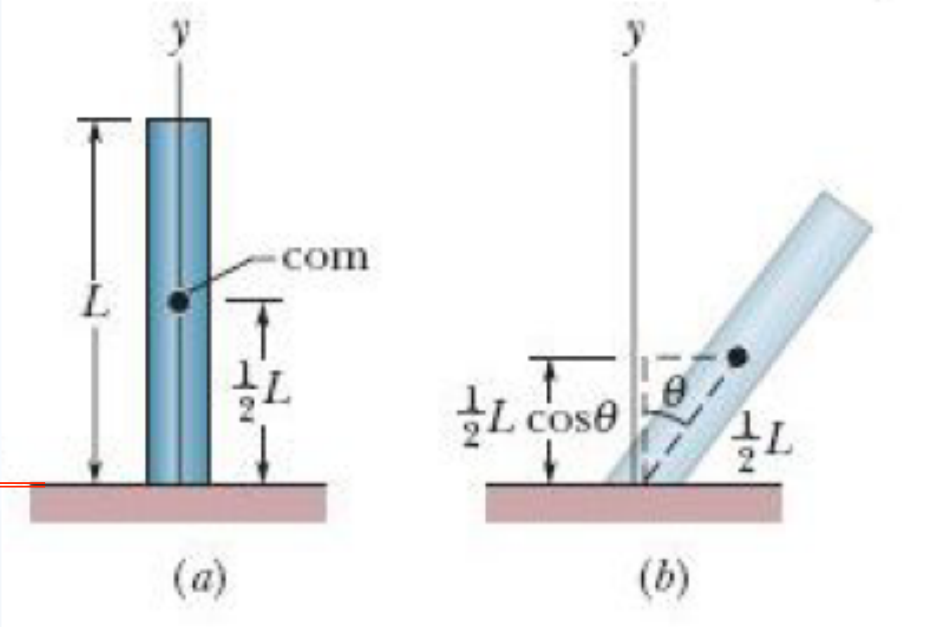
$$v = 2v_{CM} = \sqrt{3gL}$$

$$I = \tfrac{1}{3}ML^2$$

A tall, cylindrical chimney will fall over when its base is ruptured. Treat the chimney as a thin rod of length $L = 55.0$ m (Fig. 10-20*a*). At the instant it makes an angle of $\theta = 35.0°$ with the vertical, what is its angular speed ω_f?



$$K_f + U_f = K_i + U_i.$$



$$U_i = \tfrac{1}{2}mgL.$$

$$U_f = \tfrac{1}{2}mgL\cos\theta.$$

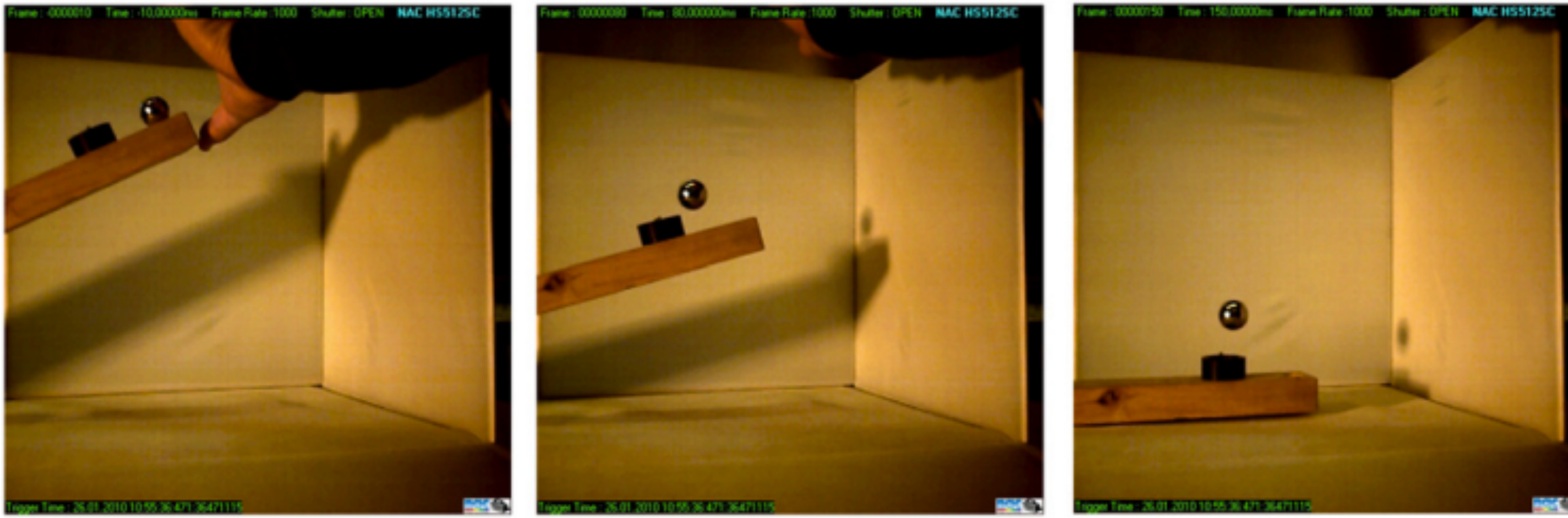
$$I = \tfrac{1}{12}mL + m\left(\frac{L}{2}\right)^2 = \tfrac{1}{3}mL^2.$$

$$K_f = \tfrac{1}{2}(\tfrac{1}{3}mL^2)\omega^2.$$

$$\omega = \sqrt{\frac{3g}{L}(1-\cos\theta)} = \sqrt{\frac{3(9.8\ \text{m/s}^2)}{55.0\ \text{m}}(1-\cos 35.0^\circ)}$$

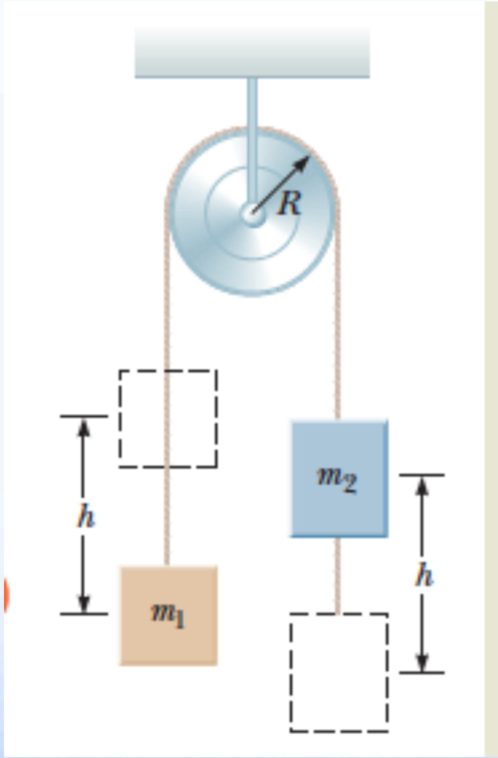
$$= 0.311\ \text{rad/s.} \qquad \text{(Answer)}$$

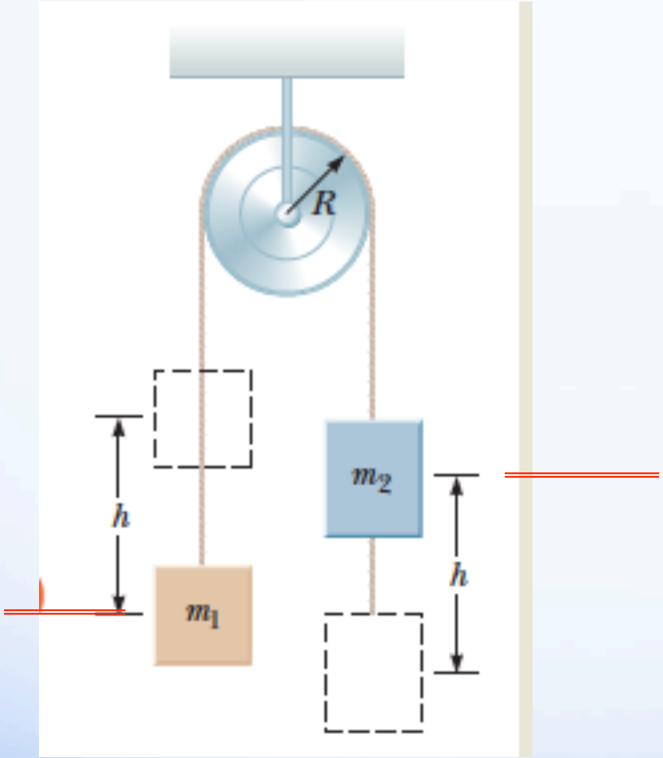
Falling chimneys (faster than g)



http://iopscience.iop.org/0143-0807/33/5/1277

Two blocks having different masses m_1 and m_2 are connected by a string passing over a pulley as shown in Active Figure 10.22. The pulley has a radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley, and the system is released from rest. Find the translational speeds of the blocks after block 2 descends through a distance h and find the angular speed of the pulley at this time.





$$K_f + U_f = K_i + U_i$$

$$\left(\tfrac{1}{2}m_1 v_f^{\,2} + \tfrac{1}{2}m_2 v_f^{\,2} + \tfrac{1}{2}I\omega_f^{\,2}\right) + (m_1 gh - m_2 gh) = 0 + 0$$

$$\tfrac{1}{2}m_1 v_f^{\,2} + \tfrac{1}{2}m_2 v_f^{\,2} + \tfrac{1}{2}I\frac{v_f^{\,2}}{R^2} = m_2 gh - m_1 gh$$

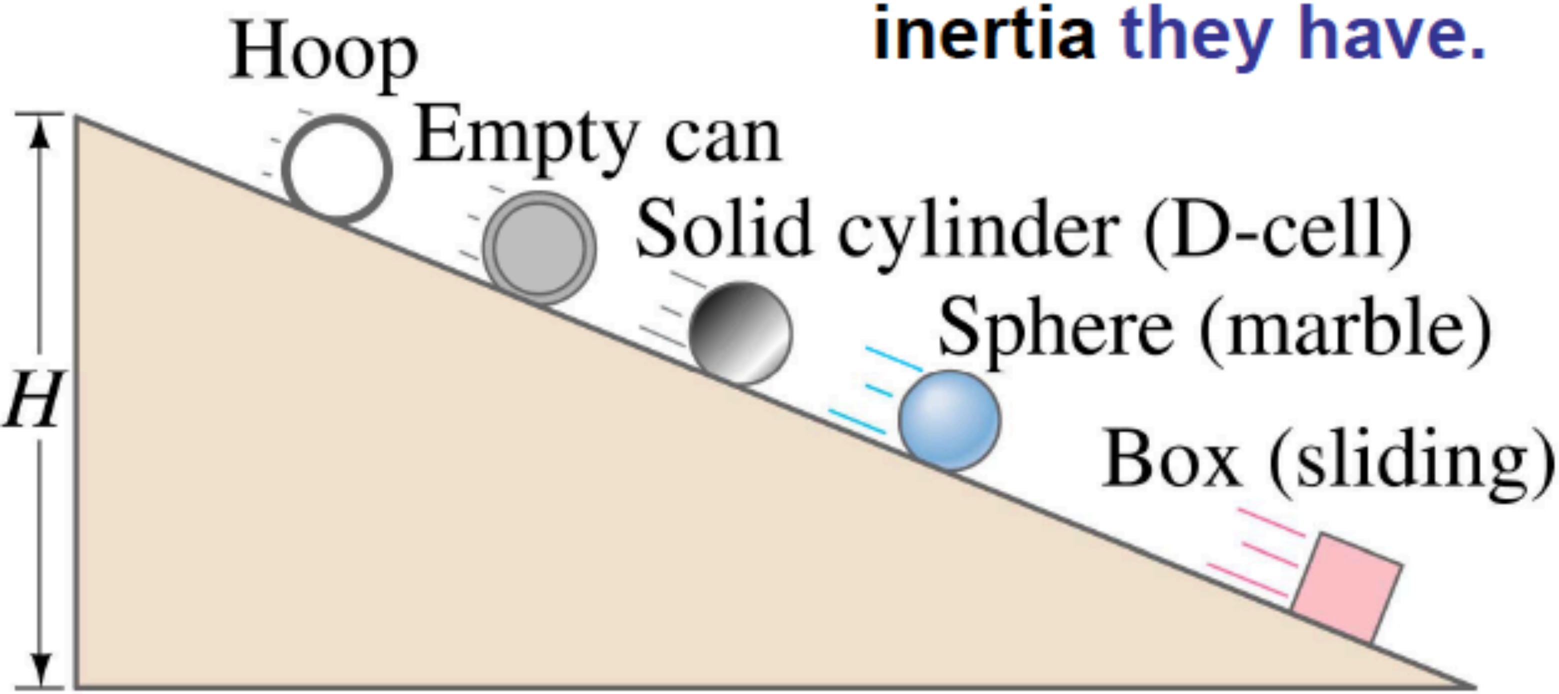
$$\tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v_f^{\,2} = m_2 gh - m_1 gh$$

$$(1) \quad v_f = \left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}$$

$$\omega_f = \frac{v_f}{R} = \frac{1}{R}\left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}$$

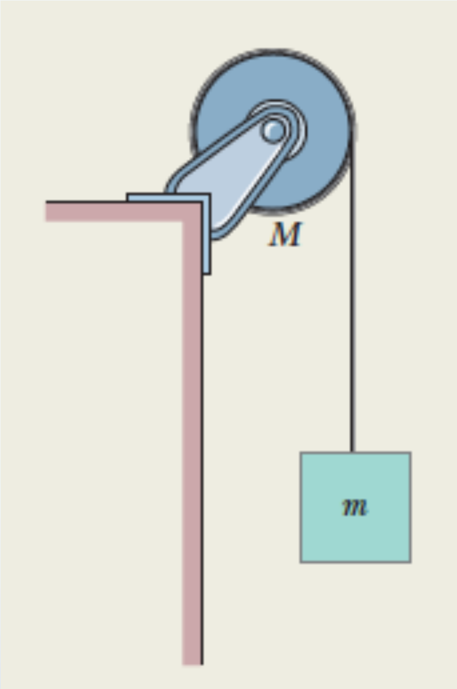
Example:

All these objects have the same potential energy at the top, but the time it takes them to get down the incline depends on how much rotational inertia they have.



Example:

Figure 10-19*a* shows a uniform disk, with mass $M = 2.5$ kg and radius $R = 20$ cm, mounted on a fixed horizontal axle. A block with mass $m = 1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.



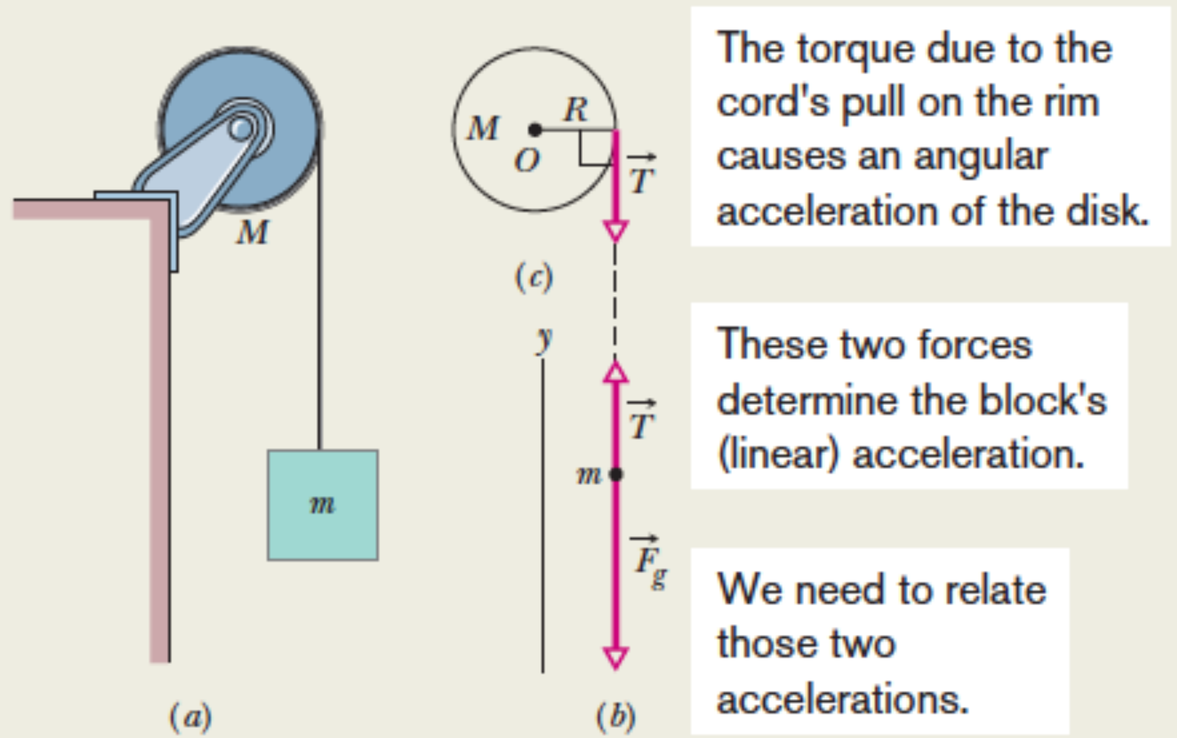


Figure 10-19 (*a*) The falling block causes the disk to rotate. (*b*) A free-body diagram for the block. (*c*) An incomplete free-body diagram for the disk.

$$T = \tfrac{1}{2} Ma.$$

$$T - mg = m(-a),$$

$$a_t = \alpha r$$

we can write the general equation $\tau_{net} = I\alpha$ as

$$-RT = \tfrac{1}{2}MR^2(-\alpha).$$

$$a = g\frac{2m}{M + 2m} = (9.8 \text{ m/s}^2)\frac{(2)(1.2 \text{ kg})}{2.5 \text{ kg} + (2)(1.2 \text{ kg})}$$

$$= 4.8 \text{ m/s}^2. \qquad \text{(Answer)}$$

$$T = \tfrac{1}{2}Ma = \tfrac{1}{2}(2.5 \text{ kg})(4.8 \text{ m/s}^2)$$

$$= 6.0 \text{ N}. \qquad \text{(Answer)}$$

$$\alpha = \frac{a}{R} = \frac{4.8 \text{ m/s}^2}{0.20 \text{ m}} = 24 \text{ rad/s}^2.$$

Example:

98 A yo-yo-shaped device mounted on a horizontal frictionless axis is used to lift a 30 kg box as shown in Fig. 10-59. The outer radius R of the device is 0.50 m, and the radius r of the hub is 0.20 m. When a constant horizontal force $\vec{F}_{app}$ of magnitude 140 N is applied to a rope wrapped around the outside of the device, the box, which is suspended from a rope wrapped around the hub, has an upward acceleration of magnitude 0.80 m/s². What is the rotational inertia of the device about its axis of rotation?

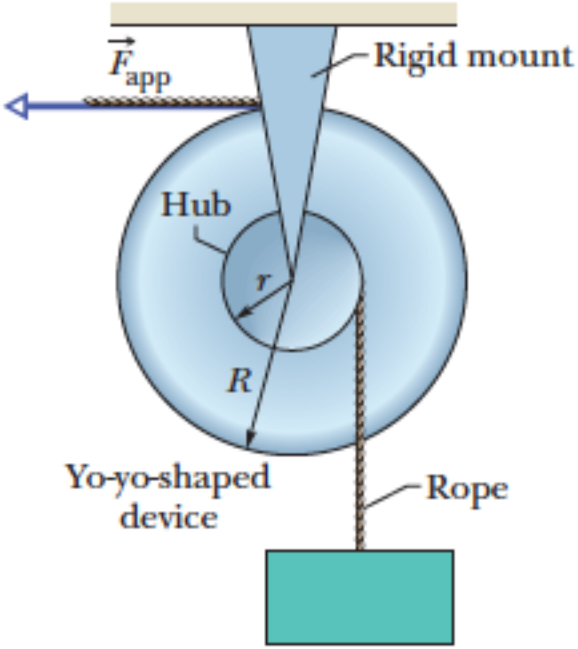
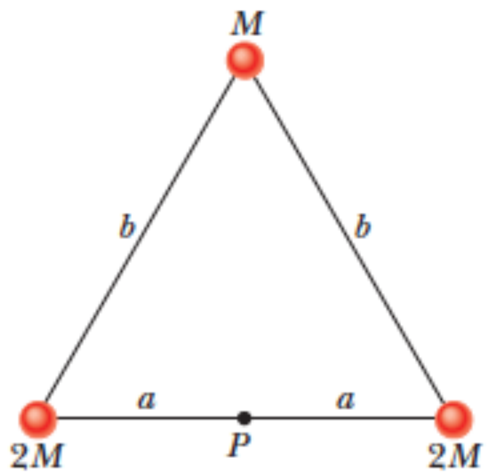


Figure 10-59 Problem 98.

Example:

95 The rigid body shown in Fig. 10-57 consists of three particles connected by massless rods. It is to be rotated about an axis perpendicular to its plane through point P. If $M = 0.40$ kg, $a = 30$ cm, and $b = 50$ cm, how much work is required to take the body from rest to an angular speed of 5.0 rad/s?



Example:

81 GO The thin uniform rod in Fig. 10-53 has length 2.0 m and can pivot about a horizontal, frictionless pin through one end. It is released from rest at angle $\theta = 40°$ above the horizontal. Use the principle of conservation of energy to determine the angular speed of the rod as it passes through the horizontal position.

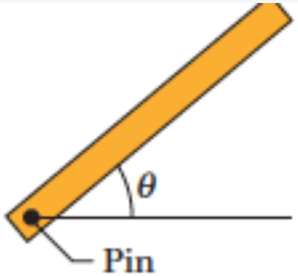


Figure 10-53 Problem 81.

Example:

78 GO A rigid body is made of three identical thin rods, each with length $L = 0.600$ m, fastened together in the form of a letter **H** (Fig. 10-52). The body is free to rotate about a horizontal axis that runs along the length of one of the legs of the **H**. The body is allowed to fall from rest from a position in which the plane of the **H** is horizontal. What is the angular speed of the body when the plane of the **H** is vertical?

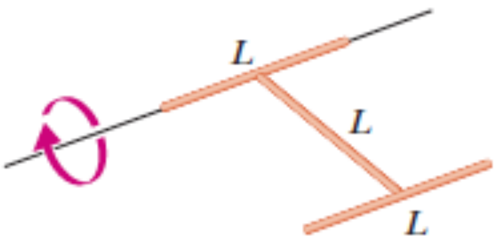


Figure 10-52 Problem 78.

Example:

•••66 GO A uniform spherical shell of mass $M = 4.5$ kg and radius $R = 8.5$ cm can rotate about a vertical axis on frictionless bearings (Fig. 10-47). A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ and radius $r = 5.0$ cm, and is attached to a small object of mass $m = 0.60$ kg. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object when it has fallen 82 cm after being released from rest? Use energy considerations.

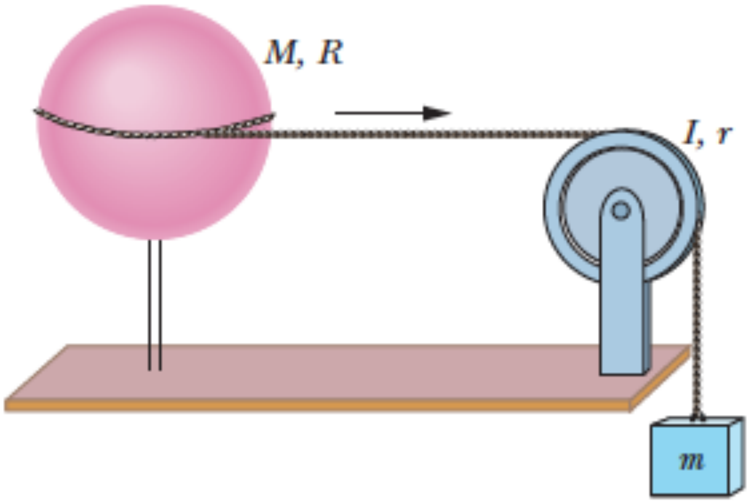


Figure 10-47 Problem 66.

Example:

86. **S** A cord is wrapped around a pulley that is shaped like a disk of mass m and radius r. The cord's free end is connected to a block of mass M. The block starts from rest and then slides down an incline that makes an angle θ with the horizontal as shown in Figure P10.86. The coefficient of kinetic friction between block and incline is μ. (a) Use energy methods to show that the block's speed as a function of position d down the incline is

$$v = \sqrt{\frac{4Mgd(\sin\theta - \mu\cos\theta)}{m + 2M}}$$

(b) Find the magnitude of the acceleration of the block in terms of μ, m, M, g, and θ.

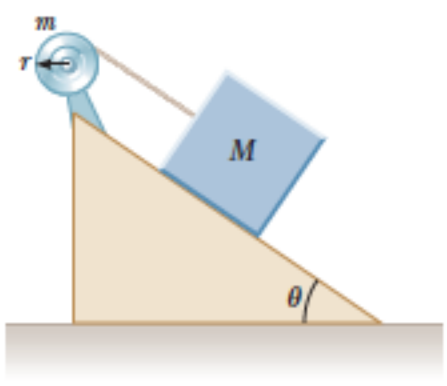


Figure P10.86

Example:

74. A common demonstration, illustrated in Figure P10.74, consists of a ball resting at one end of a uniform board of length ℓ that is hinged at the other end and elevated at an angle θ. A light cup is attached to the board at r_c so that it will catch the ball when the support stick is removed suddenly. (a) Show that the ball will lag behind the falling board when θ is less than 35.3°. (b) Assuming the board is 1.00 m long and is supported at this limiting angle, show that the cup must be 18.4 cm from the moving end.

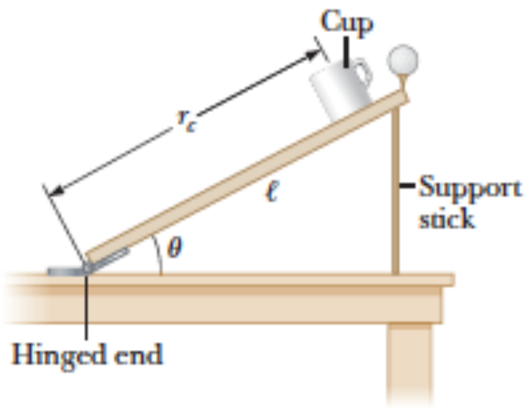


Figure P10.74

Example:

76. **S** **Review.** A spool of wire of mass M and radius R is unwound under a constant force $\vec{\mathbf{F}}$ (Fig. P10.76). Assuming the spool is a uniform, solid cylinder that doesn't slip, show that (a) the acceleration of the center of mass is $4\vec{\mathbf{F}}/3M$ and (b) the force of friction is to the *right* and equal in magnitude to $F/3$. (c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance d?

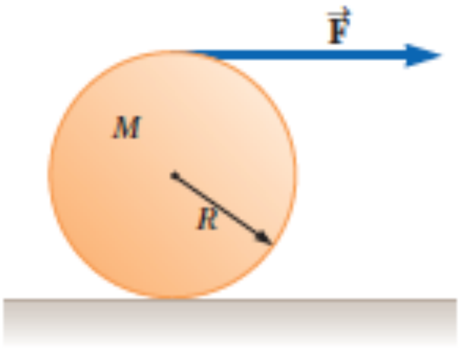


Figure P10.76

Example:

73. **S** **Review.** A string is wound around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its top end tied to a fixed bar (Fig. P10.73). Show that (a) the tension in the string is one third of the weight of the disk, (b) the magnitude of the acceleration of the center of mass is $2g/3$, and (c) the speed of the center of mass is $(4gh/3)^{1/2}$ after the disk has descended through distance h. (d) Verify your answer to part (c) using the energy approach.

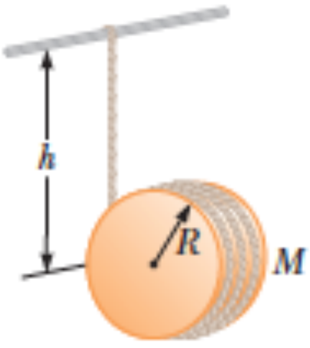


Figure P10.73

Example:

75. **S** A uniform solid sphere of radius r is placed on the inside surface of a hemispherical bowl with radius R. The sphere is released from rest at an angle θ to the vertical and rolls without slipping (Fig. P10.75). Determine the angular speed of the sphere when it reaches the bottom of the bowl.

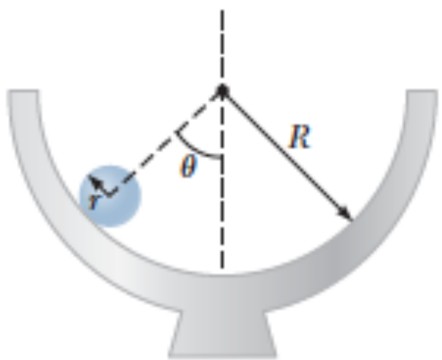


Figure P10.75

Example:

72. **S** The reel shown in Figure P10.72 has radius R and moment of inertia I. One end of the block of mass m is connected to a spring of force constant k, and the other end is fastened to a cord wrapped around the reel. The reel axle and the incline are frictionless. The reel is wound counterclockwise so that the spring stretches a distance d from its unstretched position and the reel is then released from rest. Find the angular speed of the reel when the spring is again unstretched.

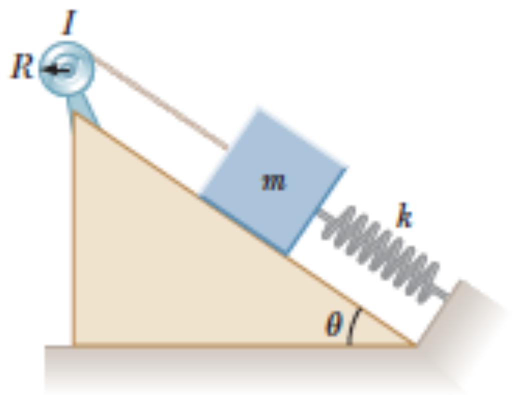


Figure P10.72

Example:

67. S A long, uniform rod of length L and mass M is pivoted about a frictionless, horizontal pin through one end. The rod is nudged from rest in a vertical position as shown in Figure P10.67. At the instant the rod is horizontal, find (a) its angular speed, (b) the magnitude of its angular acceleration, (c) the x and y components of the acceleration of its center of mass, and (d) the components of the reaction force at the pivot.

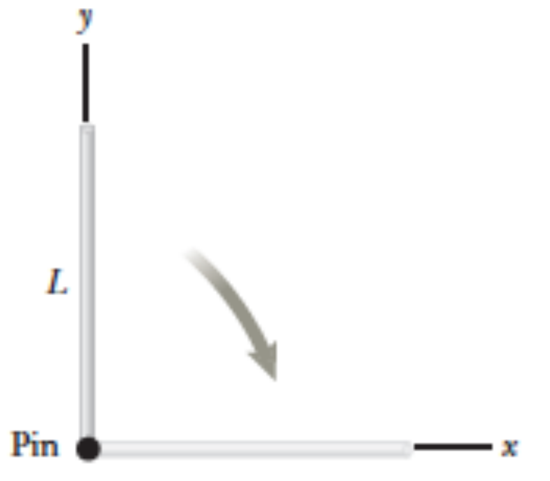


Figure P10.67

Example:

51. **M** **Review.** An object with a mass of $m = 5.10$ kg is attached to the free end of a light string wrapped around a reel of radius $R = 0.250$ m and mass $M = 3.00$ kg. The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center as shown in Figure P10.51. The suspended object is released from rest 6.00 m above the floor. Determine (a) the tension in the string, (b) the acceleration of the object, and (c) the speed with which the object hits the floor. (d) Verify your answer to part (c) by using the isolated system (energy) model.

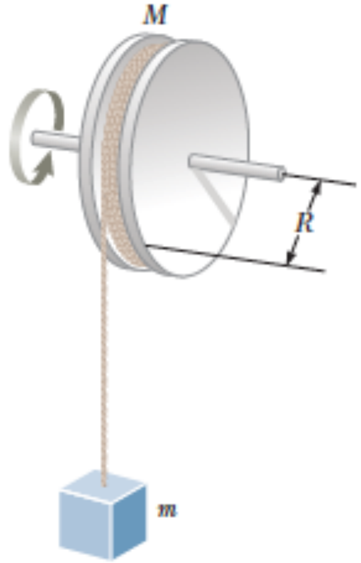


Figure P10.51

Example:

53. **S** A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (Fig. P10.53). If the disk is released from rest in the position shown by the copper-colored circle, (a) what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (b) What is the speed of the lowest point on the disk in the dashed position? (c) **What If?** Repeat part (a) using a uniform hoop.

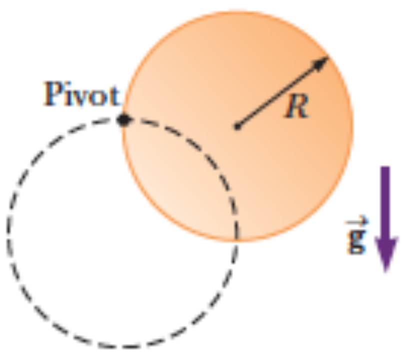


Figure P10.53

Example:

47. The top in Figure P10.47 has a moment of inertia of 4.00×10^{-4} kg · m^2 and is initially at rest. It is free to rotate about the stationary axis AA'. A string, wrapped around a peg along the axis of the top, is pulled in such a manner as to maintain a constant tension of 5.57 N. If the string does not slip while it is unwound from the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg?

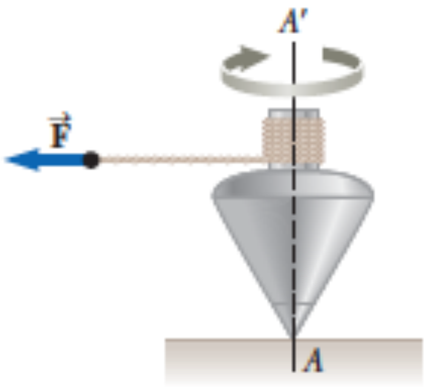


Figure P10.47

Example:

33. **S** Three identical thin rods, each of length L and mass m, are welded perpendicular to one another as shown in Figure P10.33. The assembly is rotated about an axis that passes through the end of one rod and is parallel to another. Determine the moment of inertia of this structure about this axis.

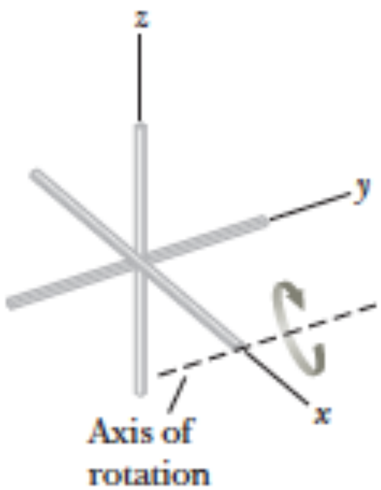


Figure P10.33

Example:

28. **S** Two balls with masses M and m are connected by a rigid rod of length L and negligible mass as shown in Figure P10.28. For an axis perpendicular to the rod, (a) show that the system has the minimum moment of inertia when the axis passes through the center of mass. (b) Show that this moment of inertia is $I = \mu L^2$, where $\mu = mM/(m + M)$.

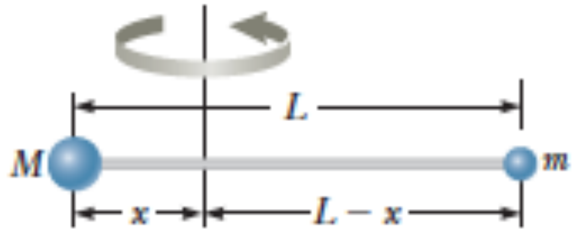


Figure P10.28

Example:

32. **S** Many machines employ cams for various purposes, such as opening and closing valves. In Figure P10.32, the cam is a circular disk of radius R with a hole of diameter R cut through it. As shown in the figure, the hole does not pass through the center of the disk. The cam with the hole cut out has mass M. The cam is mounted on a uniform, solid, cylindrical shaft of diameter R and also of mass M. What is the kinetic energy of the cam–shaft combination when it is rotating with angular speed ω about the shaft's axis?

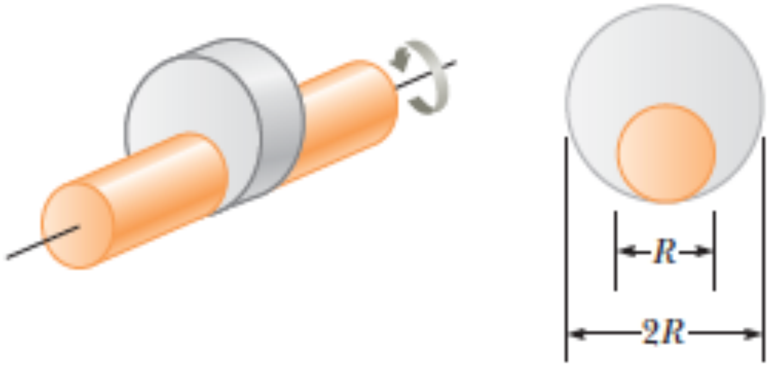


Figure P10.32

Example:

102 The rigid object shown in Fig. 10-62 consists of three balls

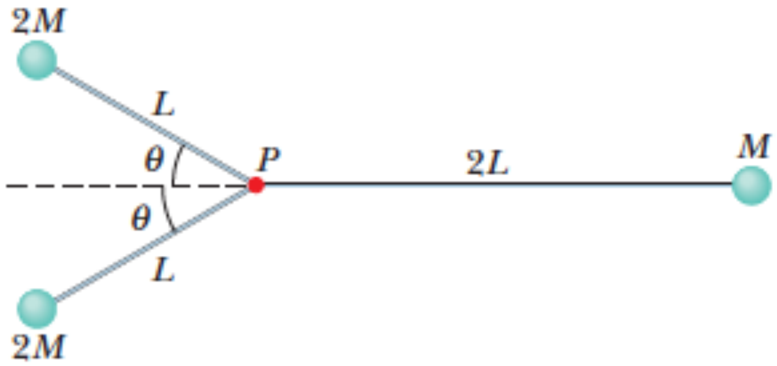


Figure 10-62 Problem 102.

and three connecting rods, with $M = 1.6$ kg, $L = 0.60$ m, and $\theta = 30°$. The balls may be treated as particles, and the connecting rods have negligible mass. Determine the rotational kinetic energy of the object if it has an angular speed of 1.2 rad/s about (a) an axis that passes through point P and is perpendicular to the plane of the figure and (b) an axis that passes through point P, is perpendicular to the rod of length $2L$, and lies in the plane of the figure.