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Scaling behavior of earthquakes' inter-events time series

Research Article

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Abstract: In this paper, we investigate the statistical and scaling properties of the California earthquakes' inter-events over a period of the recent 40 years. To detect long-term correlations behavior, we apply detrended fluctuation analysis (DFA), which can systematically detect and overcome nonstationarities in the data set at all time scales. We calculate for various earthquakes with magnitudes larger than a given M. The results indicate that the Hurst exponent decreases with increasing M; characterized by a Hurst exponent, which is given by, $H = 0.34 + 1.53/M$, indicating that for events with very large magnitudes M, the Hurst exponent decreases to 0.50, which is for independent events.

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1. Introduction

Considerable attention has been devoted in recent years to the study of the seismic data which are full of information about the reasons of occurrence of earthquakes,. The efforts have led to valuable achievements about different aspects of seismic data. Many of the important properties of this phenomenon are investigated and elucidated, such as power laws and scale-invariant properties. The Gutenberg-Richter law gives the distribution of the Magnitude of earthquakes [1, 2]. This law states that the number N_e of earthquakes with magnitude larger than M obeys the law.

$$N_e \propto 10^{-bM}, \tag{1}$$

where $b \approx 1$, [3, 4]. The other law, (modified) Omori law, says that after the main earthquake, the seismic rate, $r(M, t)$, for events or aftershocks with a magnitude more than M in a region around the main earthquake, increases suddenly, and then decreases as

$$r(M, t) = r_0(M)/(1 + t/c)^p, \tag{2}$$

where t is the time from the mainshock, $r_0(M) = r(M, t = 0)$, and c is a time constant, of the order of minutes, and describes the deviations from a power law immediately after the mainshock. In the Eq. (2) p is slightly greater than 1.

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It has been shown that the earthquake inter-events times have a universal distribution [5, 6]. Specific predictions for the probability density function (PDF), of the recurrence time between earthquakes in a homogeneous region is also derived analytically [5, 6], as well as for multiple regions, by analyzing theoretically the earthquake recurrence times using a formulation based on the probability generating functions [7], and also assuming that all earthquakes are similar (no difference between the foreshocks, main shocks, and aftershocks). It is also shown that, the scaling laws of the inter-event times do not give more information than what was found by the Gutenberg-Richter and the (modified) Omori laws. This theory accounts quantitatively for the empirical power laws suggested by Bak *et al.* and Corral [8, 9]. We expect to see fractal-like characteristics in the seismic data, because of the power law and scale-invariant properties in the time series of earthquakes.

As we see, investigations of fractal properties of the time series, especially earthquake inter-events time series is very important, these days. In this paper we study the scaling properties of the California earthquake inter events over a period of the recent 40 years. We investigate the behavior of the Hurst exponent of the spatial seismic data *versus* the magnitudes of the earthquakes. There are many methods proposed in recent years for the analysis of the different types of data. The fractal properties of the time-series are important for many reasons and give valuable information about the data, as mentioned above. A famous and straightforward technique to investigate the fractal and scaling properties of the data is the detrended fluctuation analysis (DFA) method. This method was first proposed by Peng *et al.* [10, 11]. There are several features that account for the complexity in earthquake sequences. The scale invariance and power law are the main characteristics of the earthquake data set. The Gutenberg Richter law states that the magnitude probability distribution of the earthquakes is a power law. The Omori's law states that the number of aftershocks, which follow a main event, decays as a power law with an exponent close to minus one. The maps of the faults are fractal-like (spatial) and the time series of the earthquakes are $1/f$ noise in time [12, 13]. The existence of power law and scale invariance in the spatial-temporal data series of earthquakes indicates the presence of a fractal behavior, so one of the aspects that scientists study on different data series of earthquakes is their fractal behavior. In the following sections, we first introduce the mentioned method and illustrate the mathematical details and the analysis techniques. Then we introduce the results of the process.

2. Analysis Techniques

As mentioned above, one of the well-known methods for investigation of the fractal properties of a time-series is the Detrended Fluctuation Analysis (DFA) method. Using this method, we get the Hurst exponent of the series, by which we may get information about the fractal properties of the time series. The mathematical details of this method are as follows: Consider a time-series x_k. We define the "profile" as:

$$Y(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle] . \tag{3}$$

Now divide the profile $Y(i)$ into $N_s = int(N/s)$ non-overlapping segments of equal lengths s. Since the length N of the series is often not a multiple of the considered time scale s, a short part at the end of the profile may remain. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether. Then one calculates the local trend for each of the $2N_s$ segments by a least-square fit of the series. At the last step we determine the variance as follows:

$$F^2(s,\mu) = 1/s \sum_{i=1}^{s} \{Y[(\mu-1)s+i] - y_\mu(i)\}^2 \quad \text{for} \quad \mu = 1, 2, \ldots, N_s, \tag{4}$$

$$F^2(s,\mu) = 1/s \sum_{i=1}^{s} \{Y[N-(\mu-N_s)s+i] - y_\mu(i)\}^2 \quad \text{for} \quad \mu = N_s+1, \ldots, 2N_s. \tag{5}$$

The above computation is repeated for box sizes s (different scales) to provide a relationship between $F(s)$ and s. A power law relation between $F(s)$ and s indicates the presence of scaling: $F(s) \sim s^h$.
The relation between the exponent h (DFA exponent) and the Hurst exponent is as follows. For stationary signals such as fractional Gaussian noise (FGN), $Y(i)$ in Eq. (3) will be an fractional Brownian motion (FBM) signal, so that $0 < h < 1.0$. The exponent h is known as the Hurst exponent H [10, 11, 14]. But for a non-stationary signal, such as FBM noise, $Y(i)$ will be a sum of FBM signal, so the scaling exponent of $F(s)$ is identified by $h > 1.0$ [10, 11, 14]. In this case the relation between the exponents h and H will be $H = h - 1$. In addition, it is shown

that the autocorrelation function can be characterized by a power law $C(s) \sim s^{\gamma}$ with exponent $\gamma = 2 - 2H$ and the power spectra scales by $S(\omega) \sim \omega^{-\beta}$ with frequency ω and $\gamma = 2H - 1$. For the non-stationary signals, the correlation and the power spectrum scaling exponents are $\gamma = -2H$ and $\beta = 2H + 1$, respectively [10, 11, 14].

However, the DFA method can only determine positive exponents, in order to refine the analysis near the FGN/FBM boundary or strongly anti-correlated signals when this exponent is close to zero. The simplest way to analyze such data is to integrate the time series before the DFA procedure. Hence, we replace the single summation in Eq. (3), which is describing the profile from the original data, by a double summation by using classification by the signal summation conversion method (SSC). After using the SSC method, FGN switches to FBM and FBM switches to sum-FBM. In this case the relation between the new exponent, h', and h is $h = h' - 1$ [14, 15] (recently Movahed *et al.* have proved the relation between the derived exponent from the double profile of a series in the DFA method and the h exponent in the appendix of [15]).

We apply the above method to the California earthquakes' inter-event time-series data. We use DFA method for the seismic data with different magnitudes. Changing the magnitude of the data, under investigation, gives us the following results, illustrated by the figures below.

3. Data Information

The data are from a set collected by the U.S. Geological Survey (USGS)[1], showing the earthquakes which occurred in the California region from 1973 up to the beginning of 2007. The total number of these earthquakes with the magnitude more than 3 (Richter) is (27987) and (3673) for more than 4. The number of the earthquakes with the magnitude more than M obeys the Gutenberg-Richter law which is $(magnitude > M) \sim M^{-0.9}$, for the data we have investigated. The data we have worked on are the distances of each two successive earthquakes in the mentioned region. The only restriction is that in each stage of the calculation of the Hurst exponents, we only consider the earthquakes with magnitudes greater than a specific M. Fig. 1 shows the time series of the California earthquake *versus* number of events.

[1] *www.usgs.gov*

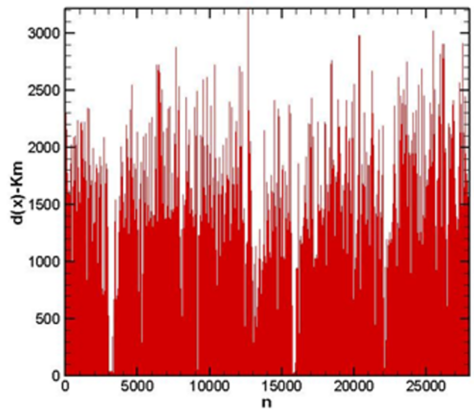

Figure 1. Time series of the California earthquake *versus* number of events. For magnitudes larger than 3 ($M > 3$).

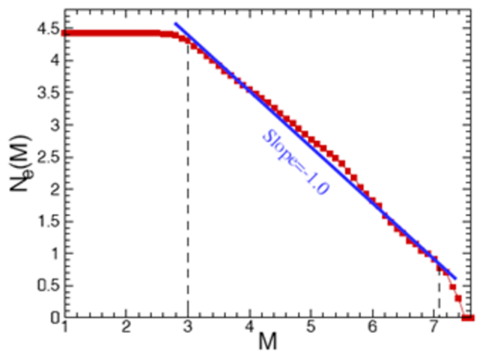

Figure 2. The number of California earthquakes $N_e(M)$ with magnitudes larger than M. The scaling region is in $M_c < M < 7$.

4. Results

Applying the DFA method, as mentioned above, gives us the scaling behavior of the California earthquakes inter-events. We repeated the calculations for various earthquakes with magnitudes more than a given M. The results indicated that the Hurst exponent decreases as M increases; as shown in Fig. 3. The following empirical formula, obtained by fitting, describes well the numerical results,

$$H = a + b/M^{c}. \quad (6)$$

With $a = 0.34 \pm 0.03$, $b = 1.53 \pm 0.13$, and $c = 1.0 \pm 0.2$. The maximum magnitude of the earthquakes (in Richter) is about $M \sim 8$. For these magnitudes the Hurst exponent H, goes to, $H \approx 0.34 + 1.53/8 = 0.53$, which is close to 0.5, So the earthquakes' inter-event spatial data with the magnitude more than M have a Hurst exponent approaching 0.50, which shows that larger earthquakes occur essentially independently (random events correspond to $H = 0.50$).

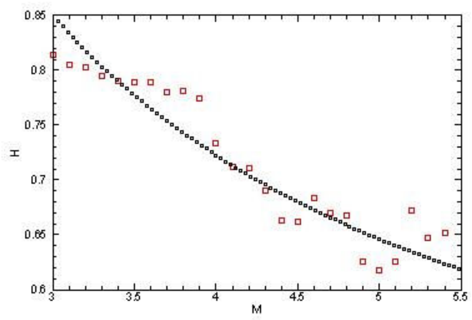

Figure 3. Hurst Exponent *Vs.* Magnitude (Richter). Considering the earthquakes' inter-event series with magnitude larger than M. (Spatial data). The red squares are the results of the calculations and the black curve is the fitted curve.

For more clarity, we have also calculated the Magnitude dependence of Hurst exponent with two other fits, which are linear and exponential. The two other fits are as follows,

$$H = 1.08 - 0.09M, \tag{7}$$

$$H = 1.19e^{-0.12M}. \tag{8}$$

Eqs. (7), (8), show the relation between the distribution of the aftershocks against their magnitude. As we see the trend of the Hurst exponent *versus* the magnitude is decreasing, regardless of the mathematical relation we propose for this trend. This reveals that the earthquakes occurring sequentially and close to each other have the lower magnitudes and they are more correlated, and for the earthquakes with more magnitudes we find them less correlated, because the Hurst exponent decreases to 0.5 as we mentioned before. For larger magnitudes, we may rarely see other earthquakes with large magnitudes after the main huge earthquake. This is because of the weak correlation between the earthquakes with large magnitudes. But after the earthquakes with small magnitudes we may see aftershocks frequently. In fact one of the reasons of the trend of Hurst exponent *versus* the magnitude may be interpreted as the aftershocks occurring frequently after the low magnitude earthquakes and rarely after the high magnitude earthquakes.

5. Conclusion

We determine that the inter-events of the earthquakes are well correlated. Our results show that increasing the magnitude of the earthquakes, leads to the decrease of the correlation of the inter-event, according to Eq. (6). We may interpret this behavior as aftershocks. In addition, it is shown that H decreases to 0.5 for earthquakes with a large magnitude ($M > 8$). As we know, $H = 0.5$, indicates the independence of the events. Therefore, the spatial data of California earthquakes, with large magnitudes, occur independently, showing that such events are weakly correlated. The suggested fitting equations could reveal how the aftershocks' distribution depends on the magnitude.

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