Physica A 391 (2012) 209–216

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The level crossing and inverse statistic analysis of German stock market index (DAX) and daily oil price time series

F. Shayeganfar [a,*], M. Hölling [b], J. Peinke [b], M. Reza Rahimi Tabar [a,c]

[a] Department of Physics, Sharif University of Technology, P.O.Box 11365-9161, Tehran 11365, Iran

[b] Carl von Ossietzky University, Institute of Physics, D-26111 Oldenburg, Germany

[c] Fachbereich Physik, Universität Osnabrück, Barbarastraße 7, 49076 Osnabrück, Germany

ARTICLE INFO

Article history:
Received 1 January 2011
Received in revised form 18 June 2011
Available online 12 August 2011

Keywords:
Level crossing
Probability
Inverse statistics
Waiting time

ABSTRACT

The level crossing and inverse statistics analysis of DAX and oil price time series are given. We determine the average frequency of positive-slope crossings, ν_α^+, where $T_\alpha = 1/\nu_\alpha^+$ is the average waiting time for observing the level α again. We estimate the probability $P(K, \alpha)$, which provides us the probability of observing K times of the level α with positive slope, in time scale T_α. For analyzed time series, we found that maximum K is about $\approx$6. We show that by using the level crossing analysis one can estimate how the DAX and oil time series will develop. We carry out the same analysis for the increments of DAX and oil price log-returns (which is known as inverse statistics), and provide the distribution of waiting times to observe some level for the increments.

1. Introduction

Stochastic processes occur in many natural and man-made phenomena, ranging from various indicators of economic activities in the stock market, velocity fluctuations in turbulent flows and heartbeat dynamics, etc. [1]. The level crossing analysis of stochastic processes has been introduced by (Rice, 1944, 1945) [2–26], and used to describe the turbulence [16], rough surfaces [27], stock markets [28], Burgers turbulence and Kardar–Parisi–Zhang equation [29,30]. The level crossing analysis of the data set has the advantage that it gives important global properties of the time series and do not need the scaling feature. The almost of the methods in time series analysis are using the scaling features of time series, and their applications are restricted to the time series with scaling properties. Our goal with the level crossing analysis is to characterize the statistical properties of the data set with the hope to better understand the underlying stochastic dynamics and provide a possible tool to estimate its dynamics. The level crossing and inverse statistics analysis can be viewed as the complementary method to the other well-known methods such as, detrended fluctuation analysis (DFA) [31], detrended moving average (DMA) [32], wavelet transform modulus maxima (WTMM) [33], rescaled range analysis (R/S) [34], scaled windowed variance (SWV) [35], Langevin dynamics [36], detrended cross-correlation analysis [37], multifactor analysis of multiscaling [38], etc.

We start with formalism of the level crossing analysis. Consider a time series of length n given by $x(t_1), x(t_2), \ldots, x(t_n)$ (here $x(t_i)$ is the log-return of DAX and oil prices). The log-return $x(t_i)$ is defined as $x(t_i) = \ln(y_i/y_{i-1})$, where y_i is the price at time t_i. Let N_α^+ denote the averaged number of *positive* slope crossing of $x(t) = \alpha$ in time scale $T = n\Delta t$ with $\Delta t = 1$ (we set also the average $\langle x \rangle$ to be zero). The averaged N_α^+ can be written as $N_\alpha^+(T) = \nu_\alpha^+ T$, where ν_α^+ is the average frequency of positive slope crossing of the level α. The positive level crossing has specific importance that it gives the next average time

* Corresponding author.
E-mail address: fshayeganfar@gmail.com (F. Shayeganfar).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.07.037

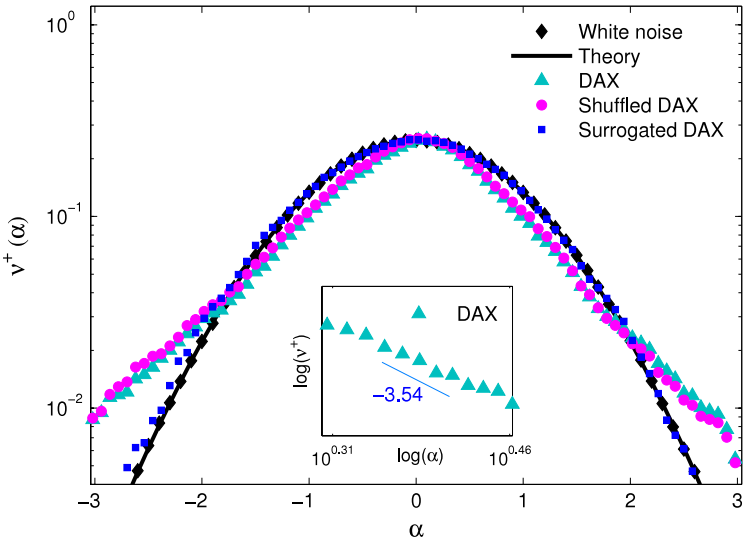

Fig. 1. The level crossing analysis of the DAX log-returns (original, shuffled and surrogated) and uncorrelated Gaussian time series. Inset: the log–log plot of level crossing frequency versus level α for DAX log-return time series. The Gaussian uncorrelated time series has exponential tails ($\sim \exp(-4\alpha^2)$), while the daily DAX time series has power-law tails with exponent $\simeq$3.5.

scale that the price y_i will be greater than the y_{i-1} again up to specific level. For narrow band processes, it has been shown that the frequency ν_α^+ can be deduced from the underlying joint probability distribution function (PDF) for x and $\mathrm{d}x/\mathrm{d}t = \dot{x}$. Rice proved that [2]

$$\nu_\alpha^+ = \int_0^\infty \dot{x} p(x = \alpha, \dot{x}) \mathrm{d}\dot{x}, \tag{1}$$

where $p(x, \dot{x})$ is the joint PDF of x and $\dot{x}$. For discrete time series (of course all of real data are discrete), the frequency ν_α^+ can be written in terms of joint cumulative probability distribution, $P(x_i > \alpha, x_{i-1} < \alpha)$ as [39],

$$\nu_\alpha^+ = P(x_i > \alpha, x_{i-1} < \alpha) = \int_{-\infty}^{\alpha} \int_{\alpha}^{\infty} p(x_i, x_{i-1}) \mathrm{d}x_i \mathrm{d}x_{i-1}, \tag{2}$$

where $p(x_i, x_{i-1})$ is the joint PDF of x_i and x_{i-1}. The inverse of frequency ν_α^+ gives the average time scale T_α that one should wait to observe the given level α again.

The rest of this paper is organized as follows. Section 2 is devoted to summary of level cross analysis of DAX and daily oil price log-returns. The inverse statistics of DAX and Oil price time series are given in Section 3. Section 4 closes with a discussion and conclusion of the present results.

2. Level crossing

Here, at first we provide the results of level crossing analysis for two normalized log-return time series, daily German stock market index (the DAX) and daily oil price. The daily fluctuations in the oil price and DAX time series were belong to the period 1998–2009. We also study the asymmetric properties of level crossing analysis for positive and negative level crossing of the time series. To have a comparison, we provide also level crossing analysis of synthesized uncorrelated noise. Also we will provide the results of level crossing analysis of high frequency data for DAX with sample rate 4 (1/min), where we have used 2511,000 data points and belongs to the period 1994–2003.

Fig. 1 shows the frequency ν_α^+ for daily log-returns of DAX and synthesized uncorrelated noise. The PDF of synthesized uncorrelated noise is Gaussian and it has white noise nature (i.e. its correlation has delta-function behavior). As shown in Fig. 1, their level crossing frequencies are almost similar near to $\alpha \simeq 0$ and have deviation for levels in the tails. The difference is related to the non-Gaussian PDF of DAX log-return time series (see below). For the normalized Gaussian uncorrelated noise, one can show that the frequency ν_α^+ is given by

$$\nu_\alpha^+ = \frac{1}{4}[1 - erf^2(\alpha/\sqrt{2})], \tag{3}$$

where $erf(U)$ is the error function. In Fig. 1, a comparison between the analytical and numerical results for uncorrelated Gaussian time series is given. For Gaussian uncorrelated data, the frequency ν_α^+ behaves as

$$\nu_\alpha^+ \simeq \frac{1}{4} \exp(-\alpha^2/2\pi) \quad \text{for } \alpha \to 0$$

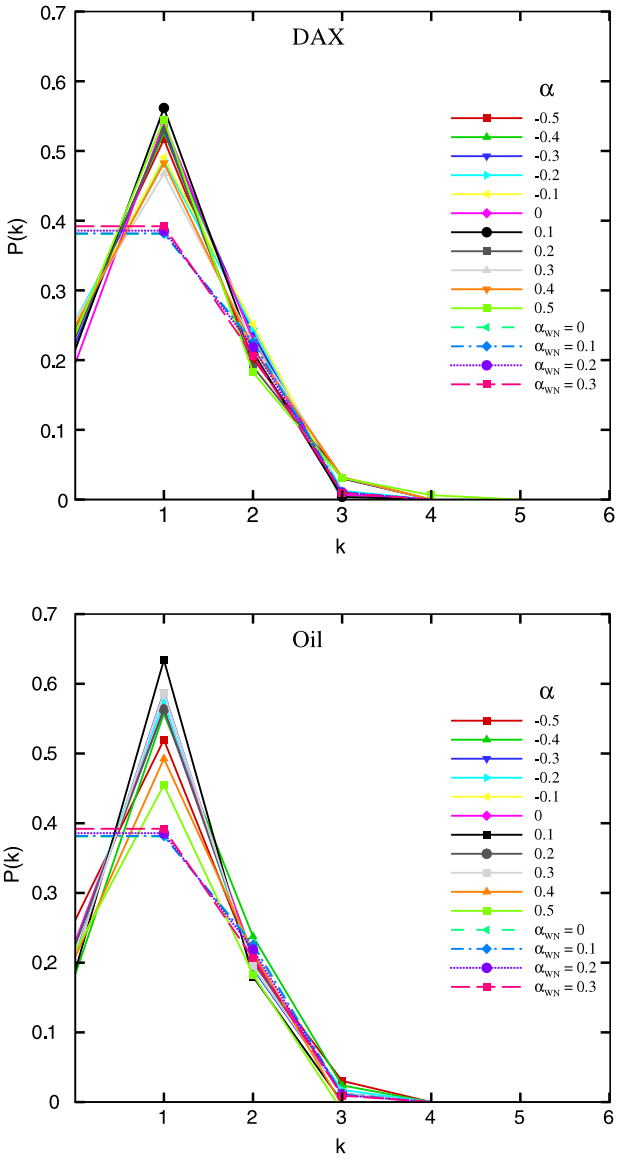

Fig. 2. The PDf $P(K, \alpha)$ versus K for different levels α for normalized log-returns time series of the daily German stock market index (DAX) (top), oil daily price (bottom) and uncorrelated synthesized data.

$$\nu_\alpha^+ \simeq \frac{1}{4}\exp(-4\alpha^2/\pi) \quad \text{for}\, \alpha \to \pm\infty, \tag{4}$$

while we have found that ν_α^+ for daily DAX and oil price time series have power-law tails $1/|\alpha|^\beta$ with exponents $\beta_{DAX} = 3.5 \pm 0.1$ and $\beta_{oil} = 3.8 \pm 0.2$, respectively (see inset of Fig. 1). It means that the DAX and oil price time series have non-Gaussian tails for their level crossing. We note that within the error bars, the exponent $\beta_{DAX} \simeq \beta_{oil}$. However, the exponent β may depend on the sample rate of data acquisition (see below). The exponent β can be estimated using the method proposed in Refs. [40–42]. If the PDF or ν_α^+ follows a power law with exponent $\beta = \kappa + 1$, one can estimate the power-law exponent κ by sorting the normalized returns or levels by their sizes, $\alpha_1 > \alpha_2 > \cdots > \alpha_N$, with the result [41] $\kappa = (N-1)[\sum_{i=1}^{N-1} \ln \frac{\alpha_i}{\alpha_N}]^{-1}$, where $(N-1)$ is the number of tail data points.

In general, there are two reasons to have non-Gaussian tails for level crossing of given time series; (i) due to the fatness of the probability density function (PDF) of the time series, in comparison with a Gaussian PDF. By definition a fat PDF is defined via the behavior of its tails. If its tail goes to the zero slower than a Gaussian PDF, then we call it as fat tail PDF. In this case, non-Gaussian tails cannot be changed by shuffling the series, because the correlations in the data set are affected by the shuffling, while the PDF of the series is invariant, (ii) due to the long-range correlation in time series. In this case, the data may have a PDF with finite moments, e.g., a Gaussian distribution. The easiest way to distinguish whether the PDF shape or

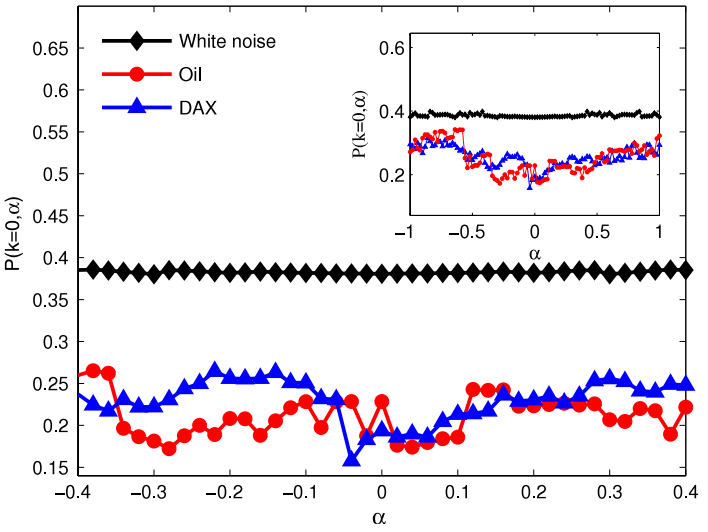

Fig. 3. The PDF $P(K = 0, \alpha)$ versus α for DAX, oil daily price and uncorrelated synthesized normalized log-return time series. The inset is the same figure with a wide range of α's. The results for uncorrelated synthesized data are plotted to have a comparison.

long-range correlation is responsible for the fastnesses of ν_α^+ for the DAX and oil log-returns time series, is by analyzing the corresponding shuffled and surrogate time series. The level crossing analysis will be sensitive to correlation when the time series is shuffled and to probability density functions (PDF) with fat tails when the time series is surrogated. The long range correlations are destroyed by the shuffling procedure and in the surrogate method the phase of the discrete Fourier transform coefficients of time series are replaced with a set of pseudo-independent distributed uniform $(-\pi, +\pi)$ quantities. The correlations in the surrogate series do not change, but the probability function changes to Gaussian distribution [43–45].

In Fig. 1, the level crossing frequency of shuffled and surrogated DAX time series are given. The figure shows that the frequency ν_α^+ has more difference for original and surrogated time series and means that the non-Gaussian tails for ν_α^+ due to the fatness of PDF is dominant [16]. We have found similar results for daily oil price log-returns.

Now let us introduce the PDF, $P(K, \alpha)$, which provides us the probability of observing K times of the level α with positive slope, in the averaged time scale T_α. By construction the average $\langle K \rangle$, i.e. $\langle K \rangle|_\alpha = \sum_{K=0}^{N} KP(K, \alpha)$ will be unity and $P(K, \alpha)$ satisfies the normalization condition $\sum_{K=0}^{N} P(K, \alpha) = 1$. In principle we can assume that the upper bound, i.e. N to be infinity. For the processes and levels that satisfy $P(0, \alpha) \ll \sum_{K=1}^{N} P(K, \alpha)$, we expect to have a good estimation about the future of process. This means that one will observe the level α with high probability in time scale T_α at least once. For the levels that the PDF $P(K, \alpha)$ satisfies $\sum_{K=1}^{N} P(K, \alpha) \approx P(0, \alpha)$, the process will be not predictable. Fig. 2 shows the PDF $P(K, \alpha)$, for daily DAX and oil price log-return time series for different levels α. We plotted also $P(K, \alpha)$ for some levels of synthesized Gaussian uncorrelated data to have a comparison. As shown in Fig. 2 the upper bound N is about 6, which means that it is almost impossible to observe same level α in average time scale T_α more than 6 times, even for the white noise. We used 10^7 data points for white noise synthesized data and found that the maximum number of observing is also about 6.

To find the best interval for the estimation of time series future, we consider the variation of the PDF, $P(0, \alpha)$ with respect to the level α. This will enable us to find the range and intervals of levels that one can estimate the future of these time series with high accuracy. In Fig. 3 the PDFs $P(K = 0, \alpha)$ for daily DAX, oil and uncorrelated synthesized time series, are given. It shows that the $P(K = 0, \alpha)$ of daily DAX and oil time series for the interval $-0.5 < \alpha < 0.5$ has smaller probability with respect to uncorrelated time series. It means that with high probability (with respect to white noise), one can observe the level α at least once in time scale T_α for αs belong to this interval. The typical time scale T_α for this interval is about 4 days for the daily DAX and oil time series, respectively. The corresponding time scales T_α for different αs are shown in Fig. 4. For the daily DAX and oil price time series (for the interval $2 > \alpha > -2$), we found the following empirical curve fittings:

$$T_\alpha(DAX) = 4.10 - 0.18\alpha + 4.90\alpha^2 + 0.21\alpha^3 + 0.94\alpha^4,$$
$$T_\alpha(oil) = 3.90 - 0.37\alpha + 5.70\alpha^2 + 0.57\alpha^3 + 1.69\alpha^4 \quad (5)$$

where T_αs have the dimension "days". To check the applicability level crossing for forecast of the time series, in Fig. 5 we plotted the daily time series of DAX and indicates the points with level $\alpha = 0$ with red points. The average T_α for this level indicated by vertical lines. We expect that in this time scale one should observe another red points with high probability. We also investigated the asymmetry properties of time series with respect to positive and negative slopes.

Finally, we have done similar analysis to the high frequency DAX time series and find that the best interval to estimate the future is $-0.01 < \alpha < 0.01$. The typical time scale belong to this level is about 75 s and the obtained exponent β

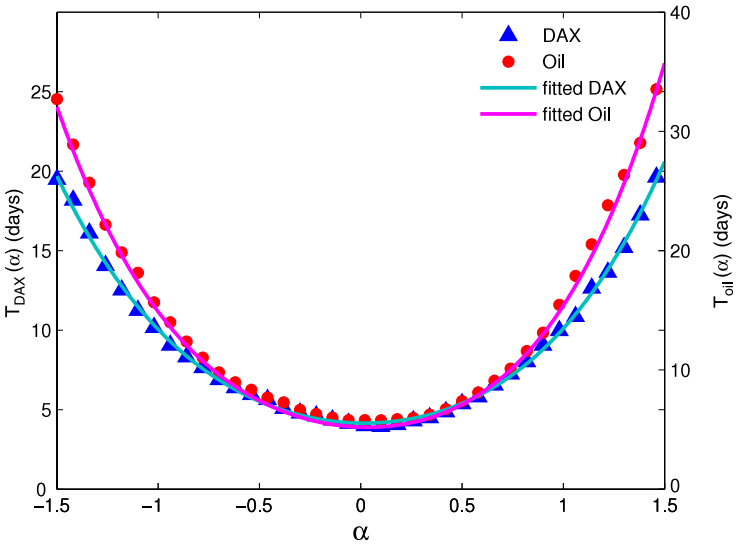

Fig. 4. The level dependence of average time T_α for daily DAX and oil price log-return time series.

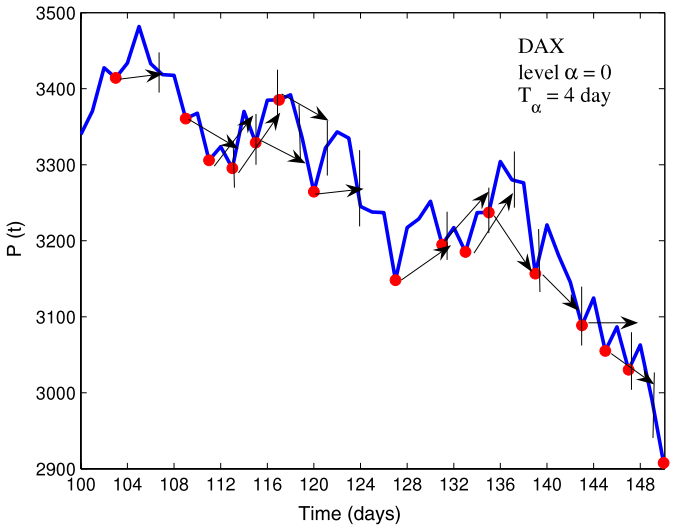

Fig. 5. The points (red) with level $\alpha = 0$ for daily DAX time series. We expect that in this time scale one should observe another red points with high probability. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

was 2.4 ± 0.1. For this time series, the averaged time scale T_α depends on the level α as

$$T_\alpha(DAX) = 72 - 3\alpha + 186\alpha^2 + \alpha^3 - 4\alpha^4, \tag{6}$$

where T_α has dimension in seconds.

3. Inverse statistics

To modeling the statistical properties of financial time series, Simonsen et al. [46] asked the "inverse" question: what is the smallest time interval needed for an asset to cross a fixed return level γ? or what is the typical time span needed to generate a fluctuation or a movement in the price of a given size [46–50]? The inverse statistics is the distribution of waiting times needed to achieve a predefined level of return obtained from every time series. This distribution typically goes through a maximum at a time the so called optimal investment horizon, which is the most likely waiting time for obtaining a given return [51]. Let $y(t)$ be the price at time t. The logarithmic return calculated over the interval Δt is, $r_{\Delta t}(t) = \ln(y(t+\Delta t)) - \ln(y(t))$. Given a fixed log-returned barrier, γ, of an index, the corresponding time span is estimated for which the log-return of index for the first time reaches the level γ. This can also be called the first passage time through the level γ for $r_{\Delta t}$. In Fig. 6, we plotted the probability distribution $p(\tau)$ of normalized waiting time τ needed to reach

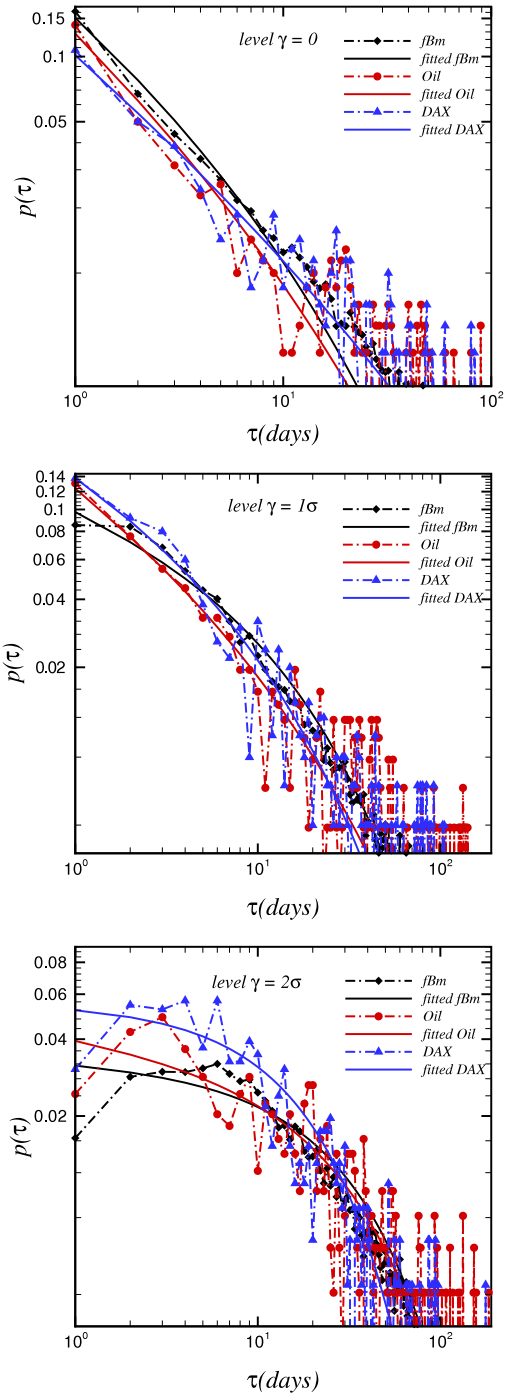

Fig. 6. The probability distribution $p(\tau)$ of normalized waiting time τ needed to reach return levels at scale τ, i.e. $\gamma = 0, 1\sigma$ and 2σ for two financial markets including oil and DAX time series. Solid curves are the fitted curve (Weibull distribution) based on Eq. (7).

return levels $\gamma = 0, 1\sigma, 2\sigma$ for daily oil, daily DAX log-returns and integrated white noise data (i.e. fractional Brownian motion, fBm).

As Fig. 6 show, for the zero level for $r_{\Delta t}$, inverse statistics of the two markets does not deviate from fractional Brownian motion while they are rather different behavior from fBm for $\gamma = 1\sigma$ and 2σ. We fit the waiting time distribution functions $p(\tau)$ for different level γ via the Weibull distribution function [52]:

$$p(\tau, T) = \frac{\delta}{T}\left(\frac{\tau}{T}\right)^{\delta-1} \exp\left[-\left(\frac{\tau}{T}\right)^{\delta}\right] \tag{7}$$

Table 1
The stretched exponent δ and characteristic time scale T fitted by Weibull distribution for various time series.

γ	Time series	δ	T
0	fBm	0.423	1.849
0	Oil	0.347	1.132
0	DAX	0.312	5.791
1	fBm	0.704	13.145
1	Oil	0.496	7.943
1	DAX	0.600	6.836
2	fBm	0.956	34.178
2	Oil	0.878	32.480
2	DAX	0.988	18.631

where δ is the stretched exponent (or shape parameter) and T is the characteristic time scale. We found δ and T for fBm and Oil and DAX time series and summarized results in Table 1.

4. Conclusion

In summary, we analyzed the DAX and oil daily price log-return time series using the level crossing method and find the average waiting time T_α for observing the level α again. This is a similar analysis as what has been done in Refs. [53,54,40]. They have been carried out the level crossing of the volatility time series, instead of the time series itself. We define and estimate the probability of observing K times of the level α, $P(K, \alpha)$ in time scale T_α. We show that by using the level crossing analysis one can estimate the future of the daily DAX and oil time series with good precision for the levels in the interval $-0.5 < \alpha < 0.5$. Also, using the inverse statistics we estimate the waiting time probability distribution for two financial markets, i.e. oil and DAX time series.

References

[1] R. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexities in Finance, Cambridge University Press, New York, 2000; R. Friedrich, J. Peinke, M. Reza Rahimi Tabar, Importance of Fluctuations: Complexity in the View of Stochastic Processes Encyclopedia of Complexity and Systems Science, vol. 3574, Springer, Berlin, 2009.
[2] S.O. Rice, Bell System Tech. J. 23 (1944) 282; S.O. Rice, Bell System Tech. J. 24 (1945) 46.
[3] W. Liepmann, Helv. Phys. Acta 22 (1949) 119; H.W. Liepmann, M.S. Robinson, NACA TN 3037 (1952).
[4] H. Steinberg, P.M. Schultheiss, C.A. Wogrin, F. Zwieg, J. Appl. Phys. 26 (1955) 195.
[5] J.A. McFadden, IEEE Trans. Inform. Theory IT-2 (1956) 146; IT-4 (1958) 14.
[6] C.W. Helstrom, IEEE Trans. Inform. Theory IT-3 (1957) 232.
[7] I. Miller, J.E. Freund, J. Appl. Phys. 27 (1958) 1290.
[8] A.J. Rainal, IEEE Trans. Inform. Theory IT-8 (1962) 372.
[9] D. Slepian, M. Rosenblatt (Ed.), Proc. Sympos. Time Series Analysis, Brown Univ., Wiley, 1963, pp. 104–115.
[10] K. Ito, J. Math. Kyoto Univ. 3 (1963/1964) 207–216.
[11] S.M. Cobb, IEEE Trans. Inform. Theory IT-11 (1965) 220.
[12] N.D. Ylvisarer, Ann. Math. Statist. 36 (1965) 1043.
[13] M.R. Leadbetter, J.D. Cryer, Bull. Amer. Math. Soc. 71 (1965) 561; R.N. Miroshin, St. Petersburg Univ. Math. 34 (2001) 30.
[14] Orey, Z. Wahrscheinlichkeitstheor, Verwandte Geb. 15 (1970) 249.
[15] J. Abrahams, IEEE Trans. Inform. Theory IT-28 (1982) 677.
[16] K.R. Sreenivasan, A. Prabhu, R. Narasimha, J. Fluid Mech. 137 (1983) 251; K.R. Sreenivasan, Annu. Rev. Fluid Mech. 23 (1991) 539.
[17] A.J. Rainal, IEEE Trans. Inform. Theory 34 (1988) 1383; A.J. Rainal, IEEE Trans. Inform. Theory 36 (1990) 1179.
[18] J.T. Barnett, B. Kedem, IEEE Trans. Inform. Theory 37 (1991) 1188; G.L. Wise, IEEE Trans. Inform. Theory 38 (1992) 213.
[19] I. Rychlik, Extremes 3 (2000) 331.
[20] J. Davila, J.C. Vassilicos, Phys. Rev. Lett. 91 (2003) 144501.
[21] St. Lück, Ch. Renner, J. Peinke, R. Friedrich, Phys. Lett. A 359 (2006) 335.
[22] D. Hurst, J.C. Vassilicos, Phys. Fluids 19 (2007) 035103.
[23] N. Mazellier, J.C. Vassilicos, Phys. Fluids 20 (2008) 015101.
[24] D. Poggi, Katul, Bound.-Layer Meteorol. 136 (2010) 219.
[25] M.F. Kratz, J.R. Len, Extremes 13 (2010) 315.
[26] N. Rimbert, Phys. Rev. E 81 (2010) 056315.
[27] F. Shahbazi, S. Sobhanian, M.R. Rahimi Tabar, S. Khorram, G.R. Frootan, H. Zahed, J. Phys. A 36 (2003) 2517.
[28] G.R. Jafari, M.S. Movahed, S.M. Fazeli, M.R. Rahimi Tabar, S.F. Masoudi, J. Stat. Mech. (2006) P06008.
[29] A. Bahraminasab, H. Rezazadeh, A.A. Masoudi, J. Phys. A: Math. Gen. 39 (2006) 3903–3909.
[30] A. Bahraminasab, M.S. Movahed, S.D. Nasiri, A.A. Masoudi, M. Sahimi, J. Stat. Phys. 124 (6) (2006) 1471–1490.
[31] C.-K. Peng, J. Mietus, J.M. Hausdorff, S. Havlin, H.E. Stanley, A.L. Goldberger, Phys. Rev. Lett. 70 (1993) 1343.
[32] E. Alessio, A. Carbone, G. Castelli, V. Frappietro, Eur. Phys. J. B 27 (2002) 197.
[33] J.F. Muzy, E. Bacry, A. Arneodo, Phys. Rev. Lett. 67 (1991) 3515.
[34] H.E. Hurst, R.P. Black, Y.M. Simaika, Long-Term Storage. An Experimental Study, Constable, London, 1965.
[35] A. Eke, P. Herman, L. Kocsis, L.R. Kozak, Physiol. Meas. 23 (2002) R1–R38.

[36] R. Friedrich, J. Peinke, Phys. Rev. Lett. 78 (1997) 863;
G.R. Jafari, S.M. Fazlei, F. Ghasemi, S.M. Vaez Allaei, M.R. Rahimi Tabar, A. Iraji Zad, G. Kavei, Phys. Rev. Lett. 91 (2003) 226101;
F. Ghasemi, J. Peinke, M. Sahimi, M. Reza Rahimi Tabar, Eur. Phys. J. B 47 (2005) 411;
P. Sangpour, G.R. Jafari, O. Akhavan, A.Z. Moshfegh, M. Reza Rahimi Tabar, Phys. Rev. B 71 (2005) 155423;
F. Ghasemi, M. Sahimi, J. Peinke, M. Reza Rahimi Tabar, J. Biol. Phys. 32 (2006) 117;
G.R. Jafari, M. Sahimi, M. Reza Rasaei, M. Reza Rahimi Tabar, Phys. Rev. E 83 (2011) 026309;
F. Shayeganfar, S. Jabbari-Farouji, M.S. Movahed, G.R. Jafari, M. Reza Rahimi Tabar, Phys. Rev. E 81 (2010) 061404;
A. Farahzadi, P. Niyamakom, M. Beigmohammadi, N. Mayer, M. Heuken, F. Ghasemi, M. Reza Rahimi Tabar, T. Michely, M. Wuttig, Europhys. Lett. 90 (2010) 10008;
S. Kimiagar, M.S. Movahed, S. Khorram, M. Reza Rahimi Tabar, J. Stat. Phys. 143 (2011) 148167.
[37] B. Podobnik, H.E. Stanley, Phys. Rev. Lett. 100 (2008) 084102.
[38] F. Wang, K. Yamasaki, S. Havlin, H.E. Stanley, Phys. Rev. E 79 (2009) 016103.
[39] F. Ghasemi, M. Sahimi, J. Peinke, R. Friedrich, G.R. Jafari, M. Reza Rahimi Tabar, Phys. Rev. E 75 (2007) 060102R.
[40] B. Podobnik, D. Horvatic, A. Peterson, H.E. Stanley, Proc. Natl. Acad. Sci. USA 106 (2009) 22079.
[41] B.M. Hill, A simple general approach to inference about the tail of a distribution, Ann. Statist. 3 (1975) 1163.
[42] A. Pagan, The econometrics of financial markets, J. Empirical Finance 3 (1996) 15.
[43] J. Theiler, D. Prichard, Fields Inst. Commun. 11 (1997) 99.
[44] T. Schreiber, A. Schmitz, Physica D 142 (2000) 346.
[45] B. Podobnik, D.F. Fu, H.E. Stanley, P.Ch. Ivanov, Eur. Phys. J. B 56 (2007) 47.
[46] I. Simonsen, M.H. Jensen, A. Johansen, Eur. Phys. J. B 27 (2002).
[47] M.H. Jensen, Phys. Rev. Lett. 83 (1999) 76.
[48] L. Bifarele, M. Cencini, D. Vergni, A. Vulpiani, Eur. Phys. J. B 20 (2001) 473.
[49] S. Karlin, A First Course in Stochastic Processes, Academic Press, New York, 1966.
[50] M. Ding, G. Rangarajan, Phys. Rev. E 52 (1995) 207.
[51] H. Ebadi, M. Bolgorian, G.R. Jafari, Physica A 389 (2010) 5530.
[52] P.C. Ivanov, A. Yuen, B. Podobnik, Y. Lee, Phys. Rev. E 69 (2004) 056107.
[53] K. Yamasaki, L. Muchnik, S. Havlin, A. Bunde, H.E. Stanley, Proc. Natl. Acad. Sci. USA 102 (2005) 9424.
[54] F. Wang, K. Yamasaki, S. Havlin, H.E. Stanley, Phys. Rev. E 73 (2006) 026117.