Localization properties of acoustic waves in the random-dimer media

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Propagation of acoustic waves in the one-dimensional (1D) random-dimer (RD) medium is studied by three distinct methods. First, using the transfer-matrix method, we calculate numerically the localization length ξ of acoustic waves in a binary chain (one in which the elastic constants take on one of two values). We show that when there exists short-range correlation in the medium—which corresponds to the RD model—the localization-delocalization transition occurs at a resonance frequency ω_c. The divergence of ξ near ω_c is studied, and the critical exponents that characterize the power-law behavior of ξ near ω_c are estimated for the regimes $\omega > \omega_c$ and $\omega < \omega_c$. Second, an exact analytical analysis is carried out for the delocalization properties of the waves in the RD media. In particular, we predict the resonance frequency at which the waves can propagate in the entire chain. Finally, we develop a dynamical method, based on the direct numerical simulation of the governing equation for propagation of the waves, and study the nature of the waves that propagate in the chain. It is shown that only the resonance frequency can propagate through the 1D media. The results obtained with all the three methods are in agreement with each other.

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I. INTRODUCTION

The scaling theory of localization[1] predicts that, for any space dimensionality $d \leq 2$, all the electronic states are localized for any degree of disorder, whereas a transition to extended states occurs for $d > 2$, depending on the strength of the disorder W. The transition between the two states—the metal-insulator transition—is characterized by the divergence of the localization length ξ according to $\xi \propto |W - W_c|^{-\alpha}$, where W_c is the critical value of the disorder intensity. A considerable number of accurate numerical simulations support this prediction; see, for example, Kramer and Mackinnon[2] for a review.

There have been several theoretical[3–8] and experimental[9,10] studies of localization of acoustic waves that have shown that such waves may be localized in disordered media. The same type of phenomena is observed in disordered optical materials. For example, experimental and numerical evidence for light localization was obtained in a random optical medium,[11] finite slabs,[12] and two-dimensional (2D) random dielectric systems.[13] Such observations have been the impetus for the study of localization properties of acoustic, electronic, and optical waves.

But as it happens quite often, there are always exceptions to the rule. It was shown by Dunlap *et al.*[14] that extended states may exist in a medium with correlated disorder, even in the one-dimensional (1D) tight-binding Anderson model. This was shown using the so-called random-dimer (RD) model (see below). More recently, a number of tight-binding[15,16] and continuous[17,18] models predicted the existence of the extended states in disordered 1D media with long- or short-range correlations. The discovery was supported by analytical, experimental, and numerical investigations.[19]

The delocalization phenomena have also been studied during propagation of acoustic waves, which are produced when long-range intersite couplings are considered,[20] or when long-range correlations are introduced in the hopping term of the Hamiltonian. For example, our group[4,21] presented the results of extensive numerical simulation of acoustic wave propagation in disordered media in one, two, and three dimensions, and showed that there can be a disorder-induced transition from delocalized to localized states of acoustic waves in *any* spatial dimension. At the same time, the existence of the extended states and, therefore, phonon transport in a 1D random mass n-mer model (a generalization of the RD model) was studied by Cao *et al.*[22] The short-range correlated disorder in such models can force the localization length to be comparable with the length of the system at the resonance frequency.[15] The n-mer model may also explain the high conductivity in some polymers.[23,24]

In this paper, we study, using three distinct methods, acoustic wave propagation in the 1D RD model in which disorder is incorporated by varying the local elastic constants. First, using the transfer-matrix (TM) method, we calculate the localization length of acoustic waves propagating in the medium, and show that when short-range correlation is imposed between the local elastic constants, phonon delocalization takes place in the model, which induces resonant transmission of the acoustic waves. Next, we derive, using an exact analytical method, the resonance frequency of the RD model. The dynamic method, based on the direct numerical simulation of the equation that govern propagation of acoustic waves, is then utilized to investigate propagation of wave packets through the RD medium, and to calculate their intensity spectrum after their propagation. As explained below, we find that the medium can filter out all the frequencies of the wave packets, except the resonance frequency.

The rest of the paper is organized as follows. In Sec. II we describe the model that we use which is based on the scalar wave equation, and discuss its details. In Sec. III we present and discuss our numerical and theoretical results. Section IV describes the dynamical method for the study of the delocalization effect. A summary of the paper is presented in Sec. V.

II. MODEL

To study acoustic wave propagation in a medium with a random distribution of elastic constants, we analyze the scalar wave equation introduced earlier.[4,21] Its 1D version is given by,

$$\frac{\partial^2}{\partial t^2}\psi(x,t) - \frac{\partial}{\partial x}\left[\lambda(x)\frac{\partial}{\partial x}\psi(x,t)\right] = 0, \quad (1)$$

where $\psi(x,t)$ is the wave amplitude, t is the time, and, $\lambda(x)=e(x)/m$ is the ratio of the stiffness $e(x)$ and the medium's mean density m (in this paper we set $m=1$). In order to calculate the localization length ξ, we use the TM method.[23] By discretizing Eq. (1) (we set the nearest-neighbor spacing, $\Delta x=a=1$), and writing down the result for site i of a linear chain, we obtain

$$\lambda_{i+1/2}(\psi_{i+1}-\psi_i) - \lambda_{i-1/2}(\psi_i-\psi_{i-1}) + \omega^2\psi_i = 0. \quad (2)$$

Setting $\lambda_{i+1/2}=\beta_i$, and $\lambda_{i-1/2}=\beta_{i-1}$, Eq. (2) is expressed in terms of the conventional TM method by the following recursive matrix form:

$$\begin{pmatrix}\psi_{i+1}\\ \psi_i\end{pmatrix} = \mathbf{M}_{i,i-1}\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix}, \quad (3)$$

where

$$\mathbf{M}_{i,i-1} = \begin{pmatrix}\dfrac{-\omega^2+\beta_{i-1}+\beta_i}{\beta_i} & -\dfrac{\beta_{i-1}}{\beta_i}\\ 1 & 0\end{pmatrix}. \quad (4)$$

The wave functions of the two ends of the chain are related together by the product of matrices, $\mathbf{M}_{N,1}(\omega) = \Pi_{i=1}^{N}\mathbf{M}_{i,i-1}$, where N is the sample length (in number of the sites), and $\mathbf{M}_{i,i-1}$ is the TM that connects the wave function of site $i+1$ to those of sites i and $i-1$. Then, the Lyapunov exponent $\gamma(\omega)$ is defined by

$$\gamma(\omega) = \lim_{N\to\infty}\frac{1}{N}\langle \ln\|\mathbf{M}_{N,1}(\omega)\|\rangle. \quad (5)$$

The localization length ξ is simply the inverse of the Lyapunov exponent, $\xi=1/\gamma$.

We now introduce a binary and correlated distribution of the disorder, i.e., one in which the elastic constants β_i take on only two values, k_A (A) and k_B (B), with the additional constraint that the k_B values appear only in pairs of neighboring sites of the chain (dimers), but distributed at random locations throughout the chain. An example is, $A,A,A,A,A,B,B,A,A,A,B,B,A,\ldots$. It should then be clear that there is correlation in the probability distribution of the nearest-neighbor site potentials.

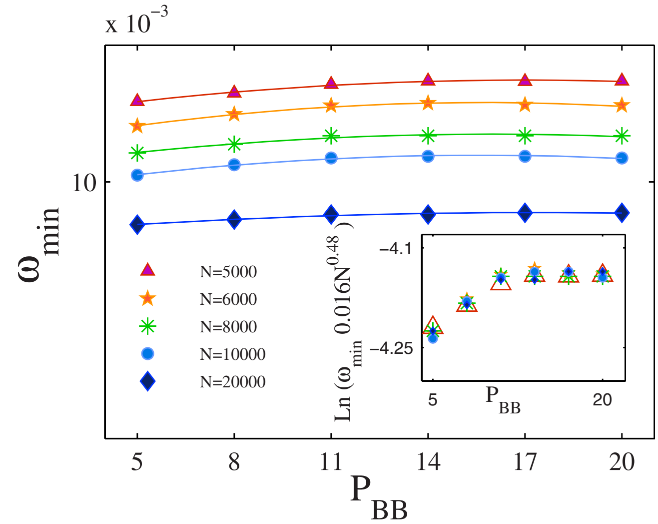

FIG. 1. (Color online) Plot of the minimum allowed frequency $\omega_{\min}$ in the RD chain, as a function of the percent P_{BB} of the paired elastic constants, distributed randomly in a chain of various sizes N. The inset shows the rescaled value of the minimum frequency. The averaging was taken over 20 000 realizations.

The range of the allowed frequencies for the prefect lattice (without disorder) is determined using $\mathrm{Tr}(\mathbf{M}_{N,1})$, where Tr denotes the trace of global transfer matrix[2] $\mathbf{M}_{N,1}$. To determine the maximum and minimum allowed frequencies in the RD model, we use direct diagonalization of the system's Hamiltonian. The results are presented in Figs. 1 and 2. For each P_{BB}, the fraction of the elastic constant pairs k_B, the averaging was taken over 20 000 realizations of the disorder. Figure 1 displays $\omega_{\min}$ as a function of the probability P_{BB}. The inset of the figure presents the scaling behavior of $\omega_{\min}$ in terms of the lattice size N. Similarly, the maximum value $\omega_{\max}$ of ω as a function of P_{BB} is presented in Fig. 2 which indicates that for large system sizes ($N\to\infty$) $\omega_{\max}$ does not change when P_{BB} is small and tends to 6.325.

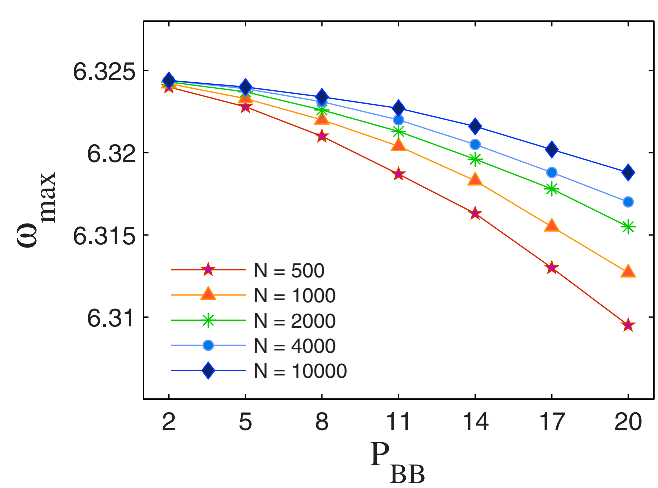

FIG. 2. (Color online) Same as in Fig. 1 but for the maximum allowed frequency $\omega_{\max}$.

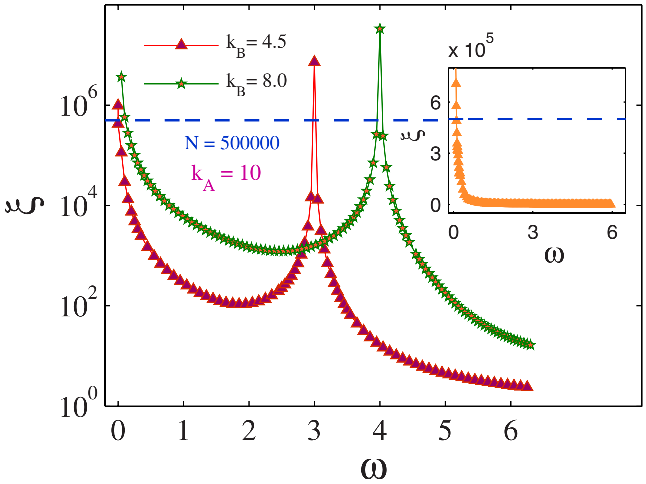

FIG. 3. (Color online) Localization length of the RD chain as a function of allowed frequencies for $k_A=10$ and two values of k_B. Dashed line indicates the localization length equal to the system size, $N=5\times10^5$. Inset: localization length of the random nonpaired binary chain as a function of allowed frequencies for two values of the elastic constant $k_A=10$ and $k_B=8$.

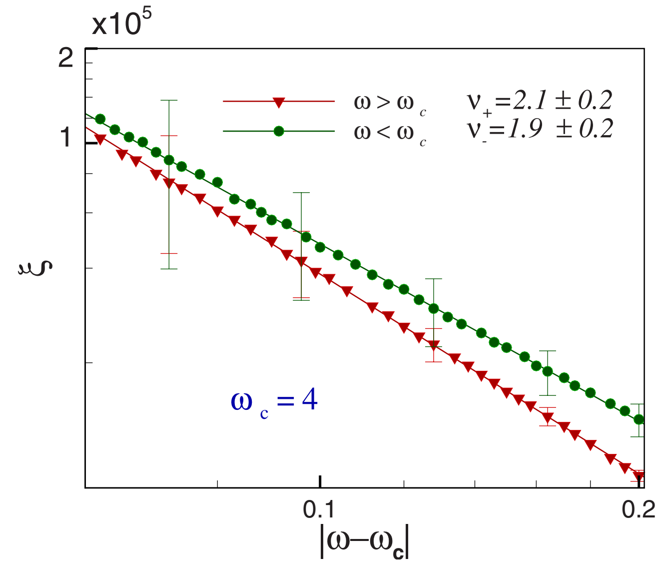

FIG. 4. (Color online) Localization length of the RD chain with $k_A=10$ and $k_B=8$ as a function of $|\omega-\omega_c|$, in the regimes $\omega<\omega_c$ and $\omega>\omega_c$, where $\omega_c=4$.

III. RESULTS

We computed the localization length of the 1D model for both the nonpaired and RD models. In addition, we carried out exact theoretical analysis of the problem, as described below.

A. Numerical results

The TM method was utilized to calculate the localization length of acoustic waves in a RD chain. First, the elastic constants $k_A=10$ and $k_B=8$ were distributed randomly in the chain. The percent of the sites having the elastic constant k_B is P_{BB}. The inset of Fig. 3 shows the localization length ξ of the waves as a function of the wave's frequency ω. The dashed line shows the localization length which is equal to the system size, $N=5\times10^5$, and the results were calculated by averaging over 5000 realizations of the disorder. The localization length diverges only when $\omega\to0$. These results agree with those presented previously.[4,21]

We now consider the RD model when the k_B's are distributed randomly in pairs. Figure 3 presents the frequency dependence of the localization length $\xi(\omega)$ for the lattice size, $N=5\times10^5$, $k_A=10$, and for two values, $k_B=4.5$ and 8.0, with $P_{BB}=0.1$. The results were obtained by averaging over 10^4 realizations of the lattice. In this case the localization length diverges, not only in the limit $\omega\to0$, but also at a second frequency which is known as the *resonance frequency* ω_c. Comparison of two figures, presented in Fig. 3, makes it clear that a phononic band for the conductivity appears in the frequency spectrum of the RD chain. The value of the resonance frequency for $k_B=8.0$ and $k_B=4.5$ are, respectively, $\omega_c=4.0$ and 3.0. We will shortly present an exact analysis that predicts these resonance frequencies.

In the RD model the localization length diverges as the frequency approaches ω_c, i.e., $\xi\propto|\omega-\omega_c|^{-\nu}$. When $\omega=\omega_c$, the localization length of the propagating wave for a finite-size chain is larger than the chain's length, i.e., $\xi(\omega_c)\gg N$. Therefore, such a mode represents a delocalized state. Using the logarithmic plot of $\xi(\omega)$ vs ω, shown in Fig. 4, we estimated the localization critical exponent ν that characterizes the power-law behavior of ξ near ω_c. Figure 4 presents such a plots for $\omega<\omega_c$ and $\omega>\omega_c$. The exponent ν_- for $\omega<\omega_c$ is estimated to be $\nu_-=1.9\pm0.2$. For the $\omega>\omega_c$ regime, we estimate that, $\nu_+=2.1\pm0.2$. Moreover, we also estimated ν_0 for the regime $\omega\to0^+$, with the result being $\nu_0=2.0\pm0.1$.

B. Exact analysis

To predict the resonance frequency ω_c, we improve upon a nonperturbative method that yields the frequency ω_c in terms of k_B. As indicated by Eq. (4), there are four different kinds of the TM in the RD chain, namely, $\mathbf{M}_{AA}$, $\mathbf{M}_{AB}$, $\mathbf{M}_{BA}$, and $\mathbf{M}_{BB}$, two of which are given by

$$\mathbf{M}_{AA}=\begin{pmatrix}\dfrac{-\omega^2+2k_A}{k_A} & -1\\ 1 & 0\end{pmatrix},$$

$$\mathbf{M}_{AB}=\begin{pmatrix}\dfrac{-\omega^2+k_A+k_B}{k_B} & -\dfrac{k_A}{k_B}\\ 1 & 0\end{pmatrix},$$

while the other two are obtained by the transformations, $A\to B$ and $B\to A$. For the RD model we have such configurations as, $A^{\eta_1}B^{2\mu_1}A^{\eta_2}\ldots A^{\eta_m}B^{2\mu_m}A^{\eta_{m+1}}\ldots$. Thus, we can write $\mathbf{M}_{N,1}$ as follows:

$$\mathbf{M}_{N,1}=(\mathbf{M}_{AA})^{\eta_1-1}\mathbf{M}_{AB}(\mathbf{M}_{BB})^{2\mu_1-1}\mathbf{M}_{BA}(\mathbf{M}_{AA})^{\eta_2-1}\ldots$$

$$(\mathbf{M}_{AA})^{\eta_m-1}\mathbf{M}_{AB}(\mathbf{M}_{BB})^{2\mu_m-1}\mathbf{M}_{BA}(\mathbf{M}_{AA})^{\eta_{m+1}-1}\ldots, \tag{6}$$

where η_m is the number of sites with elastic constant k_A and μ_m is the number of k_B dimers in their mth cluster in the binary model. According to theory of matrices,[25,26] the nth power of a 2×2 matrix with unit determinant satisfies the characteristic equation as follows:

$$\mathbf{M}_{BB}^n = p_{n-1}(\chi)\mathbf{M}_{BB} - p_{n-2}(\chi)\mathbf{I}, \tag{7}$$

where $p_n(\chi)=\sin(n+1)\theta/\sin\theta$ is the order n Chebyshev polynomials of second kind with $|\chi|\leq 1$, $\theta=\arccos\chi$, $\mathbf{I}$ is the unit matrix, and χ is given by

$$\chi = \frac{1}{2}\mathrm{Tr}(\mathbf{M}_{BB}) = \frac{1}{2}\frac{2k_B-\omega^2}{k_B}. \tag{8}$$

For a particular value of the frequency ω for which $p_{n-1}(\chi)=0$, we have $\mathbf{M}_{BB}^n=-p_{n-2}(\chi)\mathbf{I}$. If we set $n=2$ (for the dimers), we obtain, $p_{n-1}(\chi)=0$, if $\omega^2=2k_B$. For this particular frequency one finds, $p_{n-2}(\chi)=1$. Therefore, Eq. (8) reduces to, $\mathbf{M}_{BB}^2=-\mathbf{I}$, which implies that for the RD model we have, $(\mathbf{M}_{BB})^{2\mu_m-1}=(-1)^{\mu_m-1}\mathbf{M}_{BB}$. Moreover, we can also prove, using a similar analysis, relation $\mathbf{M}_{AB}\mathbf{M}_{BB}\mathbf{M}_{BA}=-\mathbf{M}_{AA}$, for $\omega^2=2k_B$. Thus, for this frequency $\mathbf{M}_{N,1}$ of the chain in Eq. (7) contains only the matrices $\mathbf{M}_{AA}$, which effectively describes an *ordered* chain. As a result, if the condition, $|\frac{1}{2}\mathrm{Tr}(\mathbf{M}_{BB})|\leq 1$, is satisfied for $\omega^2=2k_B$, then, its eigenstate is allowed to be an extended state in the chain. The wave with this frequency, which is what we refer to as the resonance frequency ω_c,

$$\omega_c^2 = 2k_B\ , \tag{9}$$

can propagate through the entire RD chain. Thus, our theoretical analysis confirms the numerical results for the RD chain presented above. As a result, if we create a 1D model with two types of dimers of elastic constants k_A and k_B, we obtain *two* resonant frequencies,

$$\omega_{c1} = \sqrt{2k_A}, \quad \omega_{c2} = \sqrt{2k_B}. \tag{10}$$

This is confirmed by the numerical results presented in Fig. 5.

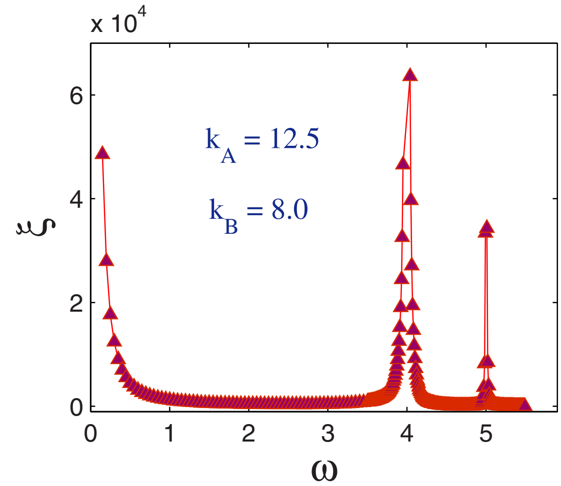

FIG. 5. (Color online) Localization length of the double RD chain (i.e., values of the k on the sites A and B appear in pairs) with $k_A=12.5$ and $k_B=8$, as a function of frequency. The localization length ξ diverges at the resonant frequencies $\omega_c=4$ and $\omega_c=5$.

IV. DYNAMICS OF THE MODEL

To study the dynamics of the model, Eq. (1) was solved directly in a 1D lattice, using the finite-difference (FD) method with second-order discretization for both the time and spatial variables. Thus, in discretized form, $\psi(x,t)$ is written as ψ_i^n, where n denotes the time step number and i is the grid point number. The second-order FD approximation accurate to $\mathcal{O}(\Delta t^2)$ to the time-dependent term of Eq. (1) is the standard form,

$$\frac{\partial^2\psi}{\partial t^2} \simeq \frac{\psi_i^{n+1} - 2\psi_i^n + \psi_i^{n-1}}{\Delta t^2}, \tag{11}$$

where Δt is the size of the time step. As for the spatial derivatives, we first expand the right side of Eq. (1) in 1D by using the second-order FD approximation,

$$\frac{\partial}{\partial x}\left[\lambda(x)\frac{\partial}{\partial x}\psi(x,t)\right] \simeq \frac{1}{\Delta x^2}\Big(\lambda_{i+\frac{1}{2}}\psi_{i+1}^n - \lambda_{i-\frac{1}{2}}\psi_i^n - \lambda_{i+\frac{1}{2}}\psi_i^n + \lambda_{i-\frac{1}{2}}\psi_{i-1}^n\Big), \tag{12}$$

where, clearly, values of ψ_i are attributed to the grid points i, whereas the λ variables are assigned to the links between the grid points and take on values k_A and k_B. Δx is the spacing between two neighboring grid points. In the simulations we set, $\Delta x=1$, and to ensure the stability of the discretized equations we set, $\Delta t=\Delta x/4$.

To investigate the propagation of the waves and its dynamics in the RD chain, we sent a wave from a source on one side of the chain, and recorded the transmitted wave at various distances L from the source. The source wave was chosen as a sine wave with frequency ω_0, and after some time we collected the numerical values of ψ_i^n at different locations of the medium. The length of the medium was $L=40\,000\Delta x$, and we used $k_A=10$ and $k_B=8$.

Figure 6 presents values of ψ at time $t=160\,000\Delta t$ in the medium, for several initial frequencies ω_0. It indicates that wave transmission is weaker for $\omega_0=3.7$ and 4.3 than for $\omega_0=4$, and that the resonance frequency $\omega_0=\omega_c=4.0$ has an extended nature. Moreover, to demonstrate the "filter" nature of the medium, one may use a source to generate a wave packet (in time) with mean frequency ω_0 and width σ_t. We use the following form of the wave source to generate the incident pulse:

$$S_0(t) = \exp[-(t-t_0)^2/2\sigma_t^2]\sin(\omega_0 t), \tag{13}$$

the shape of which is shown in Fig. 7 (inset). The spectrum of the pulse in the frequency space has the width, $\sigma_\omega=1/\sigma_t$, and the maximum amplitude is at frequency ω_0. To calculate the frequency content of the wave pulse during its propagation, we study the transmitted pulse and calculate the intensity spectrum which is defined as

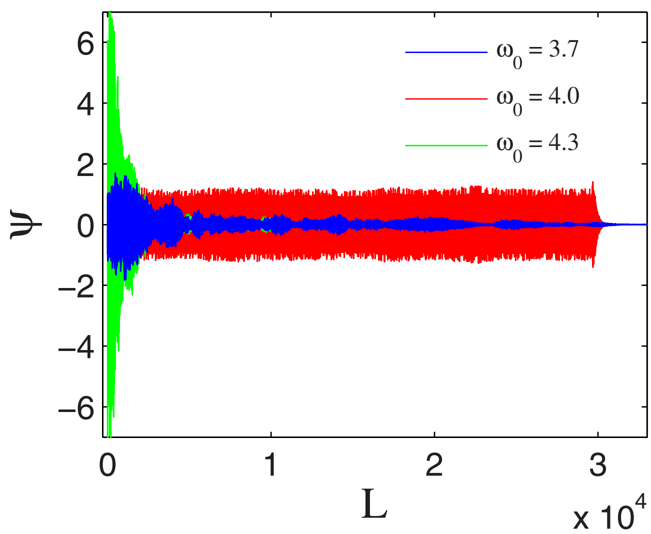

FIG. 6. (Color online) The amplitude of the wave during propagation through the RD model. The incident wave is sine wave with different frequencies ω_0.

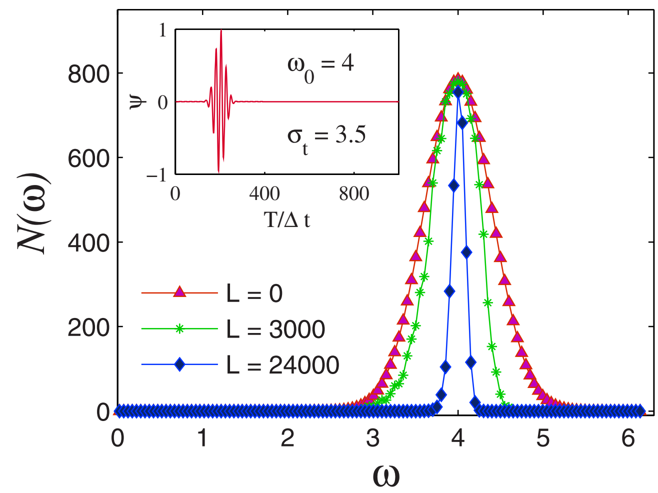

FIG. 7. (Color online) Intensity spectrum of the transmitted wave pulse through the RD medium at several receivers that are at a distance L from the source. Inset: the wave packet inserted in the medium.

$$N(\omega) = \frac{1}{2}|\psi(\omega)|^2. \tag{14}$$

Here, $\psi(\omega)$ is the Fourier transform of $\psi(t)$ at distance L from the source. In the simulation we recorded the wave pulse at many receivers at various distances L from the source, and calculated the intensity spectrum of received wave pulse.

Figure 7 presents the resulting frequency dependence of the wave pulse amplitudes collected at several receivers. Here, we used $k_A=10$, $k_B=8$, $\sigma_t=3.5$, and $\omega_0=4$. The intensity spectrum was computed for the averages of 20 realizations of the disorder. The results did not change when we used a larger number of realizations. As shown in the figure, all the modes with $\omega > \omega_c$ and $\omega < \omega_c$ decay, and the medium behaves as a filter to transmit only the frequency ω_c. With the value of k_B that we used, the resonance frequency is $\omega_c = 4$. These results confirm those obtained by the exact analytical analysis and the TM method described earlier.

V. SUMMARY

We studied the localized and delocalized states of acoustic waves in the random-dimer chain using the transfer-matrix method, exact analytical analysis, and direct numerical simulation of the scalar wave equation. The minimum and maximum values of the allowed frequencies were first computed and, then, the localization length ξ of the acoustic waves in the random-dimer chain was computed. We showed that there exists a resonance frequency ω_c at which the localization length of the acoustic waves diverges, when the frequency of the waves approaches ω_c. The resonance frequency ω_c depends on the value of paired elastic constant. The critical exponent ν that characterizes the power-law behavior of ξ near ω_c was also estimated.

We also carried out an exact analysis in order to predict the dependence of resonance frequency ω_c on the value of the paired elastic constant. It was shown that, at ω_c, the random-dimer chain behaves as an ordered chain. Using the dynamical method, based on directly solving the scalar wave equation for propagation of a wave packet with a wide spectral density, we showed that the chain localizes all the frequency content of the wave pulse, except that for the resonance frequency.

As discovered recently, symmetry of a random potential [for example, the mirror symmetry, $v(x)=v(-x)$, in the 1D Anderson model] gives rise to a nontrivial mechanism of tunneling at macroscopic scales for a localized wave packet.[27] Unlike quantum tunneling through a regular potential barrier, which occurs only at the energies lower than the barrier's height, the proposed mechanism of tunneling exists even for weak white-noise-like scattering potentials. The connection between the resonance frequency of acoustic waves in the disordered random dimmer, calculated and determined exactly in the present paper, and the symmetry of the random elastic stiffness is an open question, to be studied in the future.

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